Solitons from BCS to BEC:

Oscillations, decay, collisions and the general characteristics of dark solitons across the BCS-BEC crossover, and their future detection in experiment.

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(with F. Dalfovo, L. Pitaevskii and S. Stringari)

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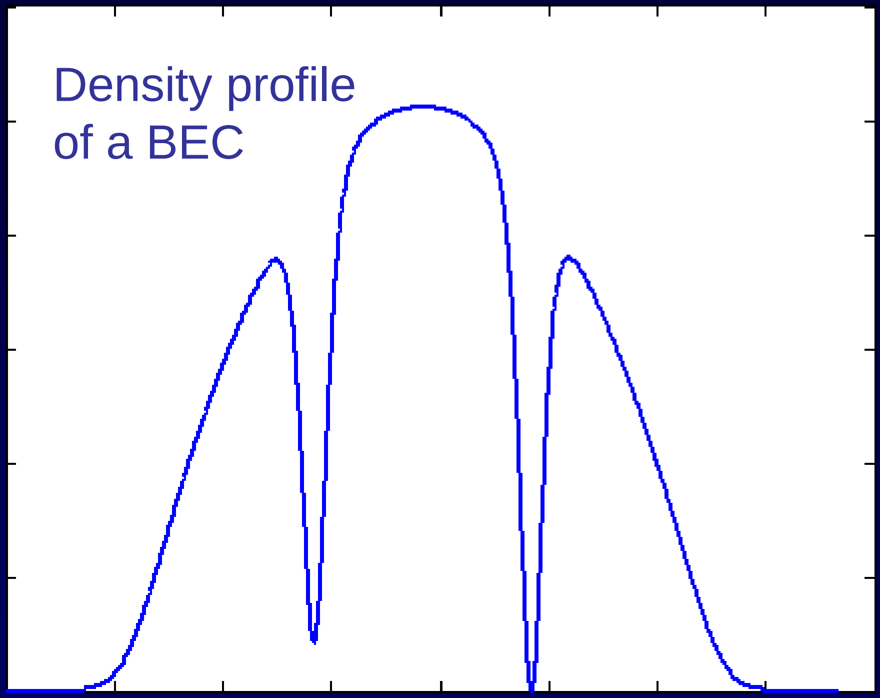
Soliton basics

Solitons are excitations of a repulsive BEC.

In BECs, they….

- are characterised by a density minimum and a phase jump
- have a maximum speed given by the speed of sound
- oscillate in traps (the period is √2 * the trap period)
- are robust against collisions



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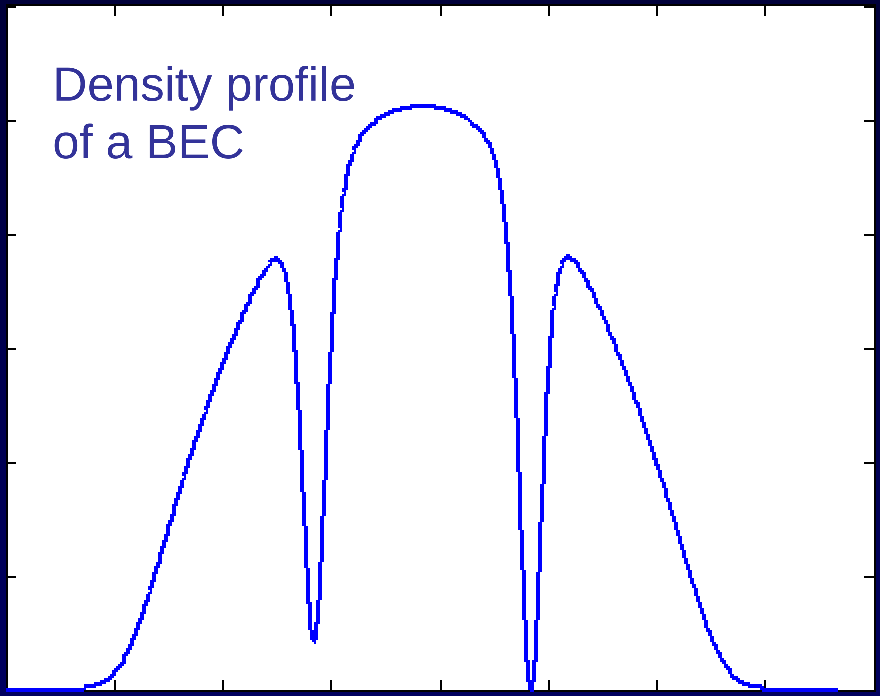

They are well understood in BECs!

The Gross-Piiaevskii equation has analytic solitonic solutions, enabling us to calculate the phase jump, density profile, etc. as a function of velocity.

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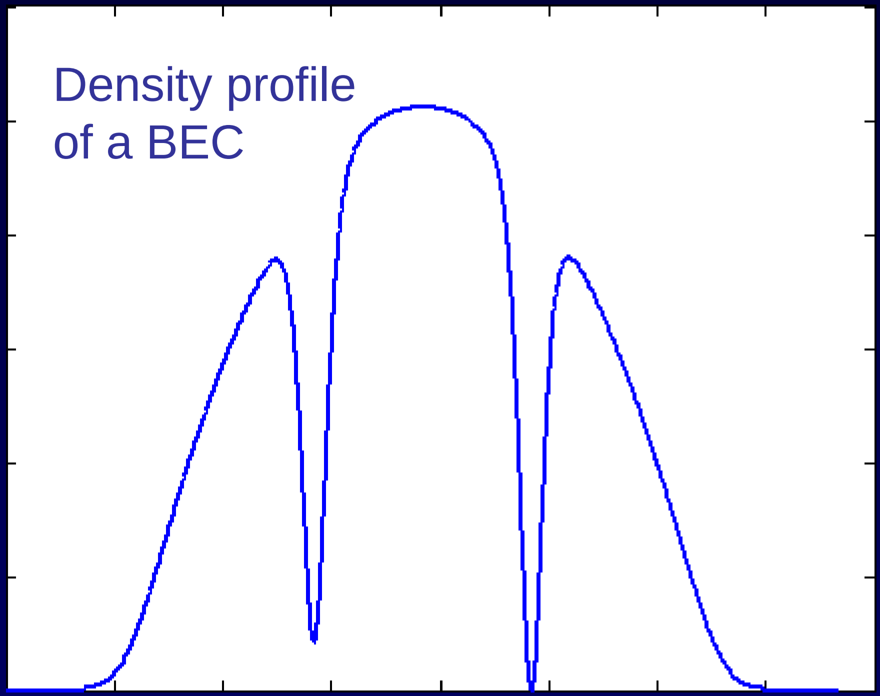

They are well understood in BECs!

Solitonic solution of the GP equation: $$\psi = \sqrt{n_0}\left[i\frac{V}{s} + \sqrt{1-\frac{V^2}{s^2}}\tanh\left(\frac{x-Vt}{\sqrt{2}\xi_V}\right)\right] e^{-i\mu t/\hbar}$$

• Tanh form, and • Constant imaginary component

$$\xi_V = 1/\sqrt{8\pi n_0 a\left(1-V^2/s^2\right)}$$

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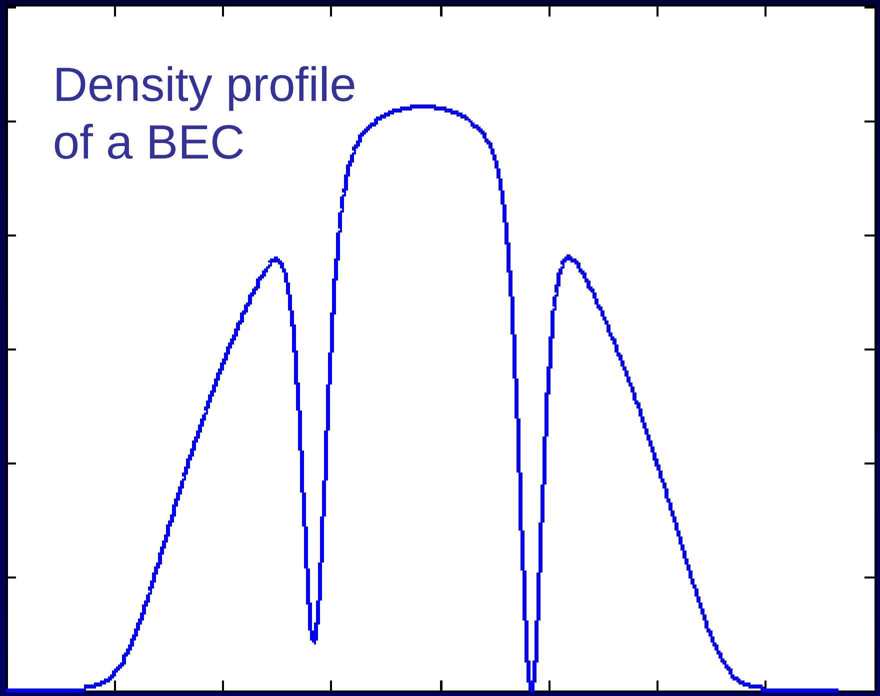
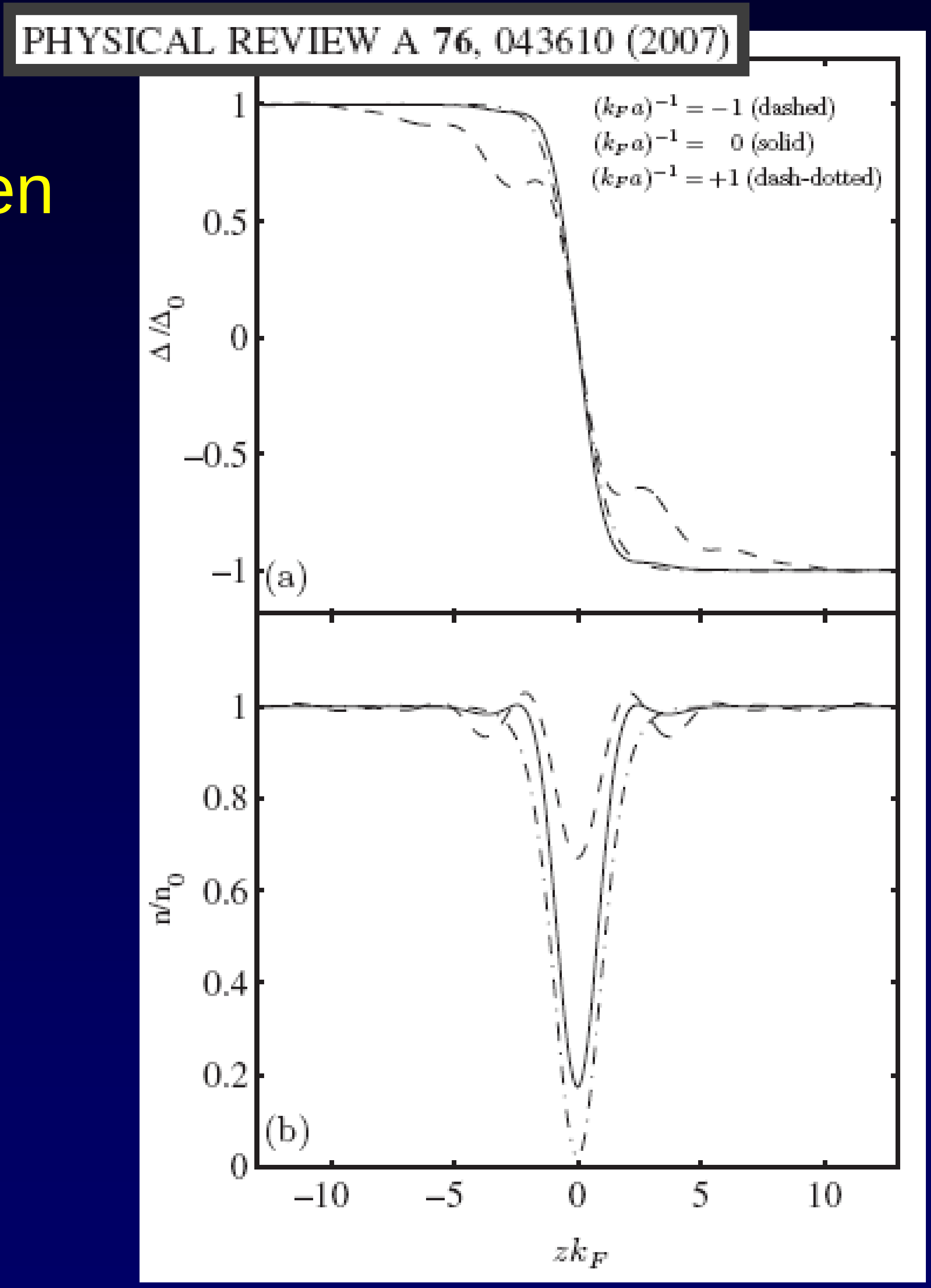


What are their properties in the BEC-BCS crossover?

What is their phase jump/density profile as a function of velocity? What is their oscillation period? Are they robust objects which are easily formed, or are they fragile objects destroyed by a tiny breath of sound?

Black solitons have been investigated across the crossover….



Black solitons have been investigated across the crossover….

Time-independent Bogoliubov-de Gennes equations:

$$\epsilon_{\mathbf{k}} \begin{bmatrix} u_{\mathbf{k}}(\mathbf{r}) \\ v_{\mathbf{k}}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} \mathcal{L}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -\mathcal{L}(\mathbf{r}) \end{bmatrix} \begin{bmatrix} u_{\mathbf{k}}(\mathbf{r}) \\ v_{\mathbf{k}}(\mathbf{r}) \end{bmatrix}$$

$$\mathcal{L}(\mathbf{r}) = \frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r}) - E_{\mathrm{F}}$$

$$\Delta(\mathbf{r}) = -V_{\mathrm{int}} \sum_{\mathbf{k}} u_{\mathbf{k}}(\mathbf{r}) v_{\mathbf{k}}^*(\mathbf{r})$$

Order parameter

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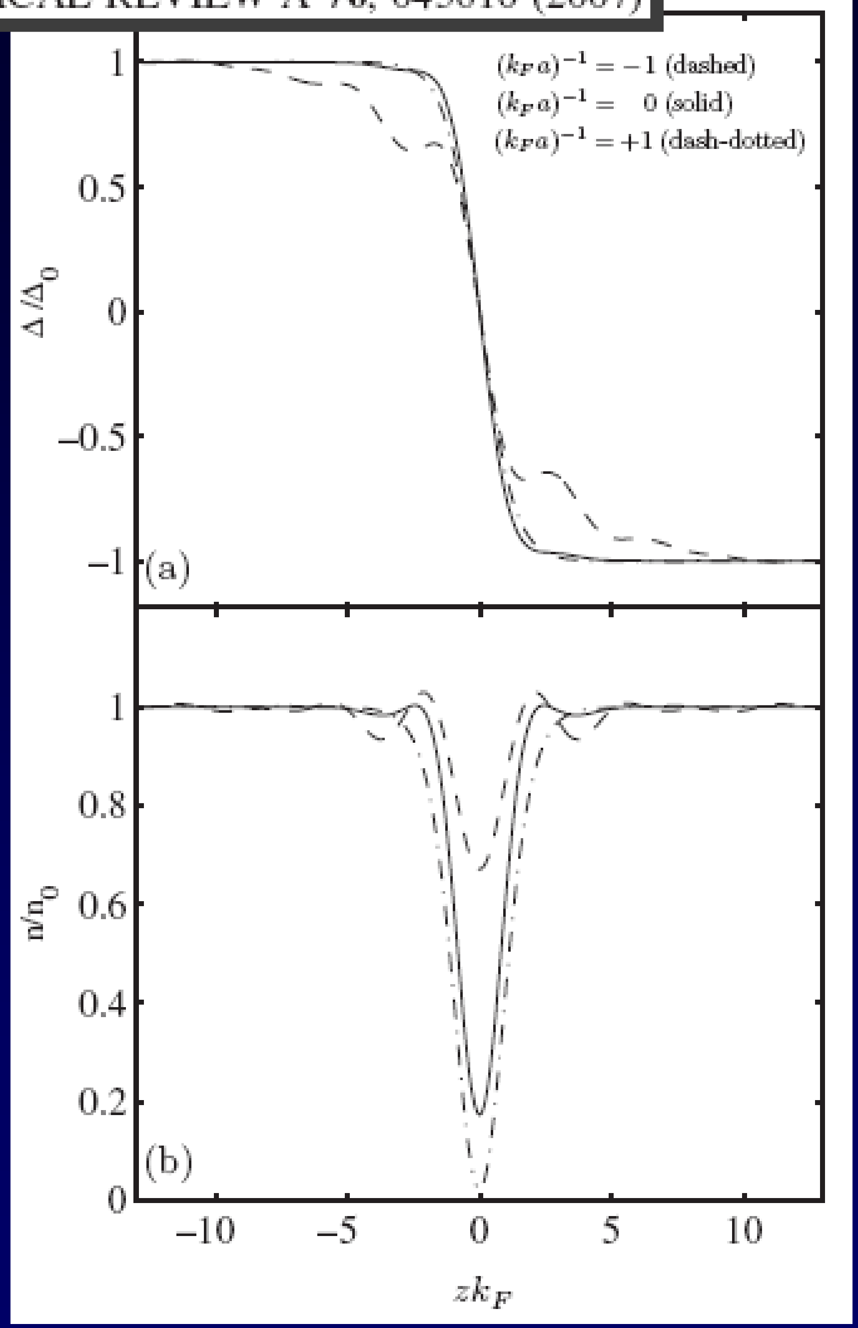

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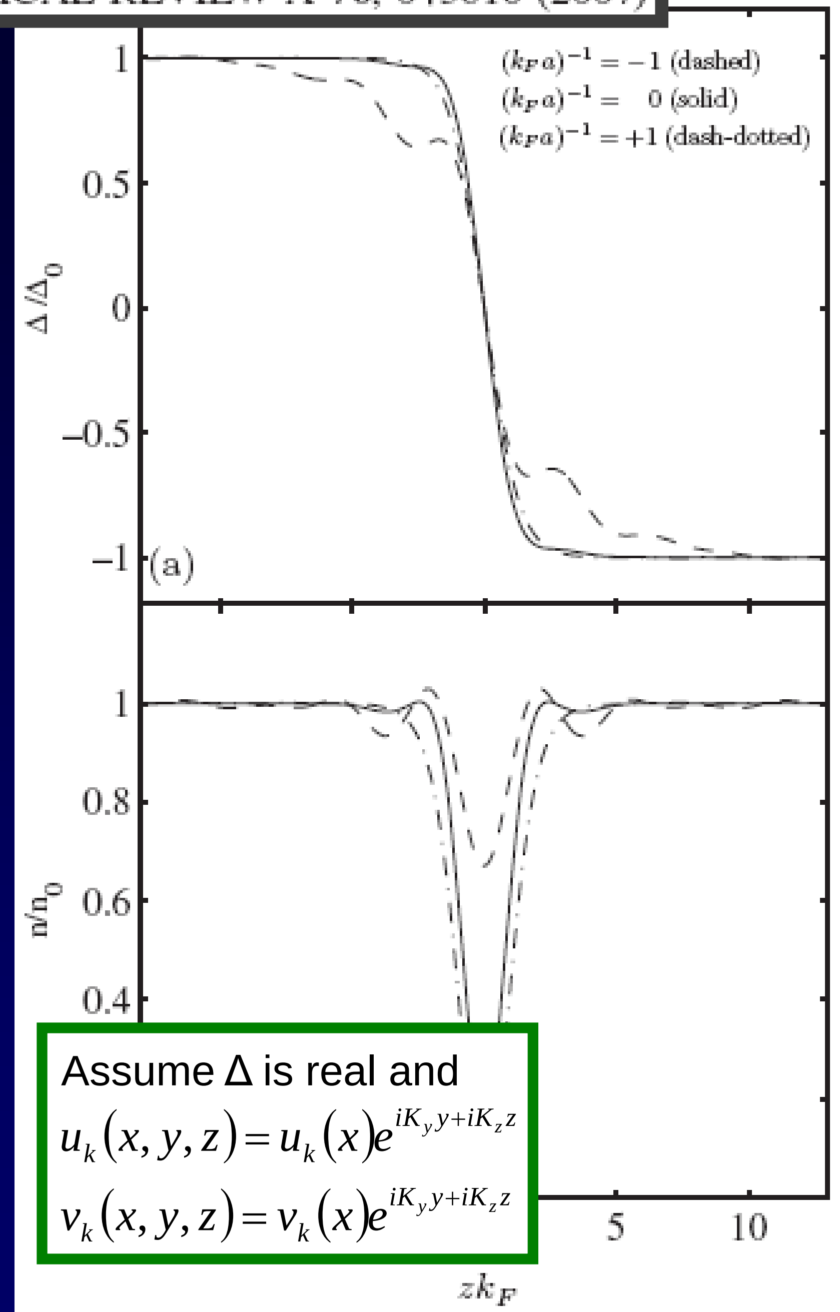

Assume Δ is real and

$$u_k(x,y,z) = u_k(x)e^{iK_y y + iK_z z}$$

$$v_k(x,y,z) = v_k(x)e^{iK_y y + iK_z z}$$

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Order parameter

k typically goes up to ~10000

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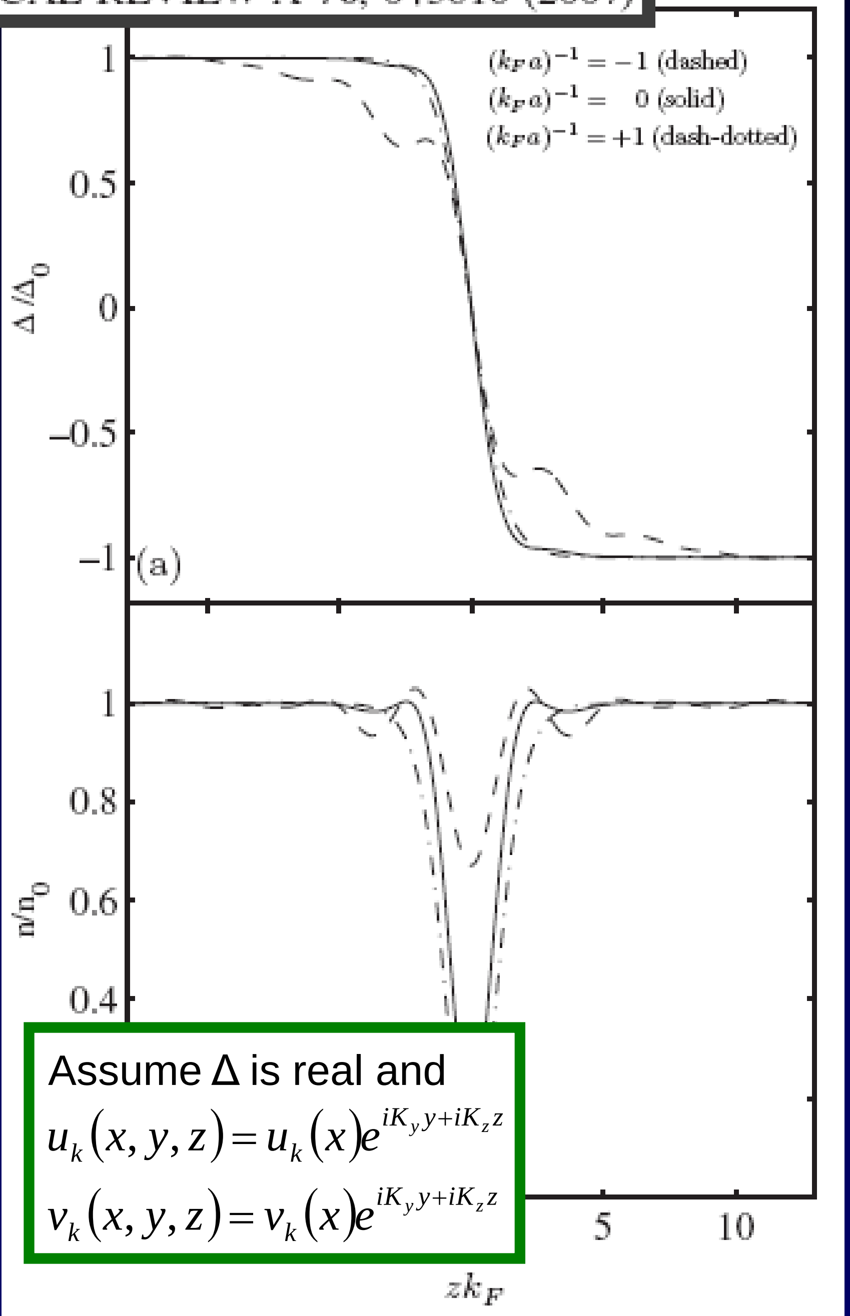

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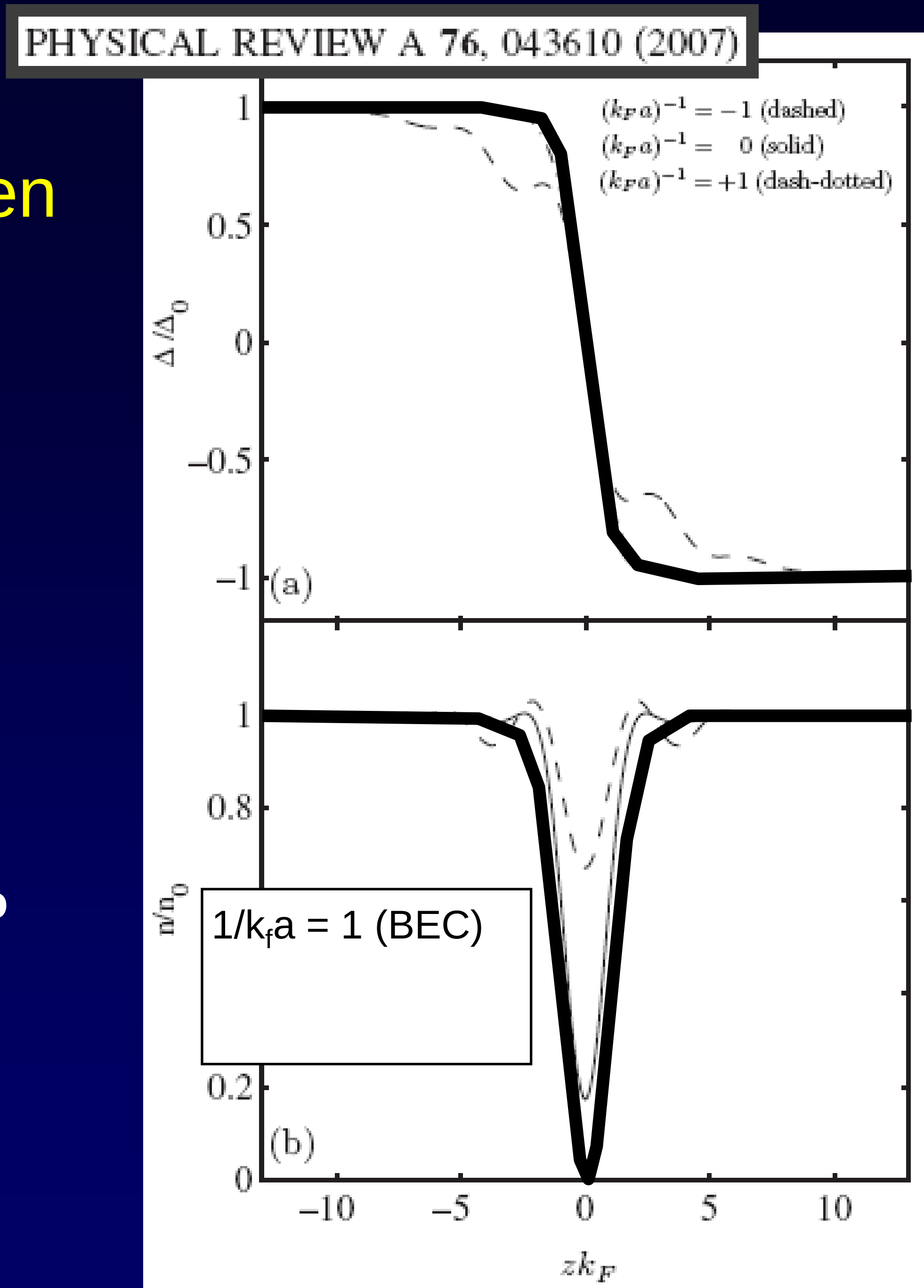
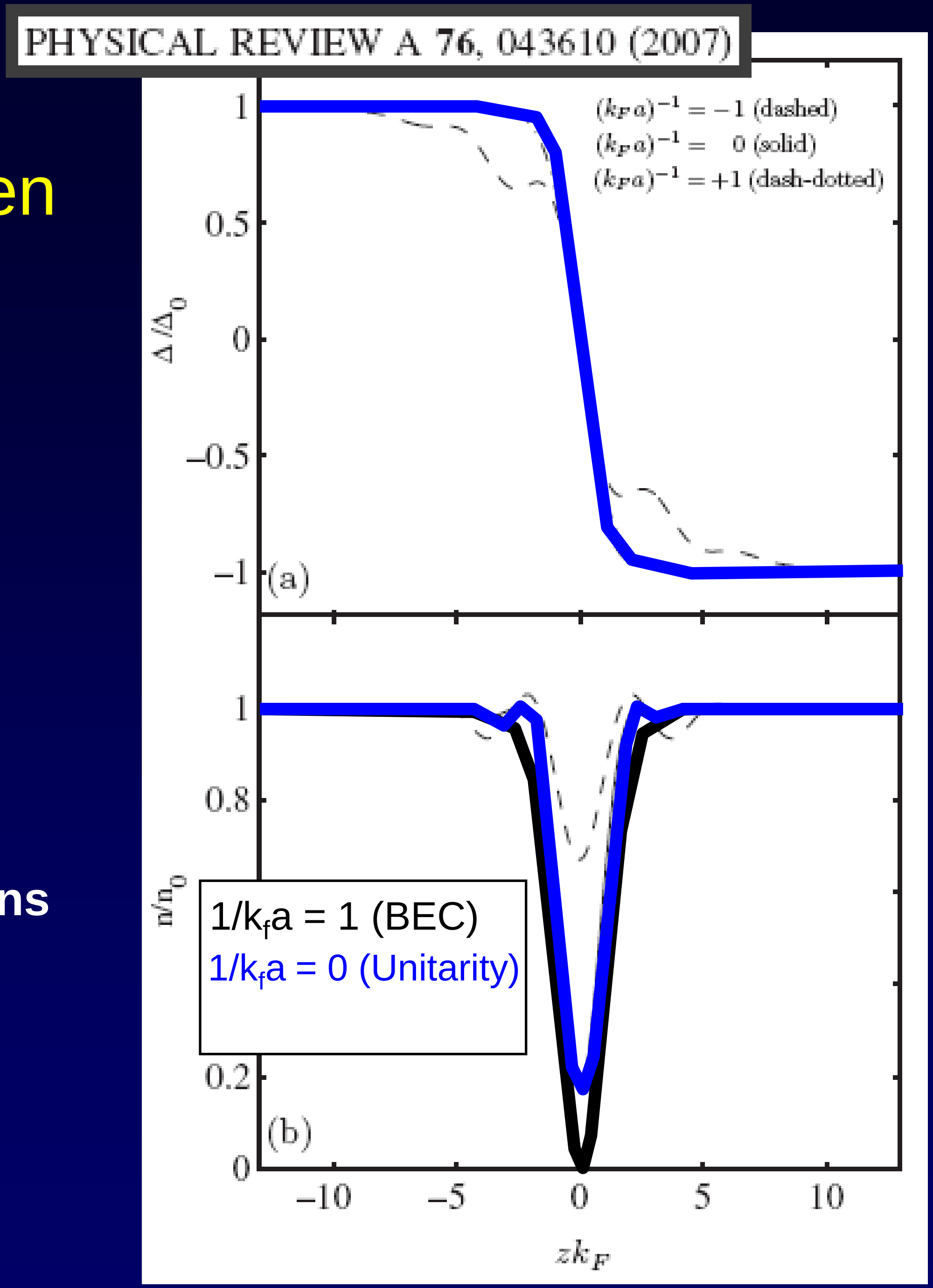


In the BEC regime the soliton has a minimum density of zero and a tanh^2 density profile.

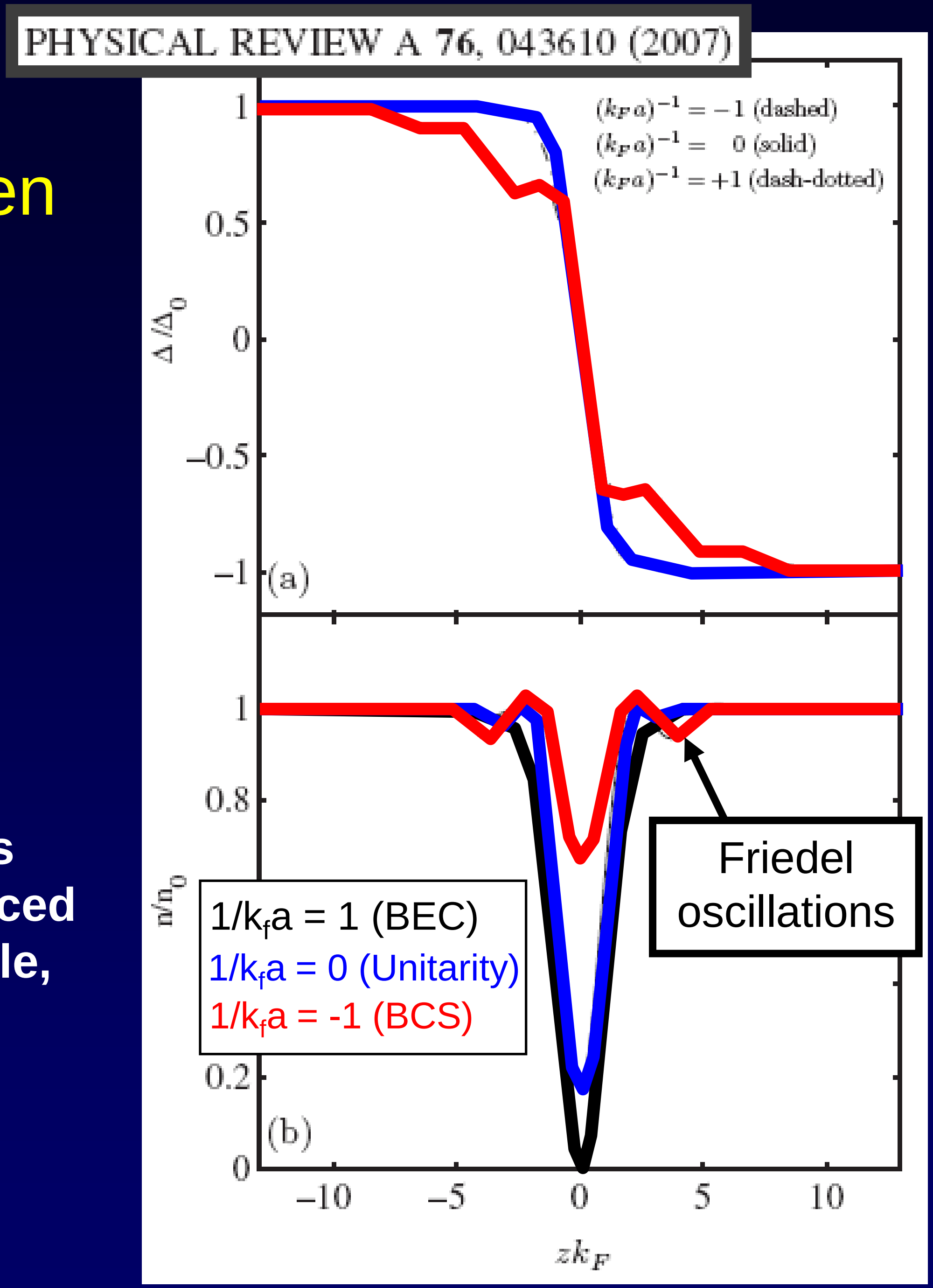
Black solitons have been investigated across the crossover….



At unitarity the soliton is shallower, and small oscillations appear in the density profile.

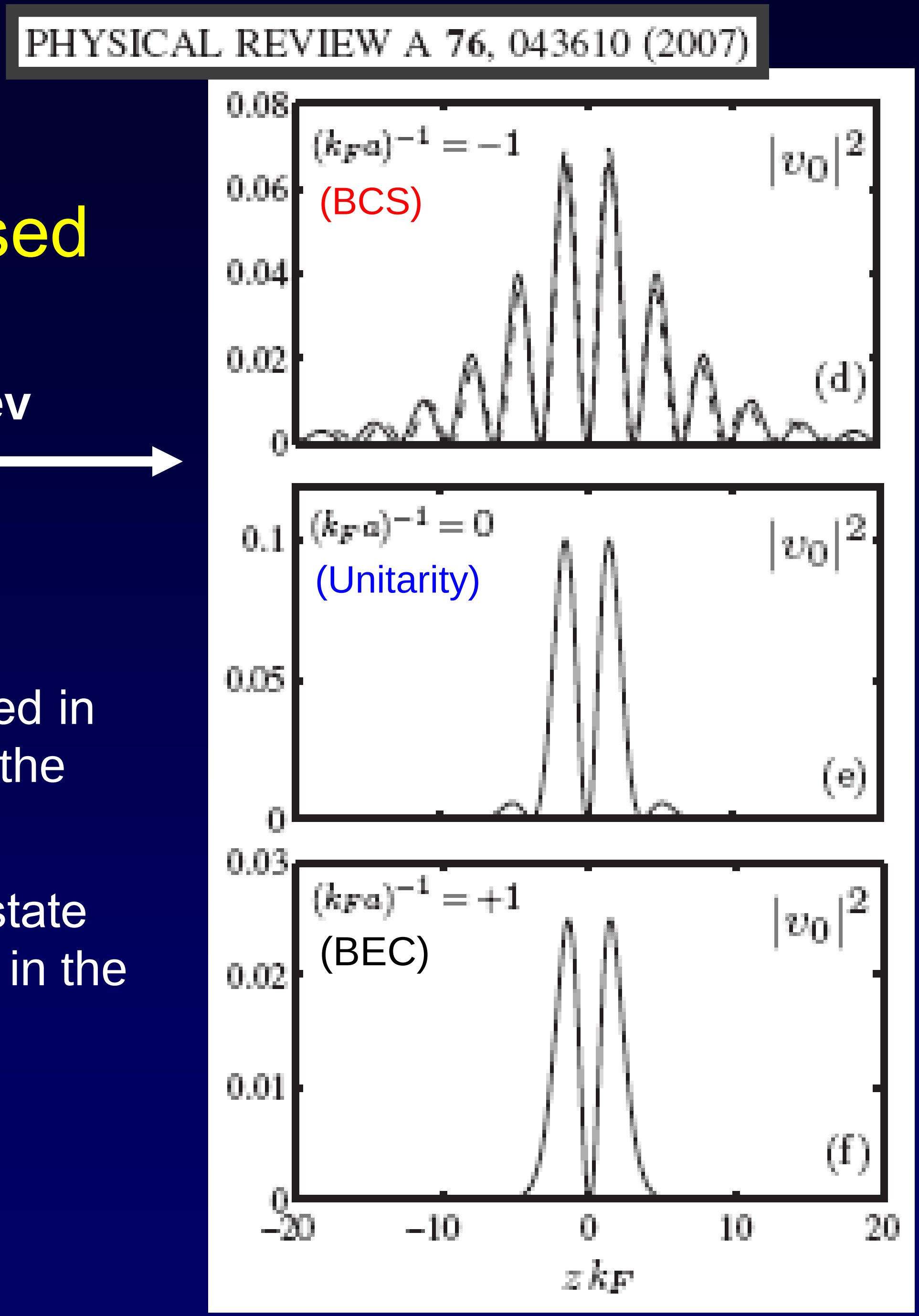
Black solitons have been investigated across the crossover….

In the BCS regime the soliton is very shallow, and has pronounced oscillations in the density profile, which are Friedel oscillations.



The solitons contain "Andreev states" localised within the soliton.

Density profile of lowest Andreev state in different regimes.



- The Andreev state is very localised in the BEC regime, and very wide in the BCS regime.
- The contribution of the Andreev state to the density becomes very small in the BEC regime.

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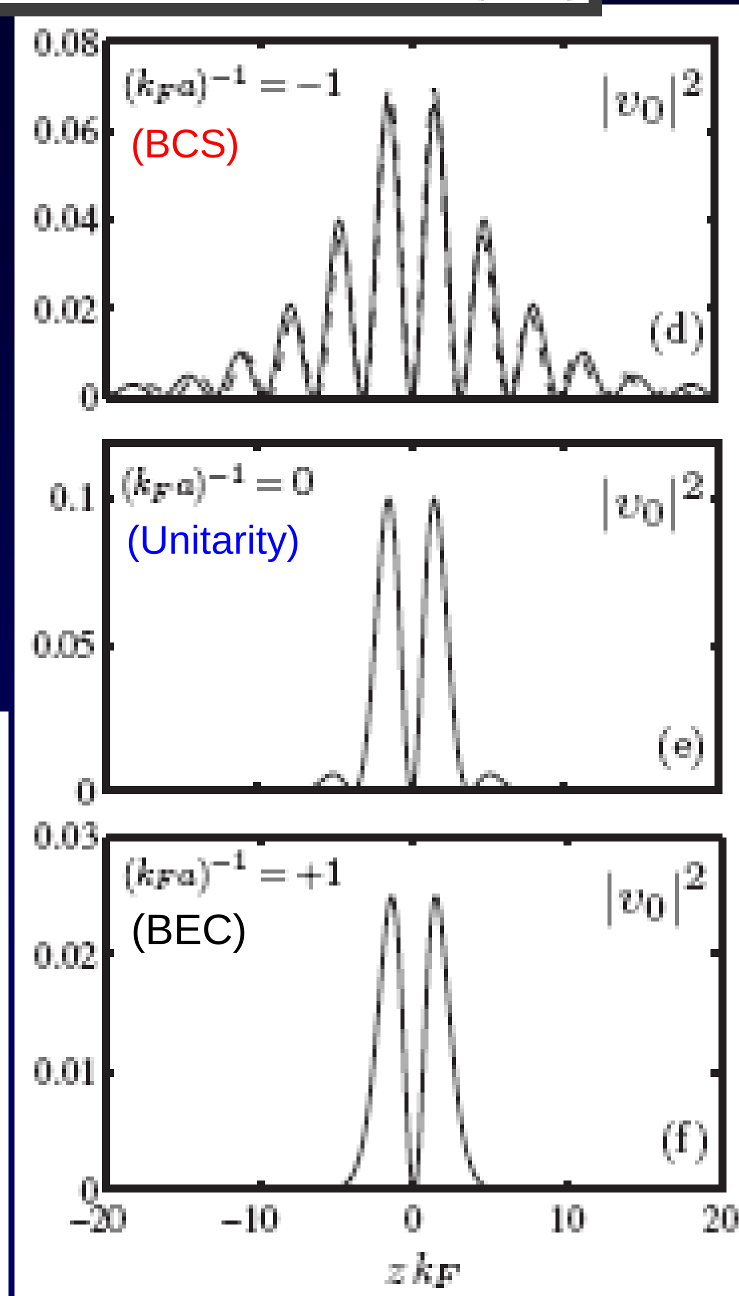
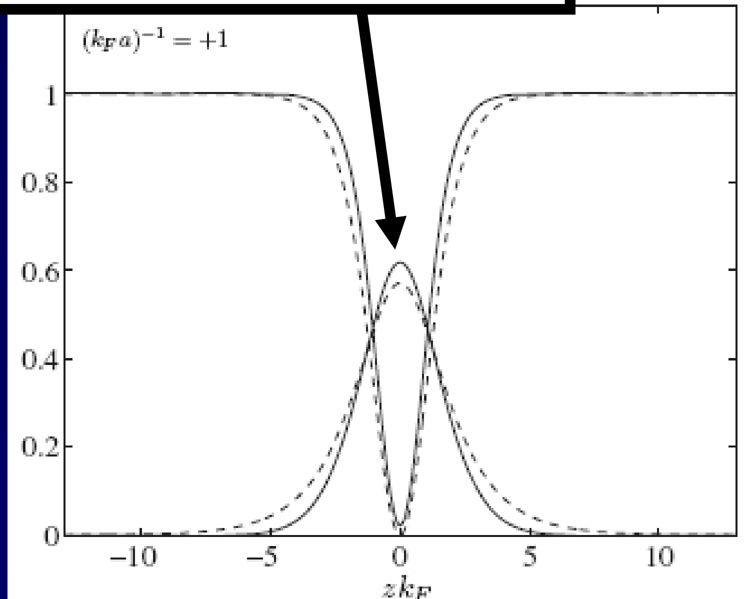


In the BEC limit the lowest Andreev bound state becomes equivalent to the lowest "impurity" bound state



Imagine a soliton in a superfluid,
which may be Bosonic or Fermionic.

- Soliton Energy $E_S(\mu, V^2)$
- Soliton speed $V = \frac{dX}{dt}$
- Soliton position X
- Chemical potential $\mu(X)$

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- Current j
- Current in soliton frame $\bar{j} = j - nV = -n_0 V$
- Phase jump J in superfluid phase φ

Why? Because the soliton is localised in terms of the density, but the phase jump J stretches to ∞.

$$P_c \neq P_s$$

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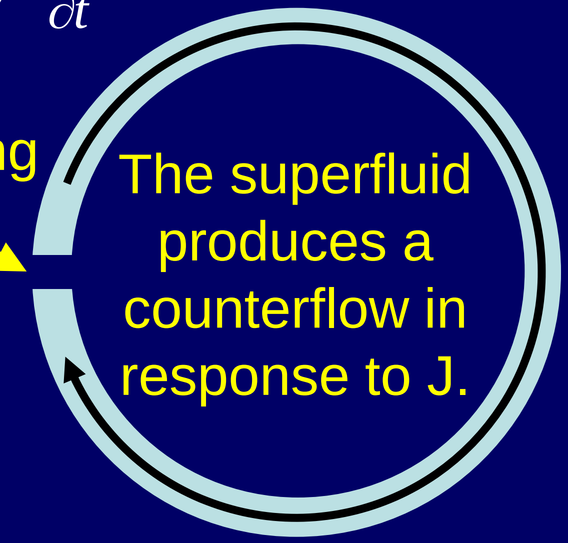

The difference between P_C and P_S is the “counterflow”.

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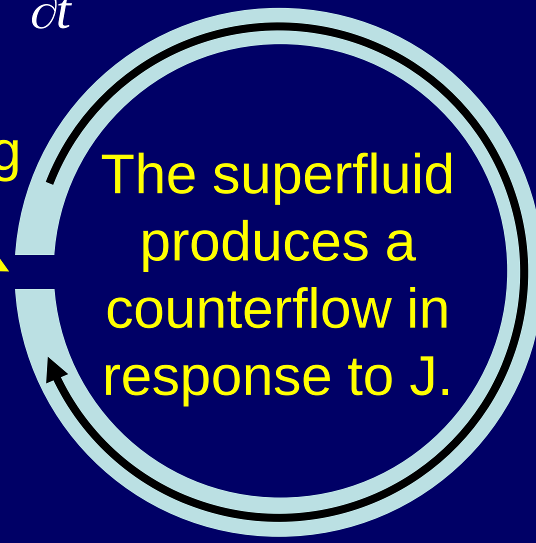
- Phase jump J in superfluid phase φ
- Velocity field v

Soliton on a ring



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$$P_c = P_s + n_0 m\int v\, dx$$

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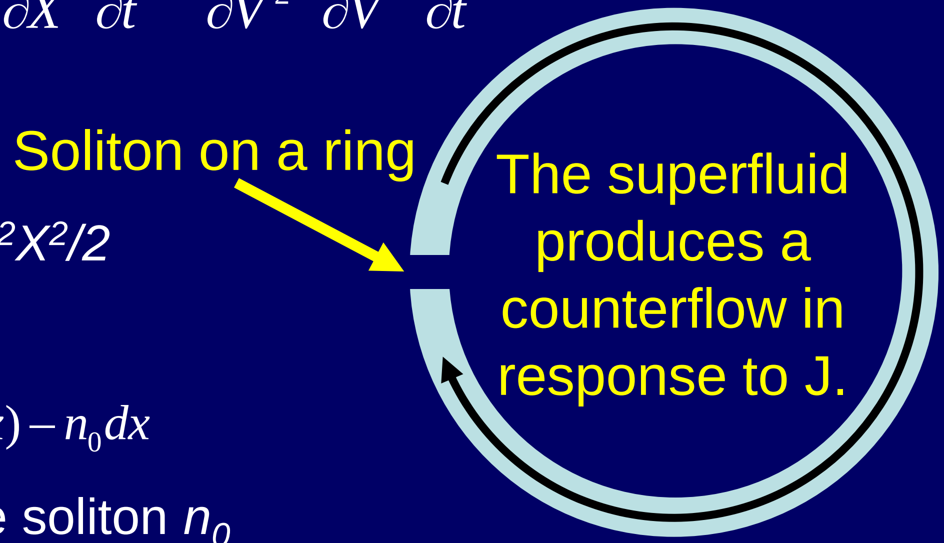
- $m_B = m$ for Bosons and $2m$ for Fermions



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$$P_c = P_s + \hbar n_0 Jm / m_B$$

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$$\left(\frac{T_S}{T_x}\right)^2 - 1 = \frac{\hbar n_0}{m_B N_S}\frac{dJ}{dV}$$

$$P_c = P_s + n_0 m \int \hbar \nabla \phi / m_B\, dx$$

$$P_c = P_s + \hbar n_0 J m / m_B$$

Analytic result, general for any superfluid:

$$\left(\frac{T_S}{T_x}\right)^2 - 1 = \frac{\hbar n_0}{m_B N_S} \frac{dJ}{dV}$$

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Numerical approach: time-dependent Bogoliubov-de Gennes equations

$$i\hbar\frac{\partial}{\partial t}\begin{bmatrix} u_{\mathbf{k}}(\mathbf{r},t) \\ v_{\mathbf{k}}(\mathbf{r},t)\end{bmatrix} = \begin{bmatrix} \mathcal{L}(\mathbf{r},t) & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -\mathcal{L}(\mathbf{r},t)\end{bmatrix}\begin{bmatrix} u_{\mathbf{k}}(\mathbf{r},t) \\ v_{\mathbf{k}}(\mathbf{r},t)\end{bmatrix}$$

$$\mathcal{L}(\mathbf{r}) = \frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r}) - E_{\mathrm{F}}$$

$$\Delta(\mathbf{r}) \quad = \quad -V_{\mathrm{int}}\sum_{\mathbf{k}} u_{\mathbf{k}}(\mathbf{r})v^*_{\mathbf{k}}(\mathbf{r})$$

Order parameter Not real!

k typically goes up to ~10000

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Order parameter Not real!

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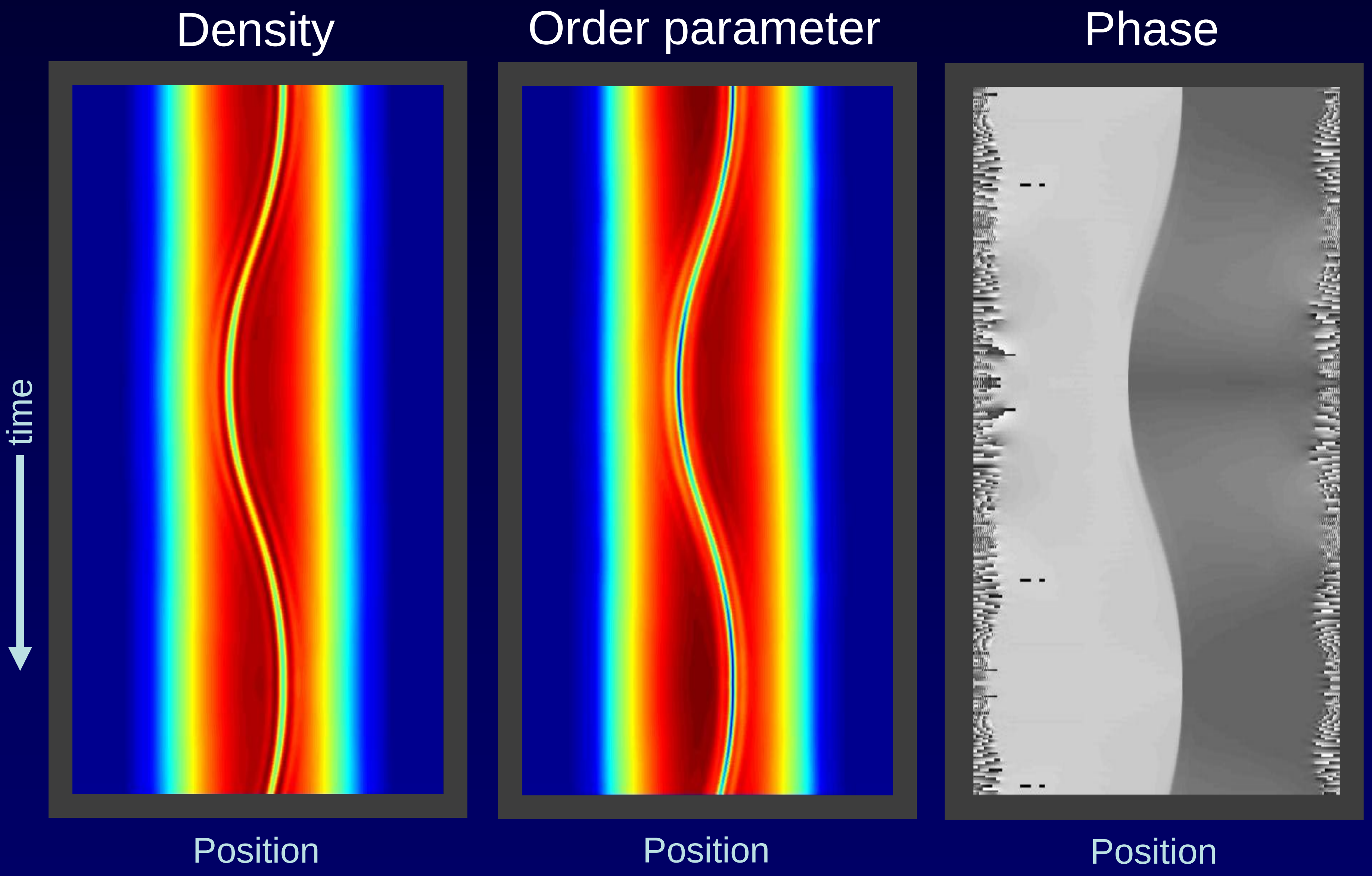
We may say

$$u_k(x,y,z) = u_k(x)e^{iK_y y + iK_z z}$$

$$v_k(x,y,z) = v_k(x)e^{iK_y y + iK_z z}$$

Soliton oscillations in a trap: e.g. $1/k_f a = -0.5$ (BCS)



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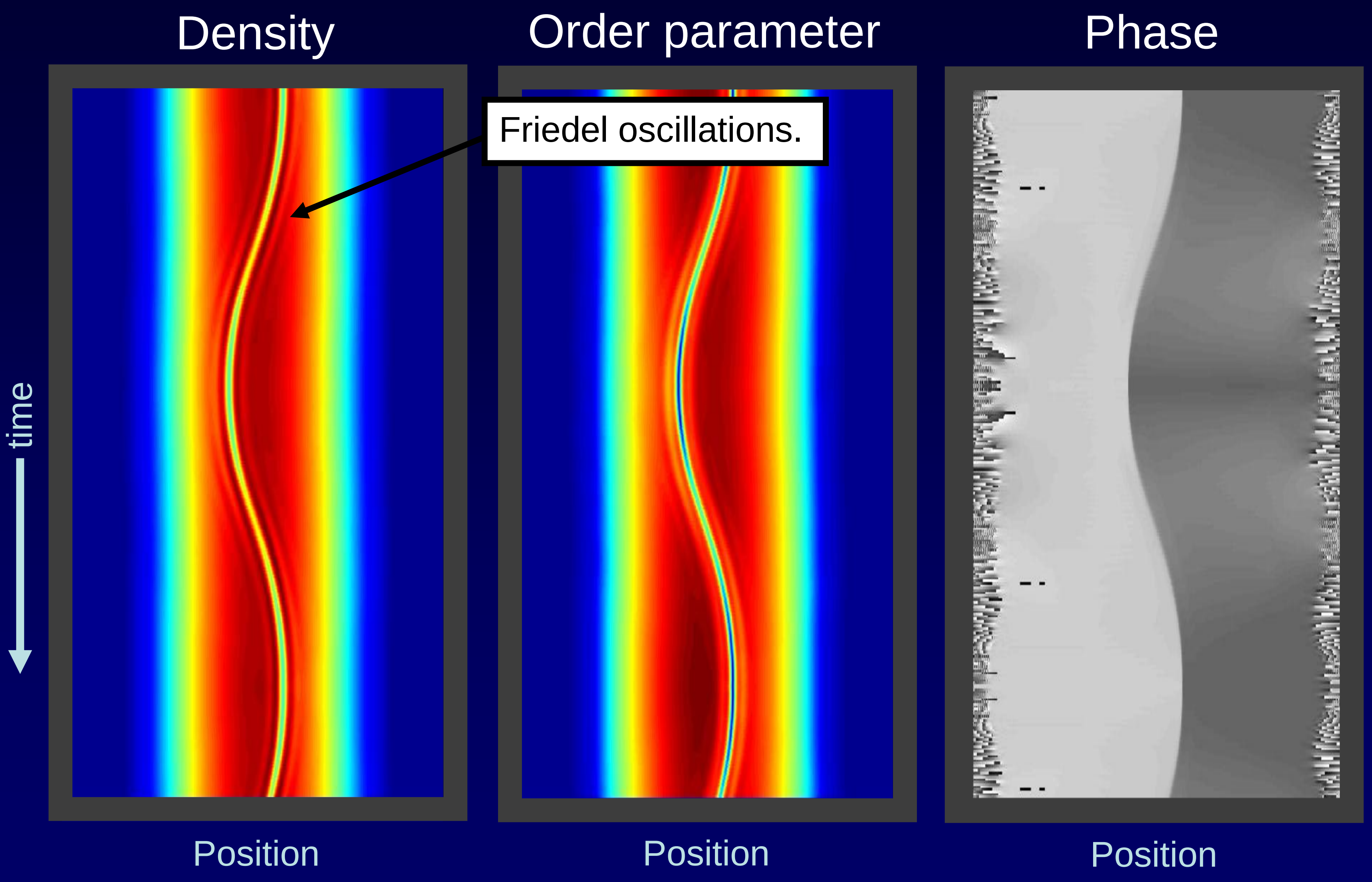

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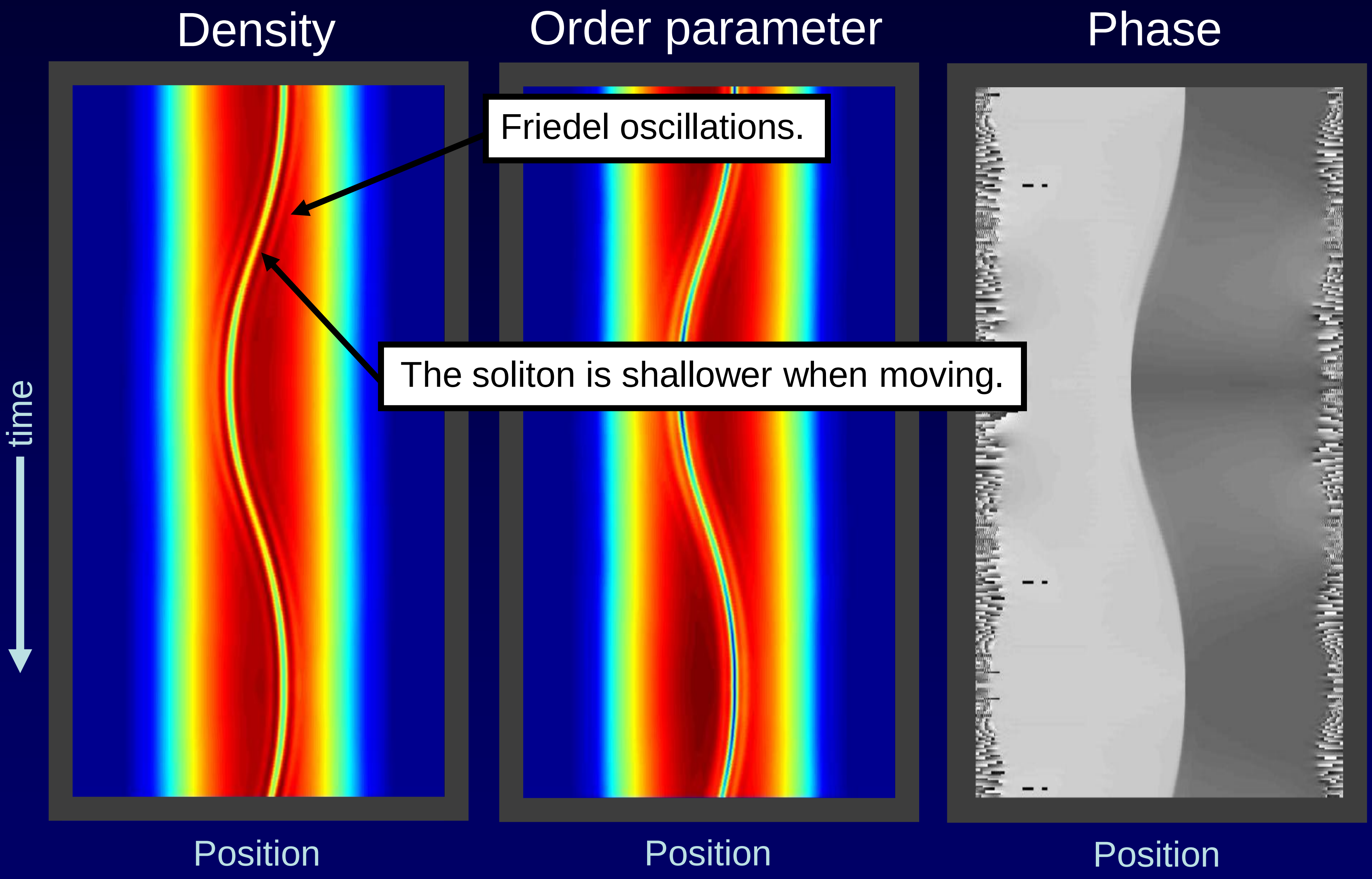

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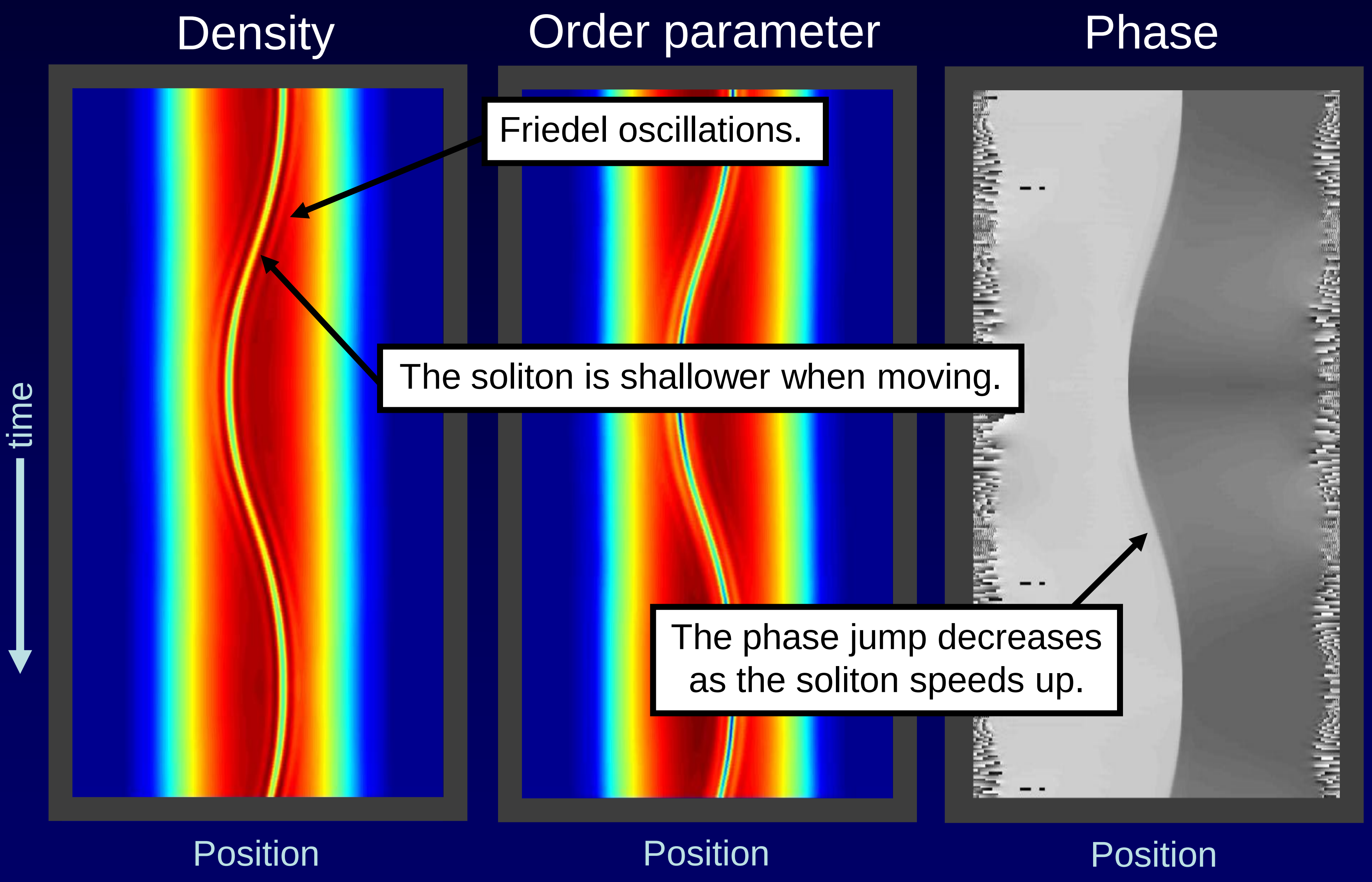

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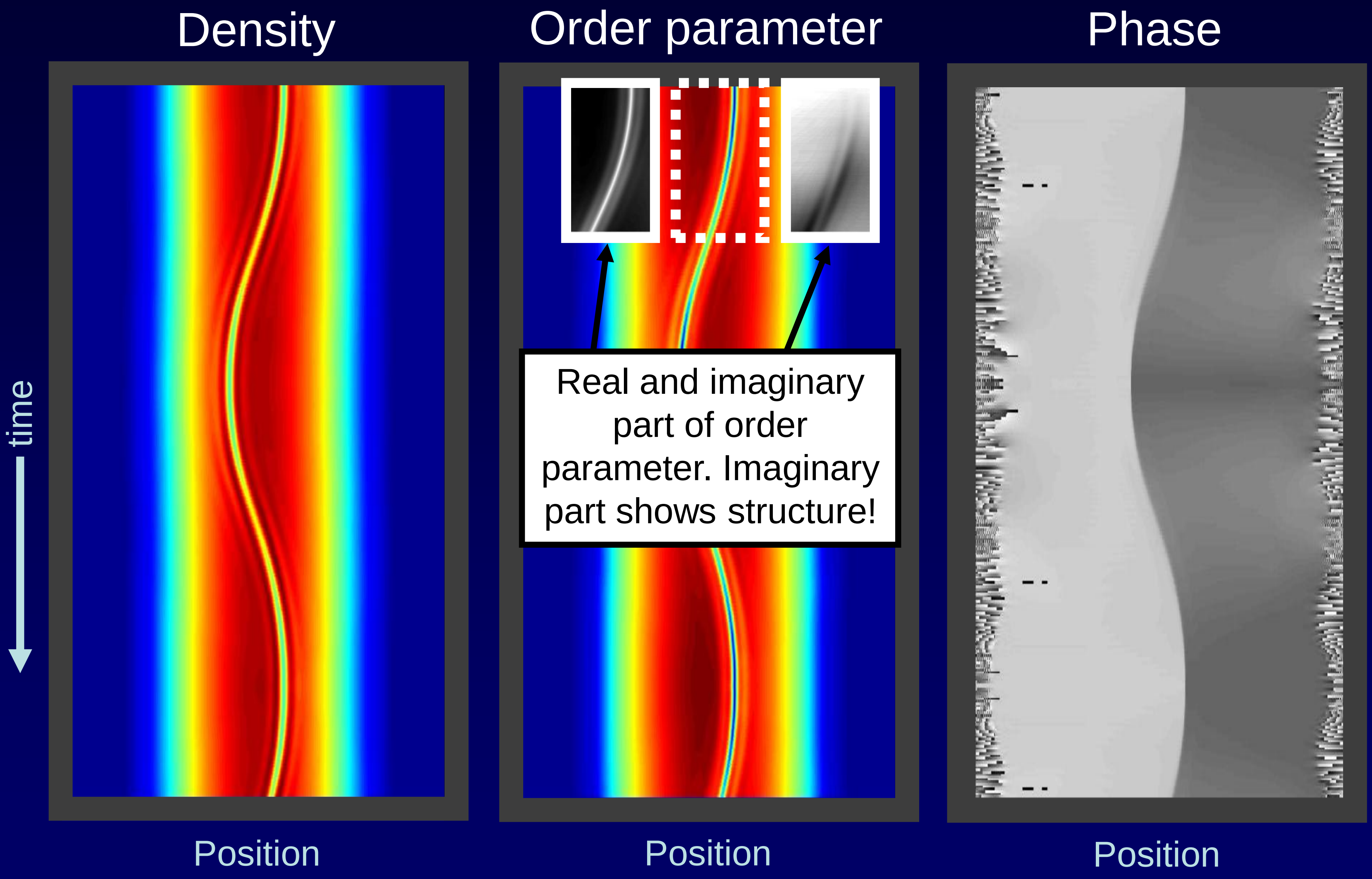
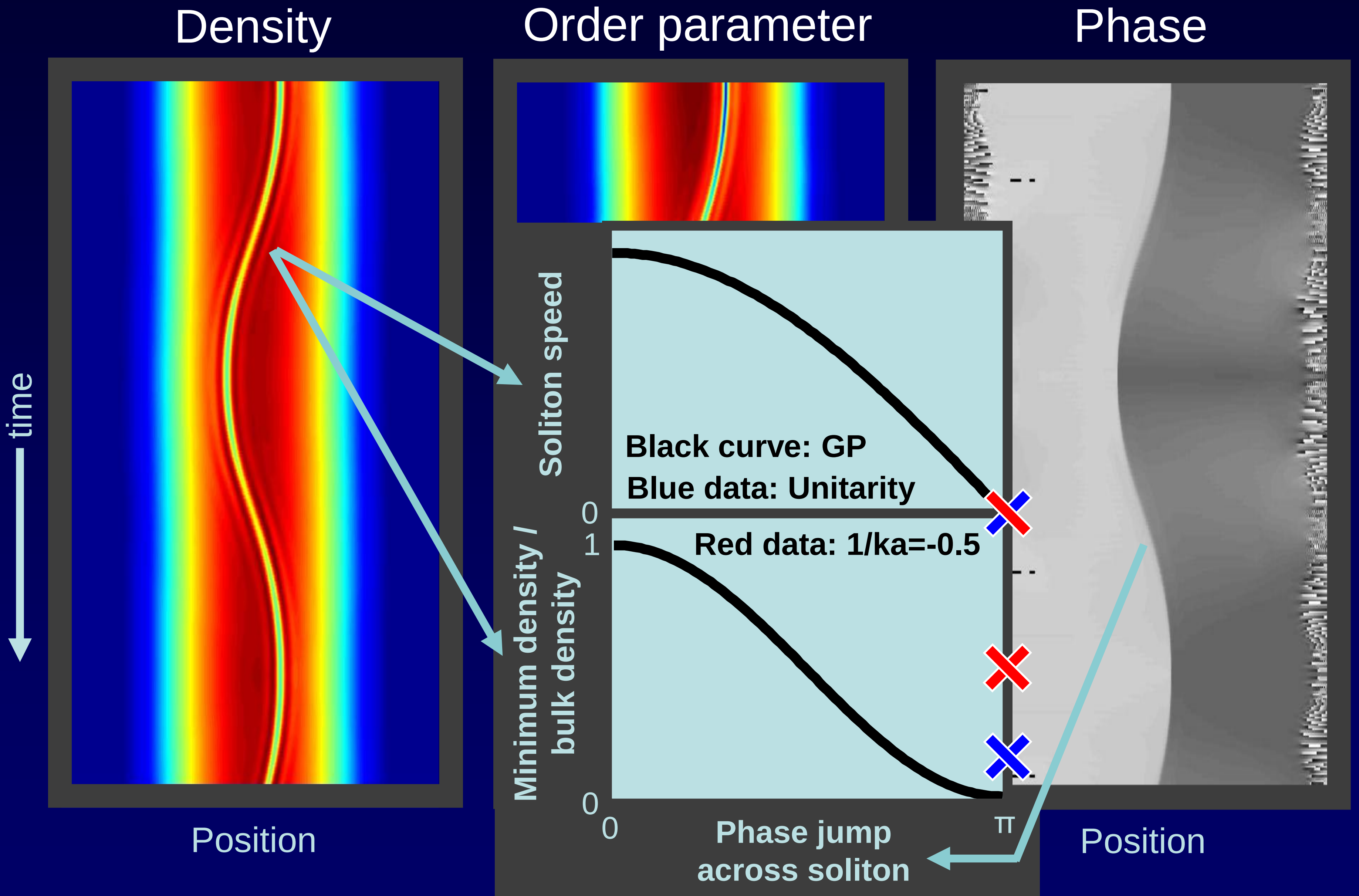
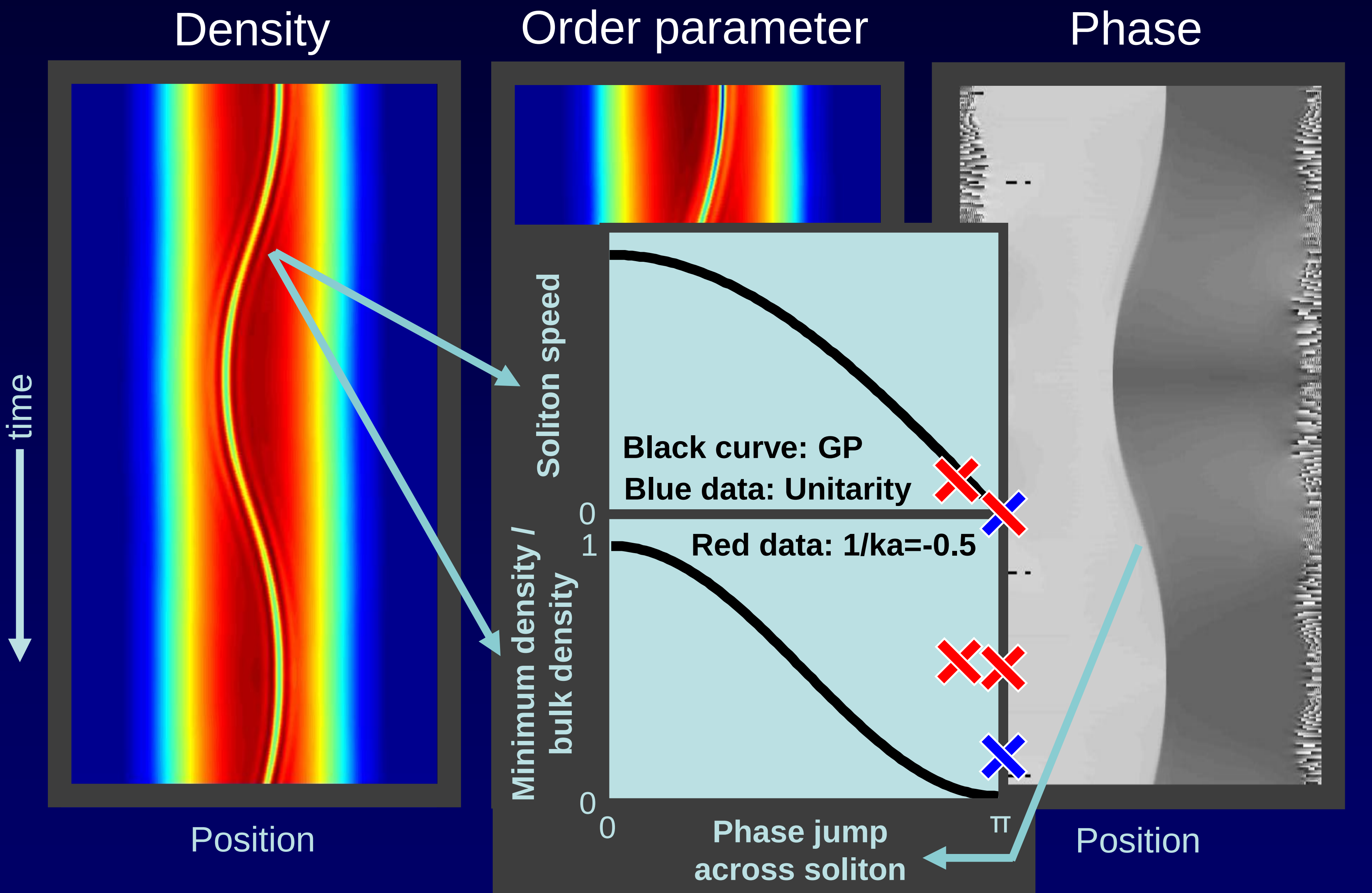


Soliton oscillations in a trap: e.g. $1/k_f a = -0.5$ (BCS)



Soliton oscillations in a trap: e.g. $1/k_fa$ = -0.5 (BCS)



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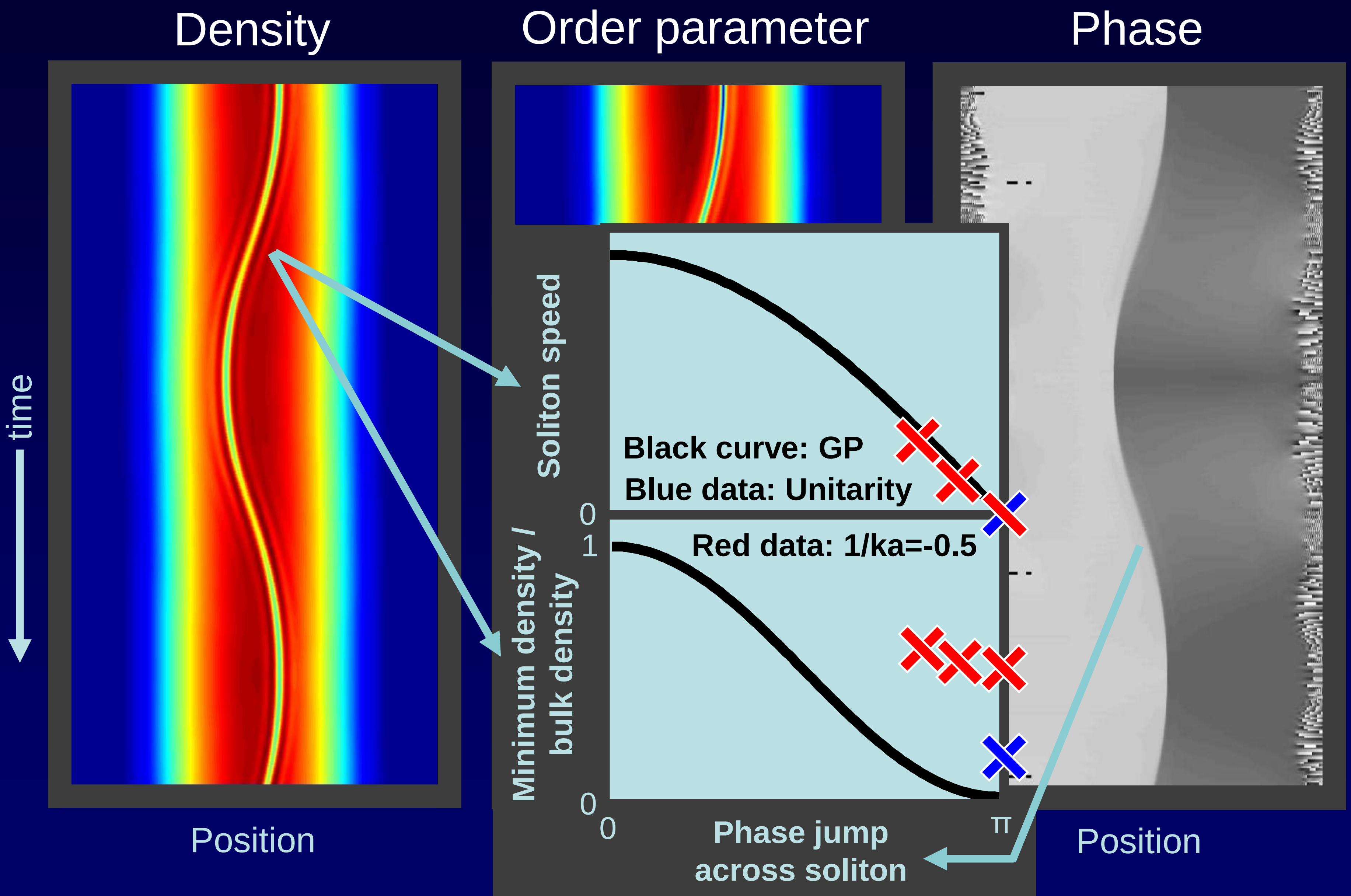
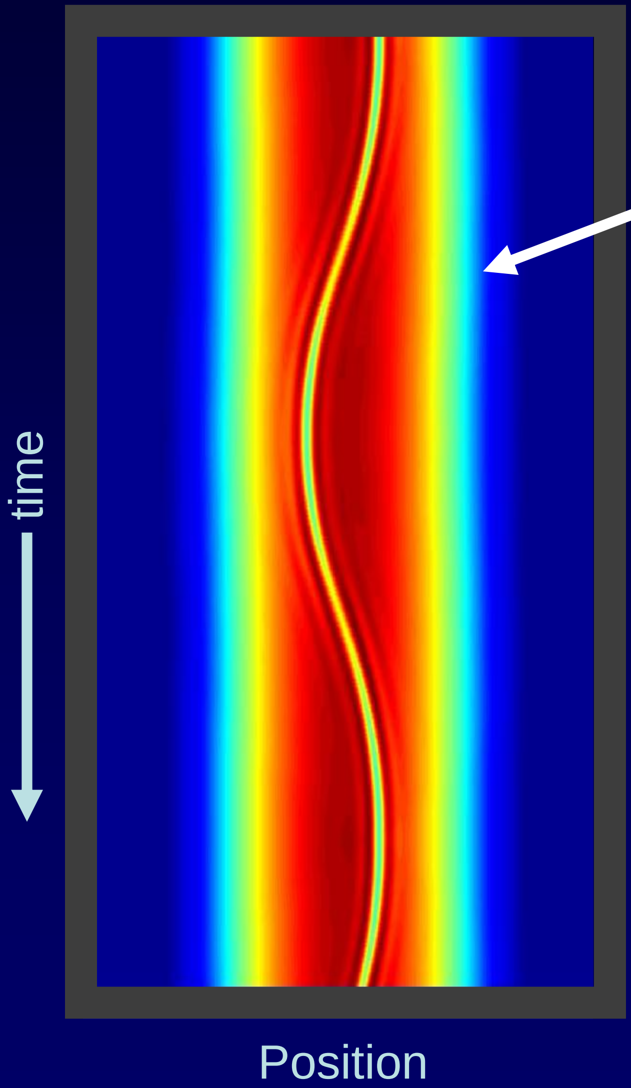


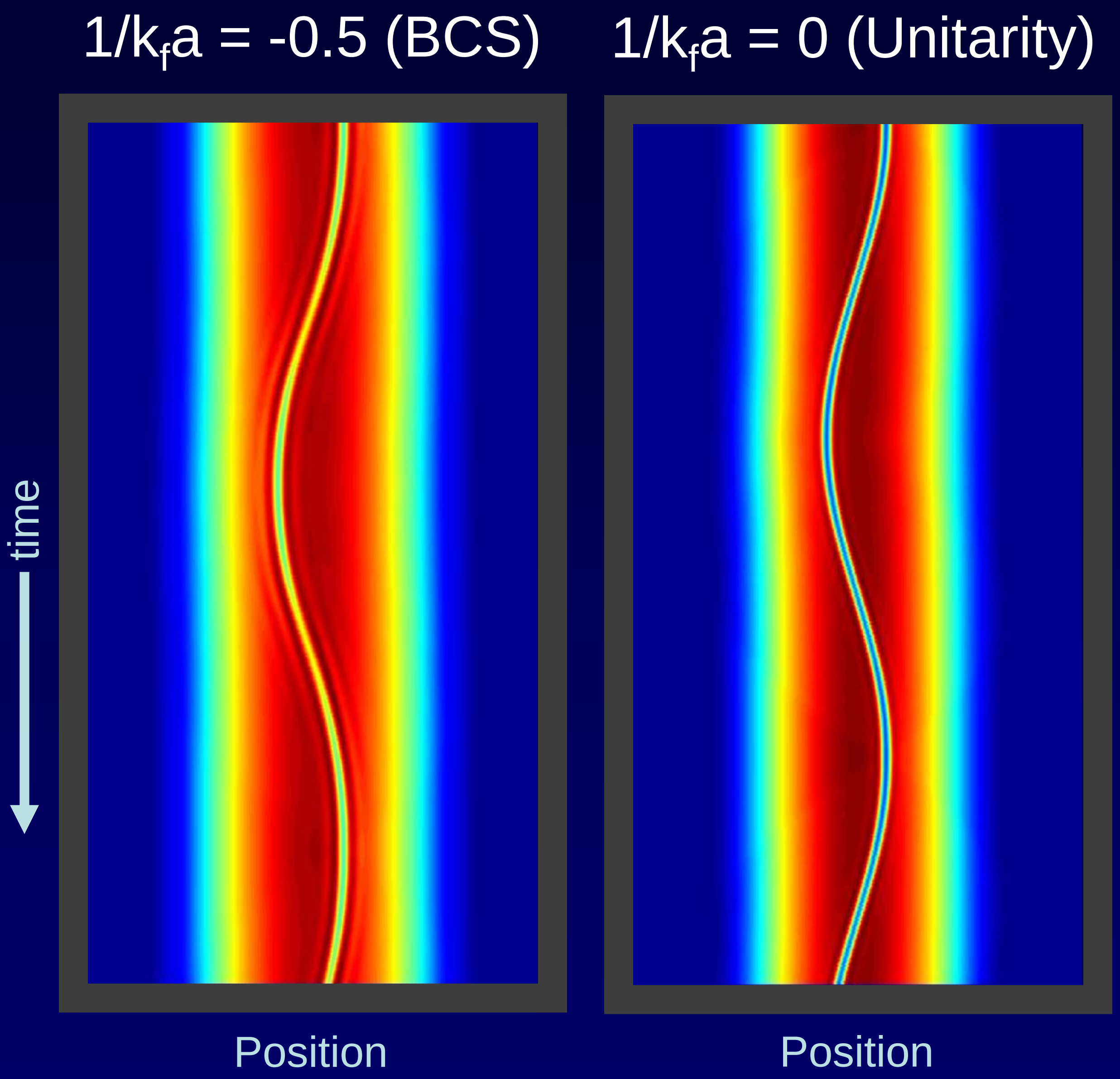
Soliton oscillations in a trap across the crossover

$1/k_f a = -0.5$ (BCS)

Atom density profile

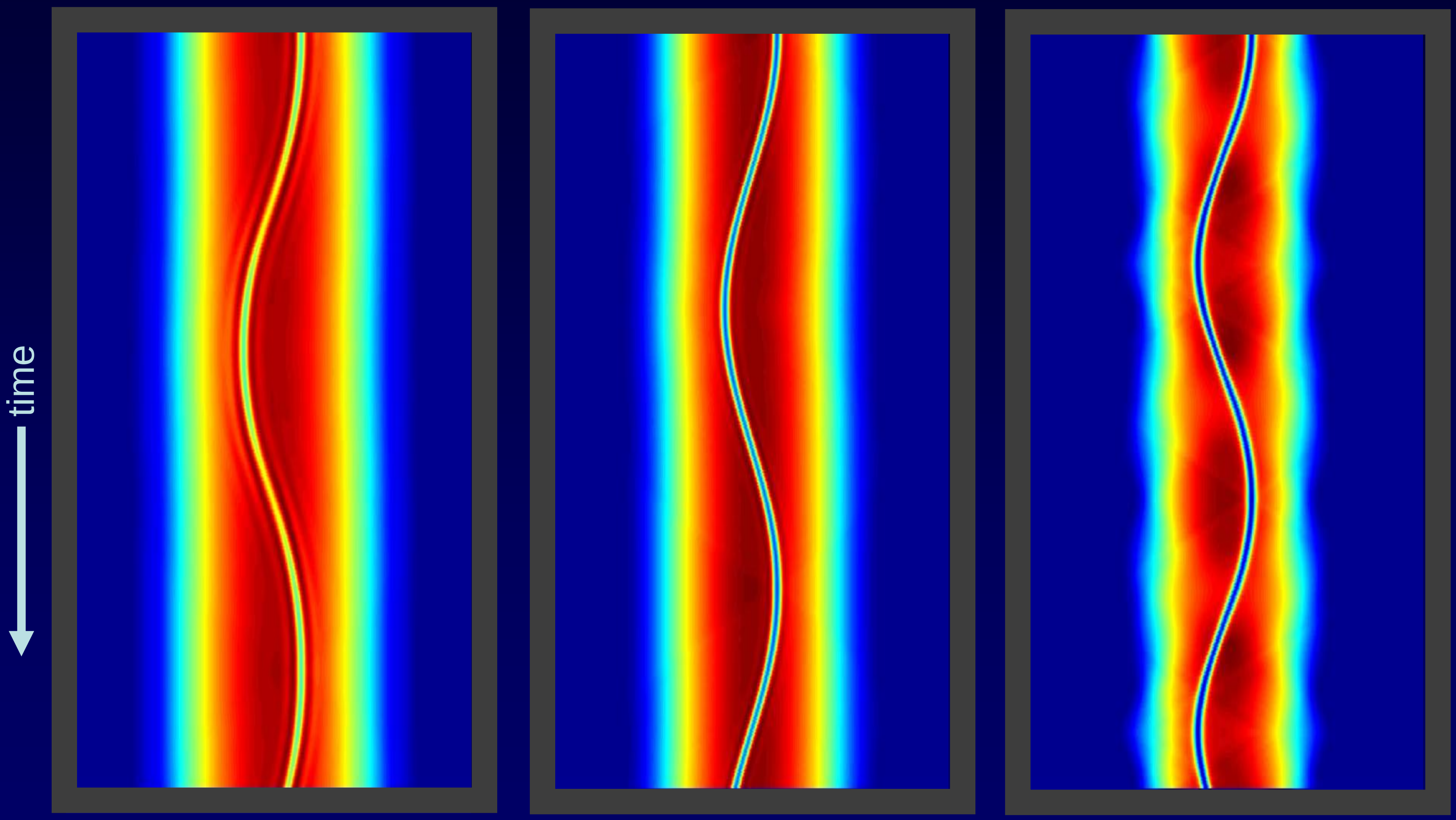


Soliton oscillations in a trap across the crossover



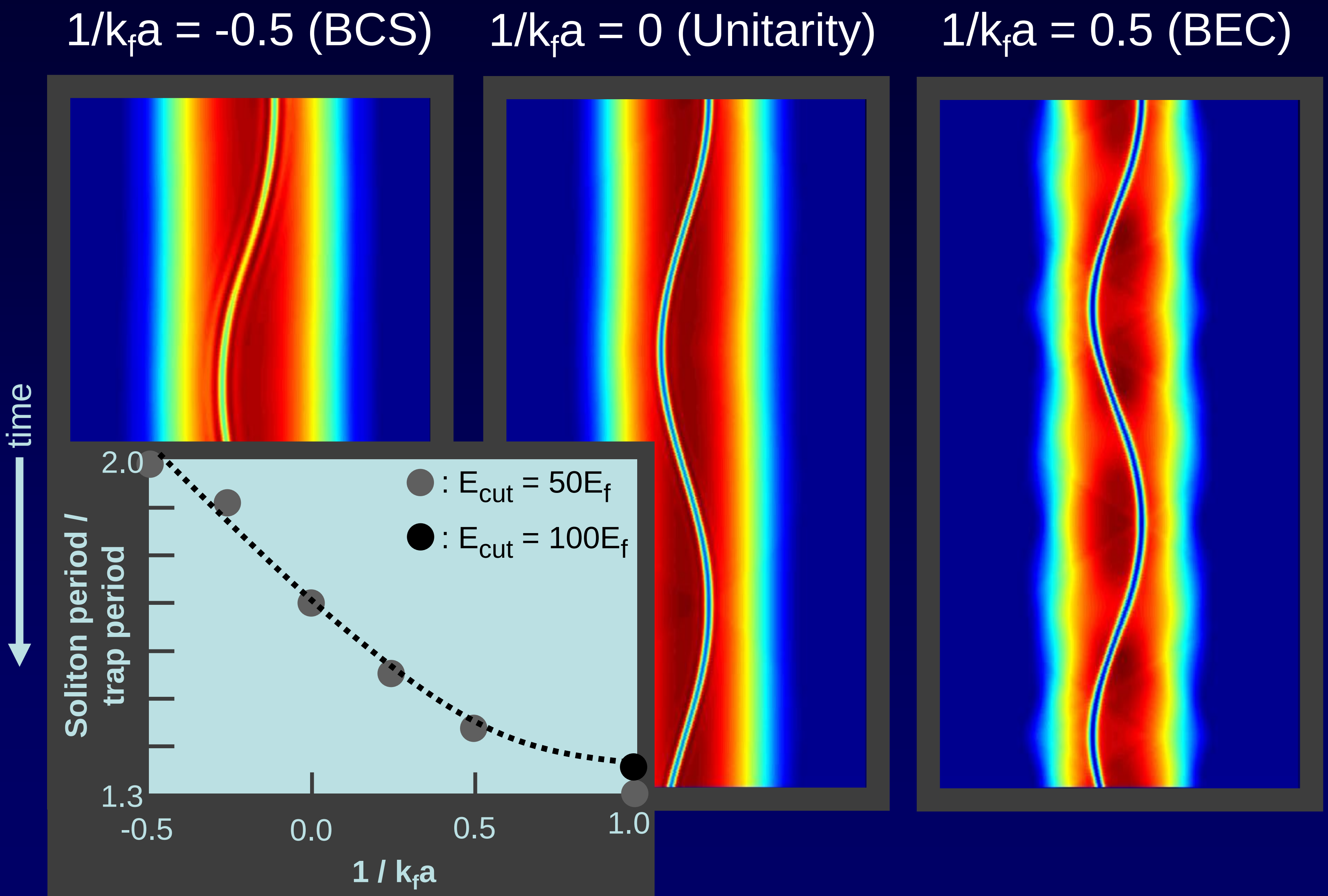
Soliton oscillations in a trap across the crossover

$1/k_f a = -0.5$ (BCS) $1/k_f a = 0$ (Unitarity) $1/k_f a = 0.5$ (BEC)



Position Position Position

Soliton oscillations in a trap across the crossover



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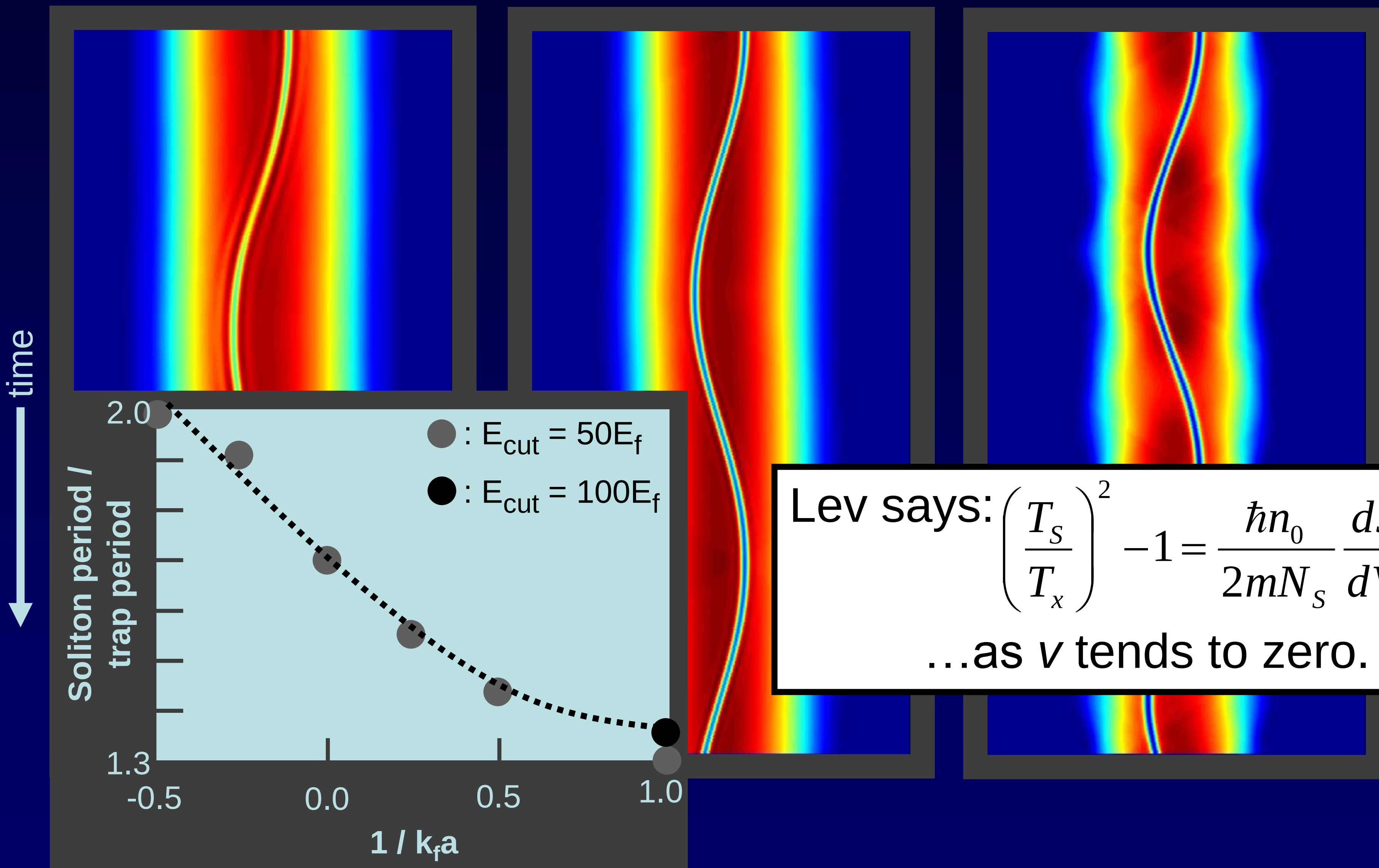

Lev says: $$\left(\frac{T_S}{T_x}\right)^2 - 1 = \frac{\hbar n_0}{2mN_S}\frac{dJ}{dV}$$

…as *v* tends to zero.

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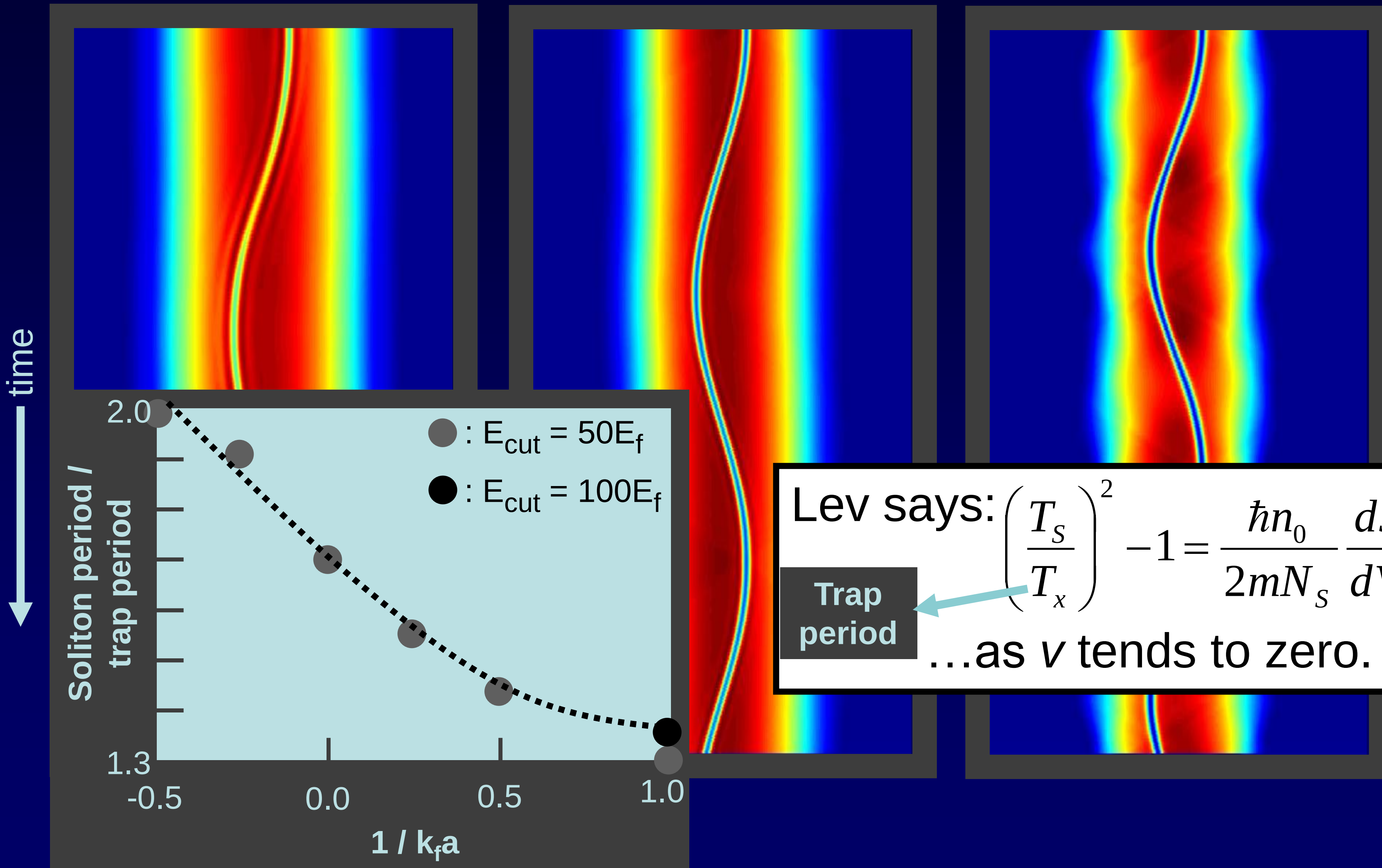

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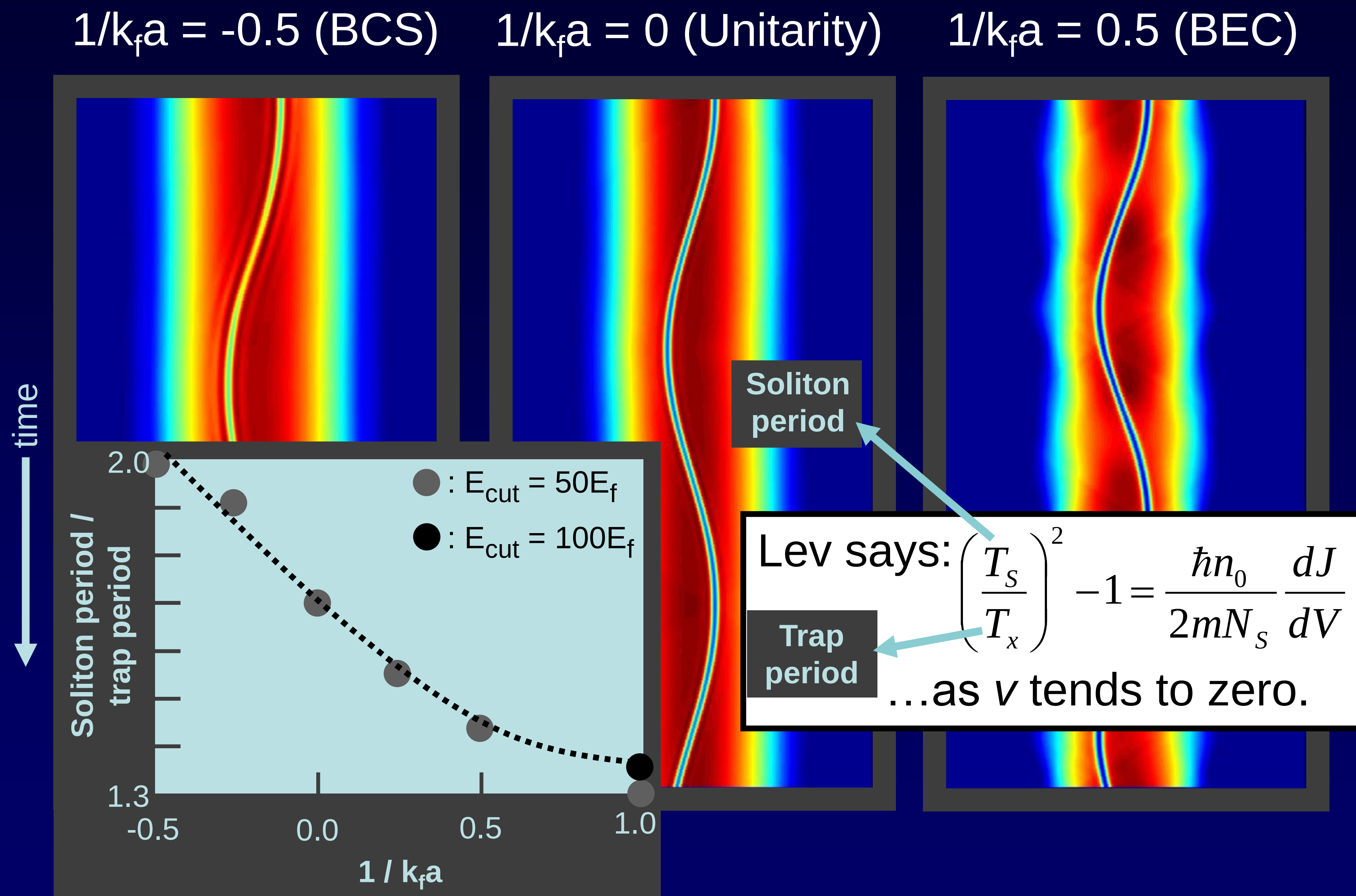

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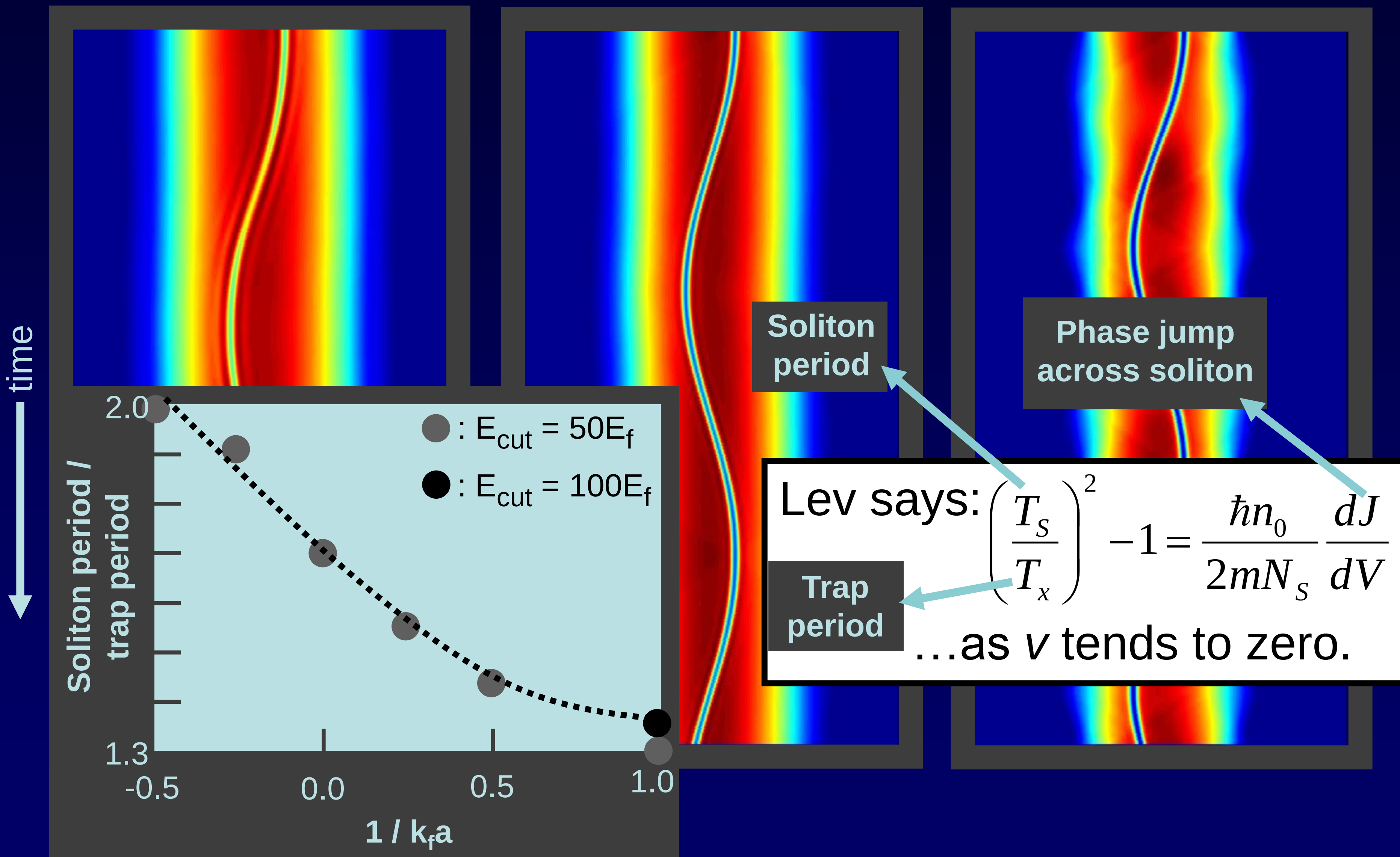

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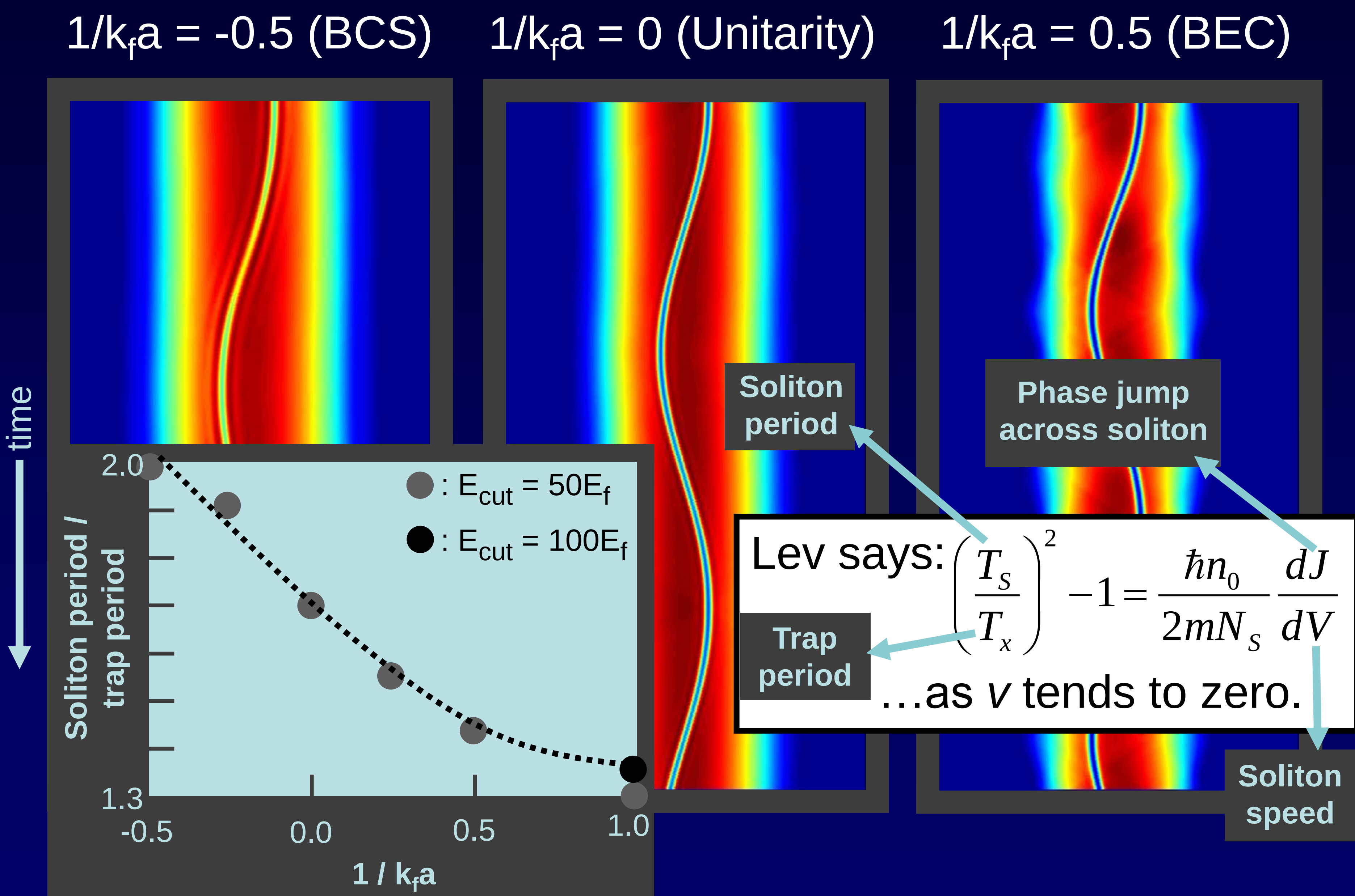

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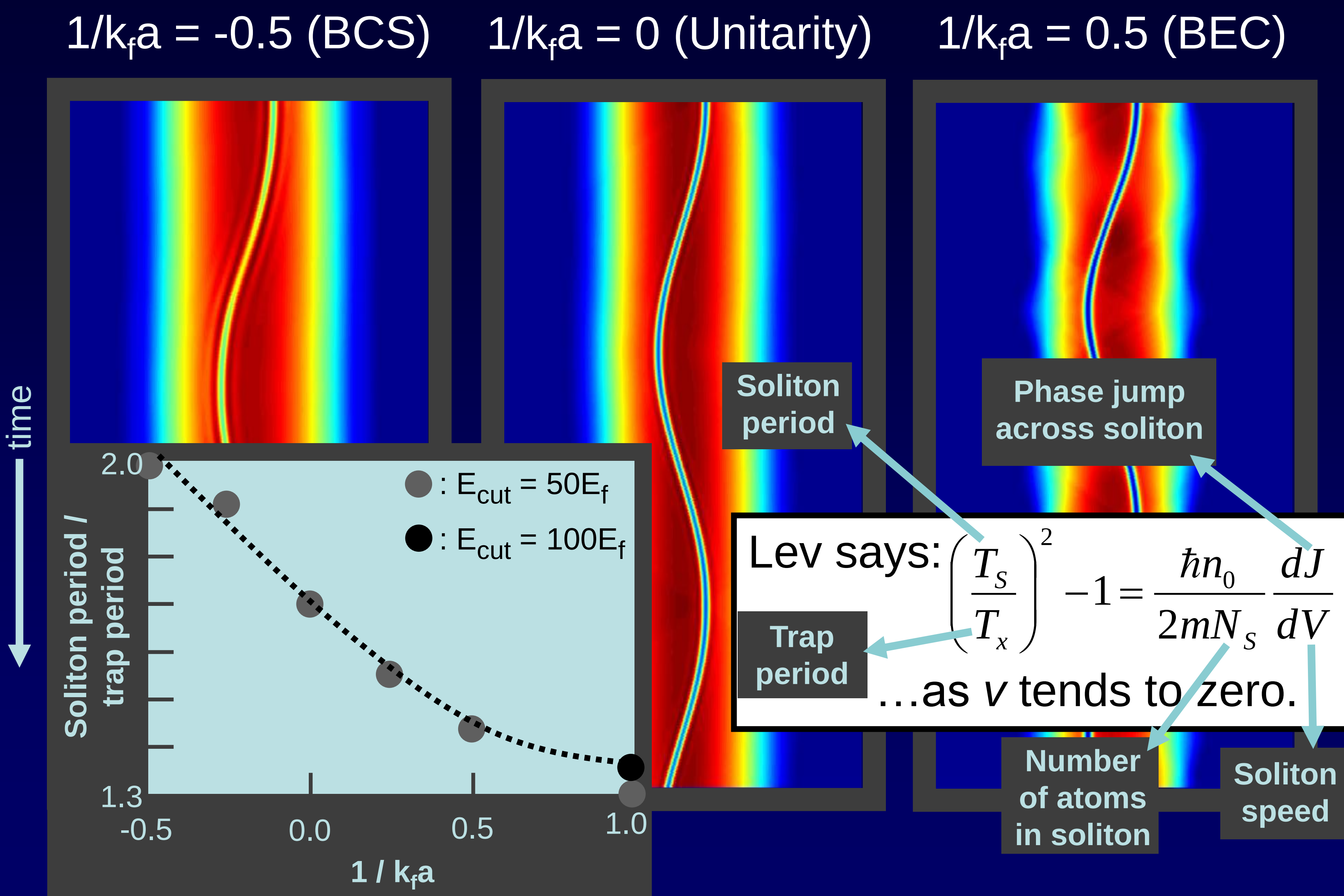

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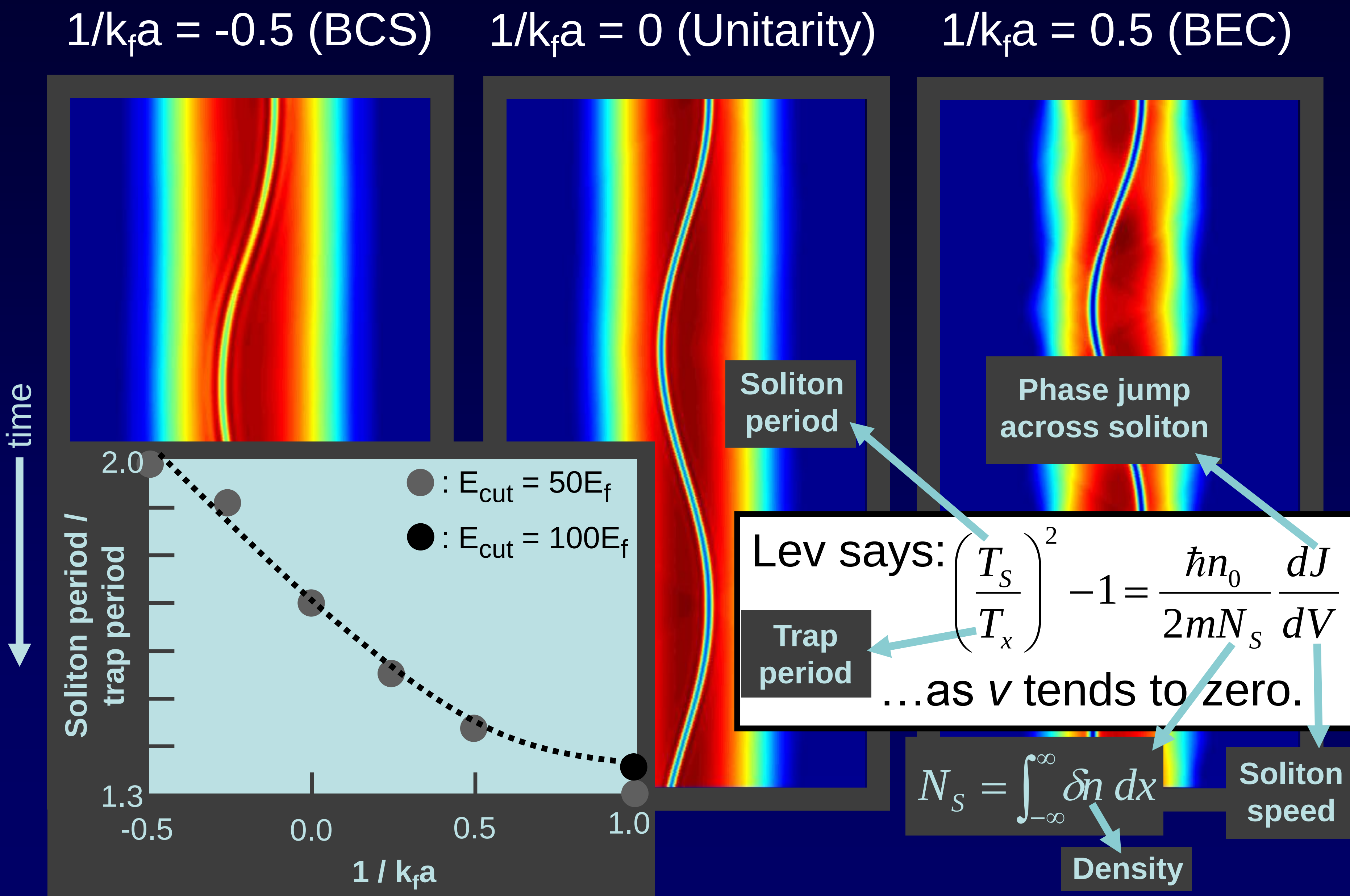

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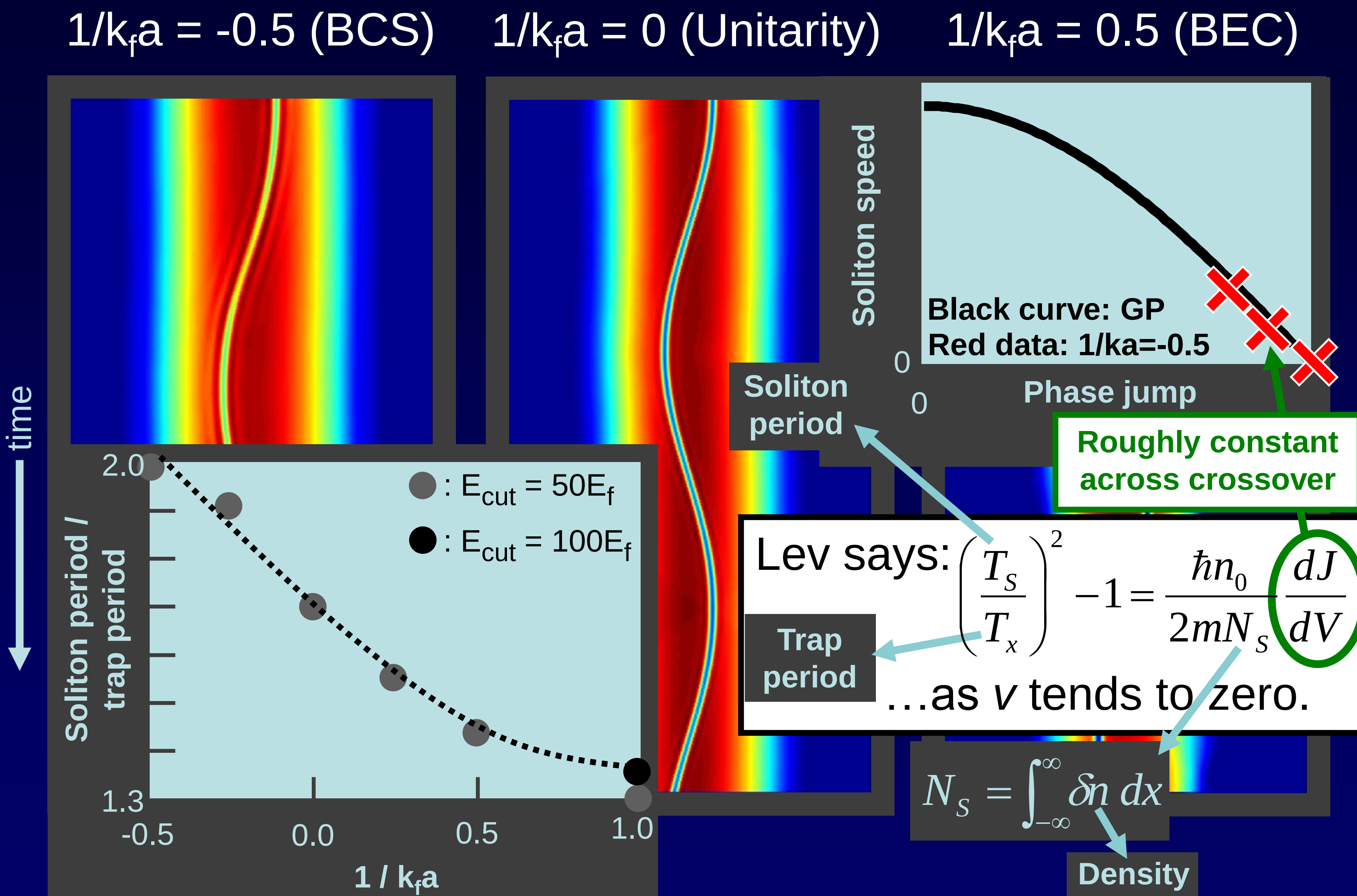
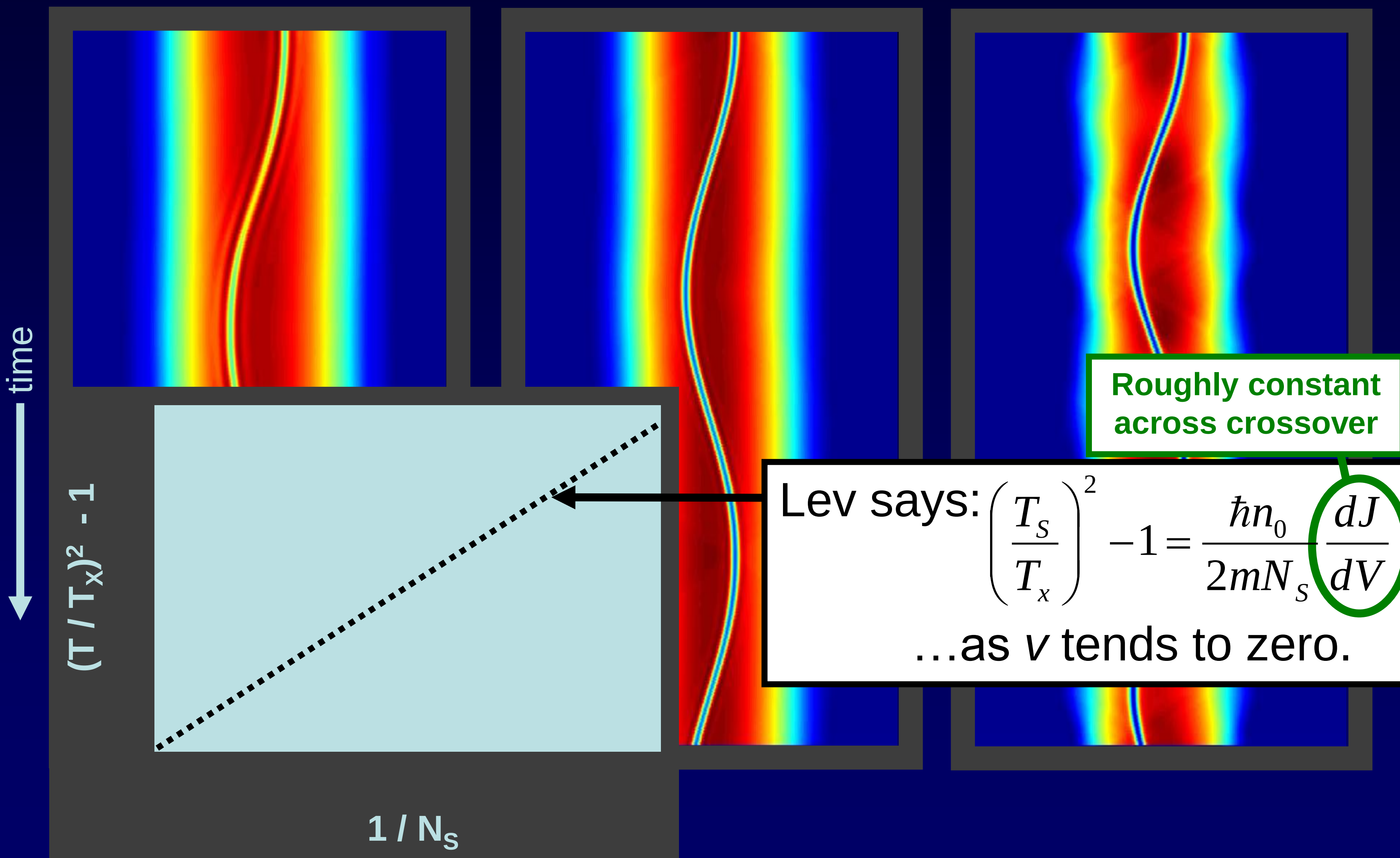


Soliton oscillations in a trap across the crossover

$1/k_f a = -0.5$ (BCS) $1/k_f a = 0$ (Unitarity) $1/k_f a = 0.5$ (BEC)



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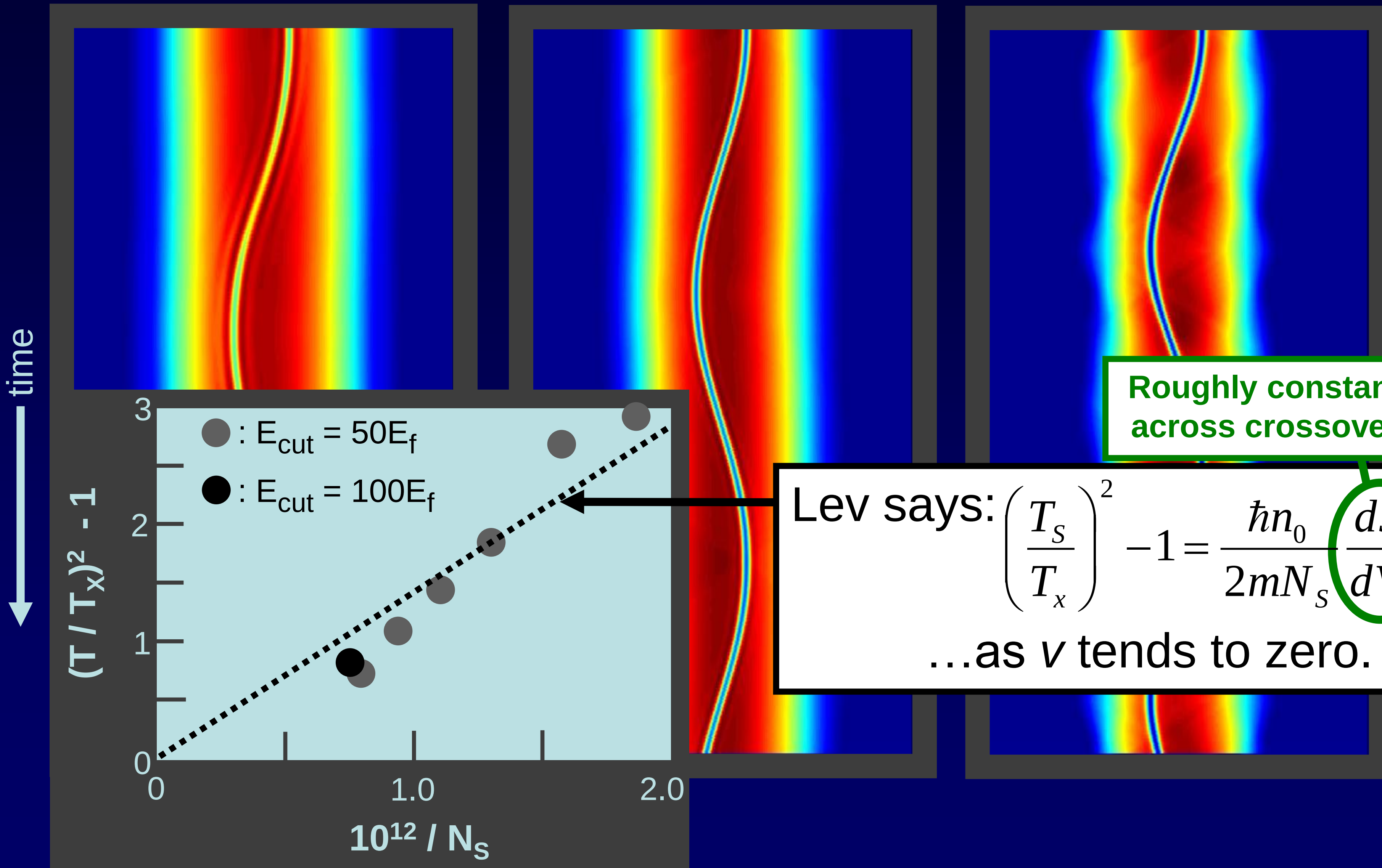
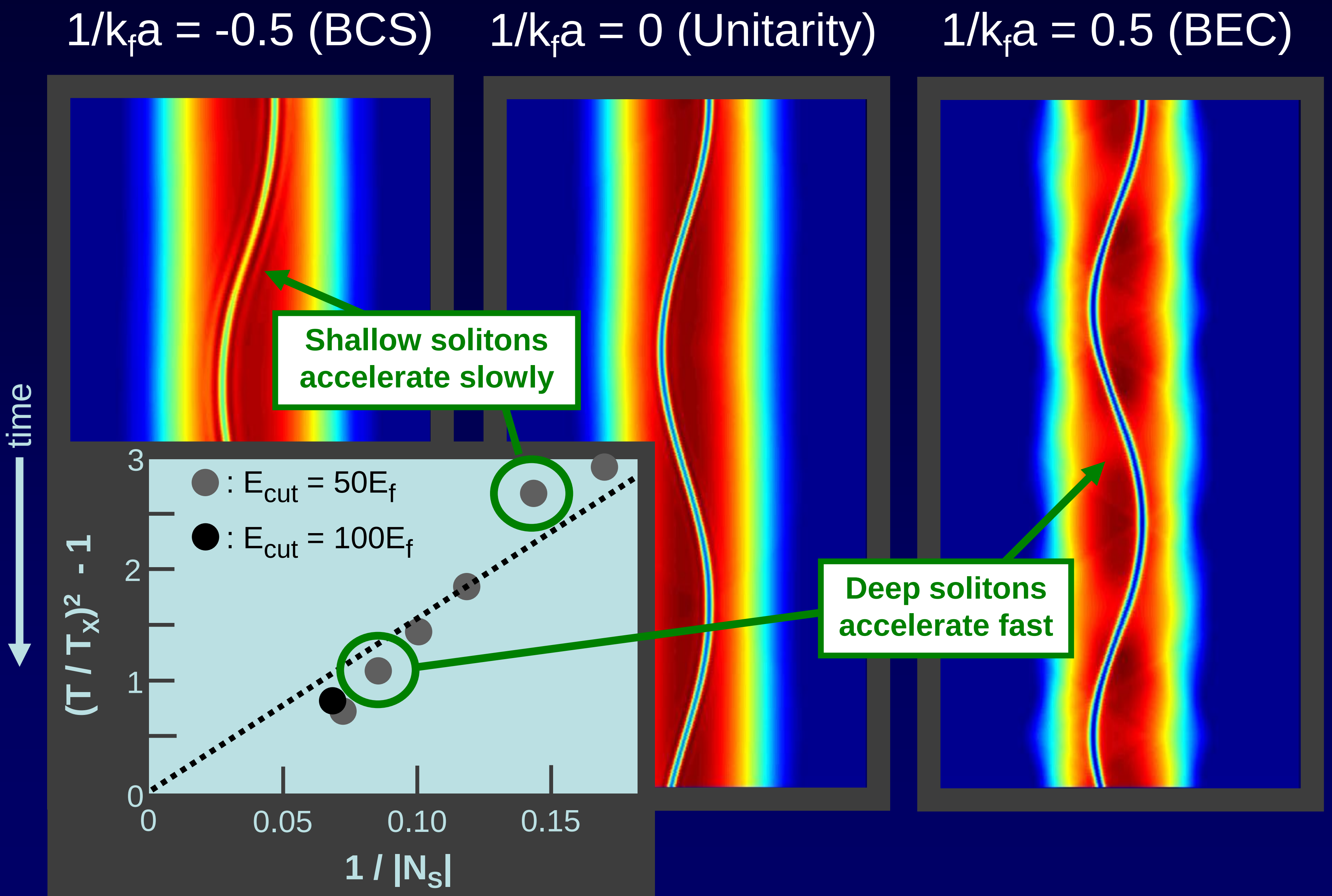


Roughly constant across crossover

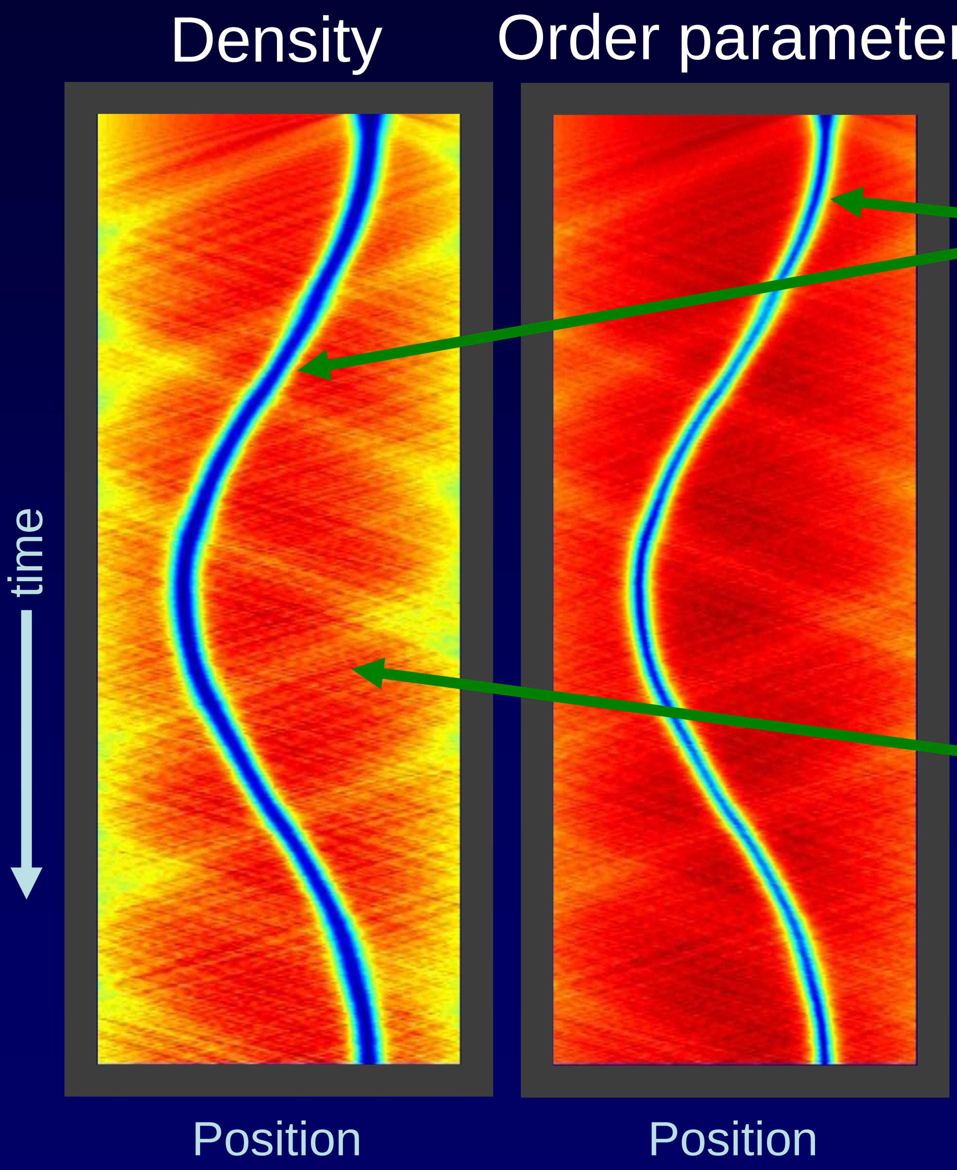
Lev says: $$\left(\frac{T_S}{T_x}\right)^2 - 1 = \frac{\hbar n_0}{2mN_S}\frac{dJ}{dV}$$

…as *v* tends to zero.

Soliton oscillations in a trap across the crossover



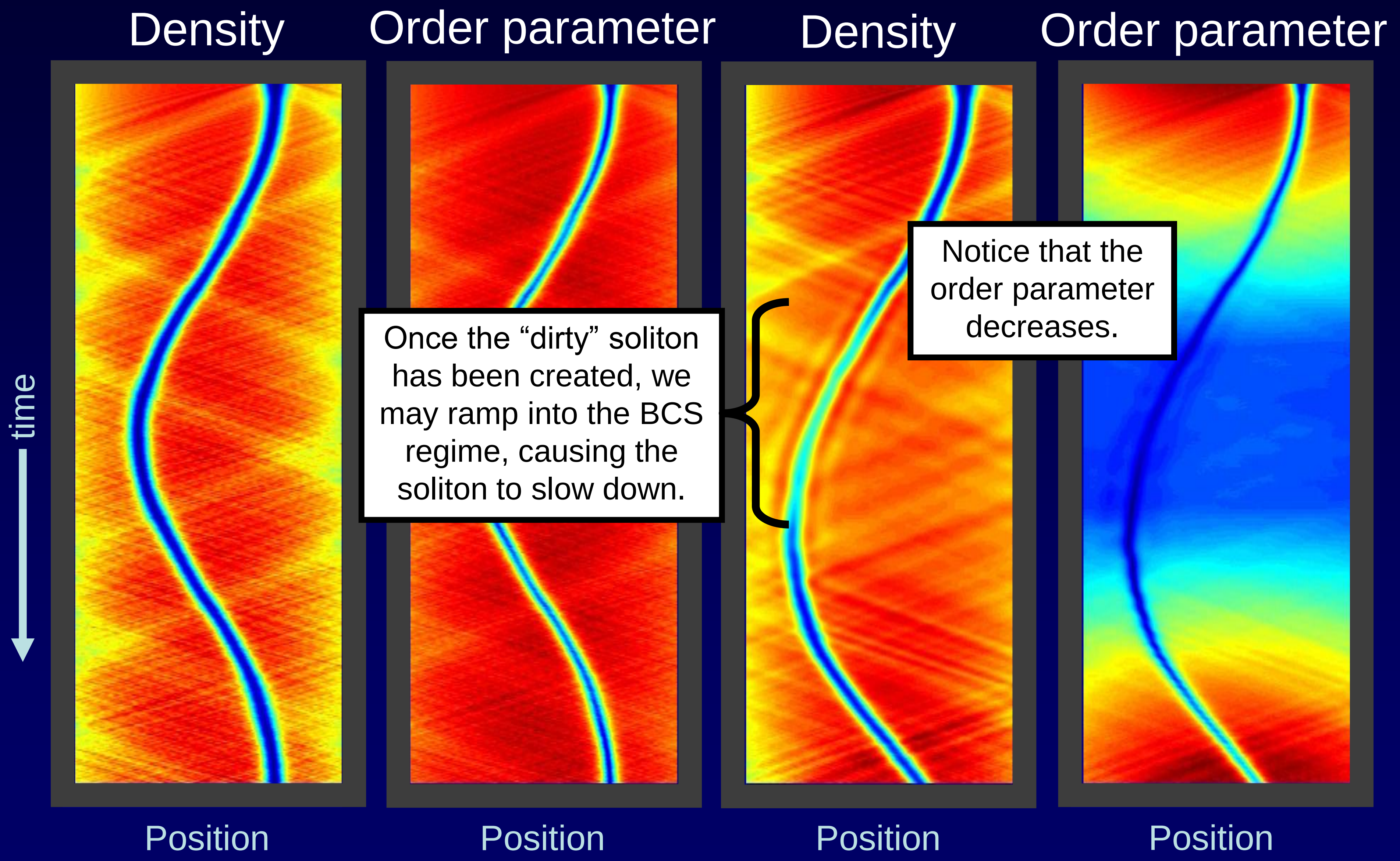
Formation and detection of solitons



"Dirty" soliton created ($1/k_f a=1$) by a combination of density imprinting (creating a hole) and phase imprinting (creating a phase jump).

Sound is also created.

Formation and detection of solitons



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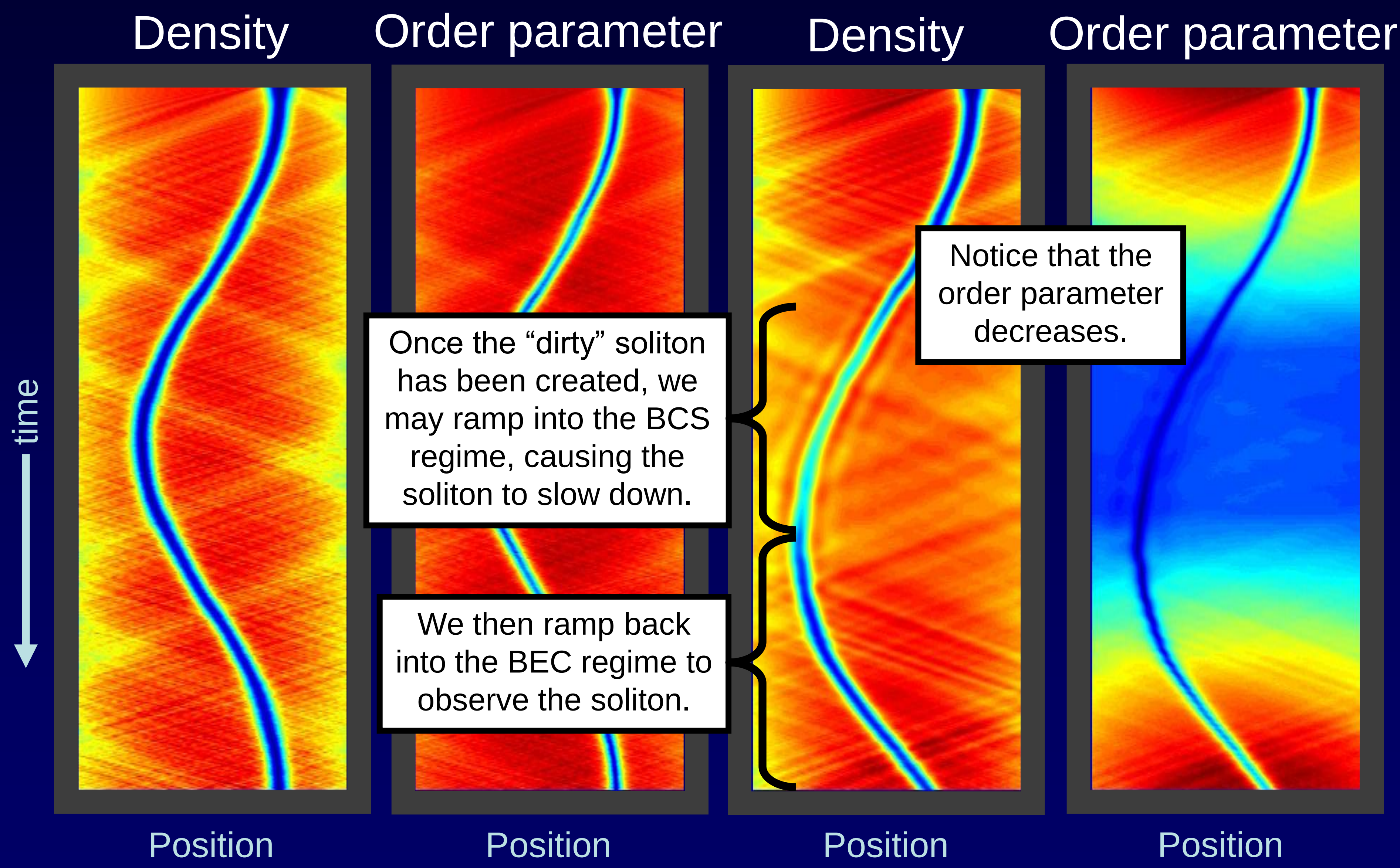
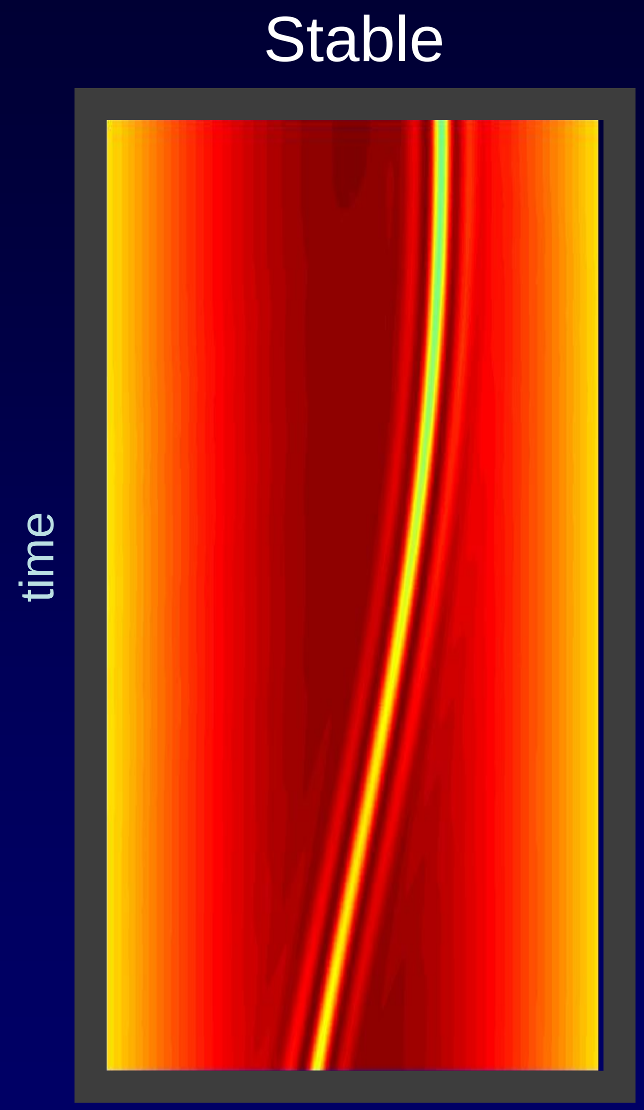
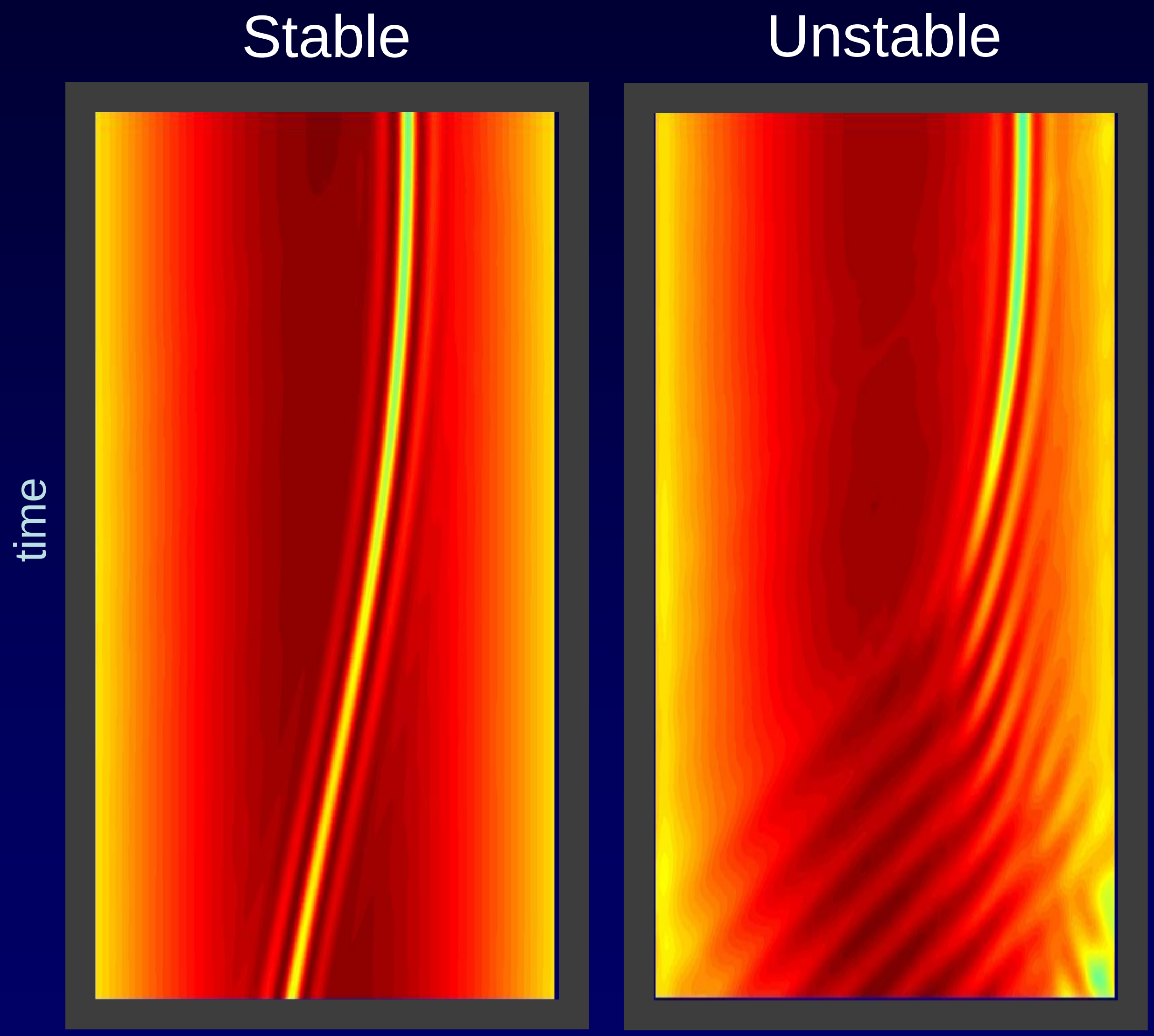


Soliton decay in the BCS regime ($1/k_f a = -0.5$)



Soliton decay in the BCS regime ($1/k_fa = -0.5$)



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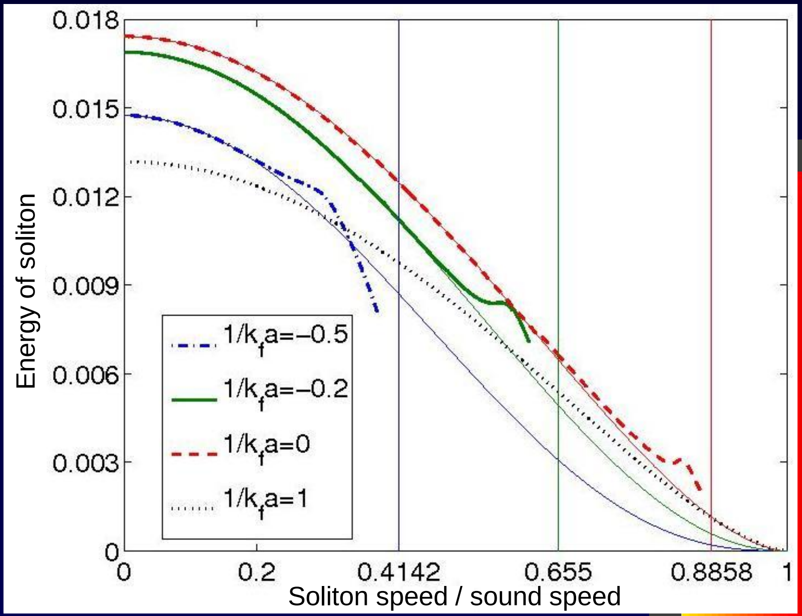

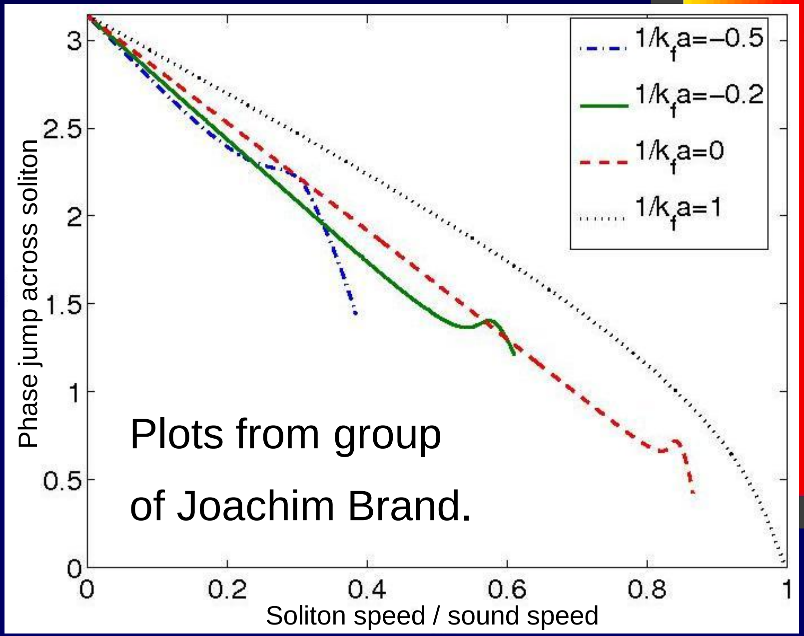

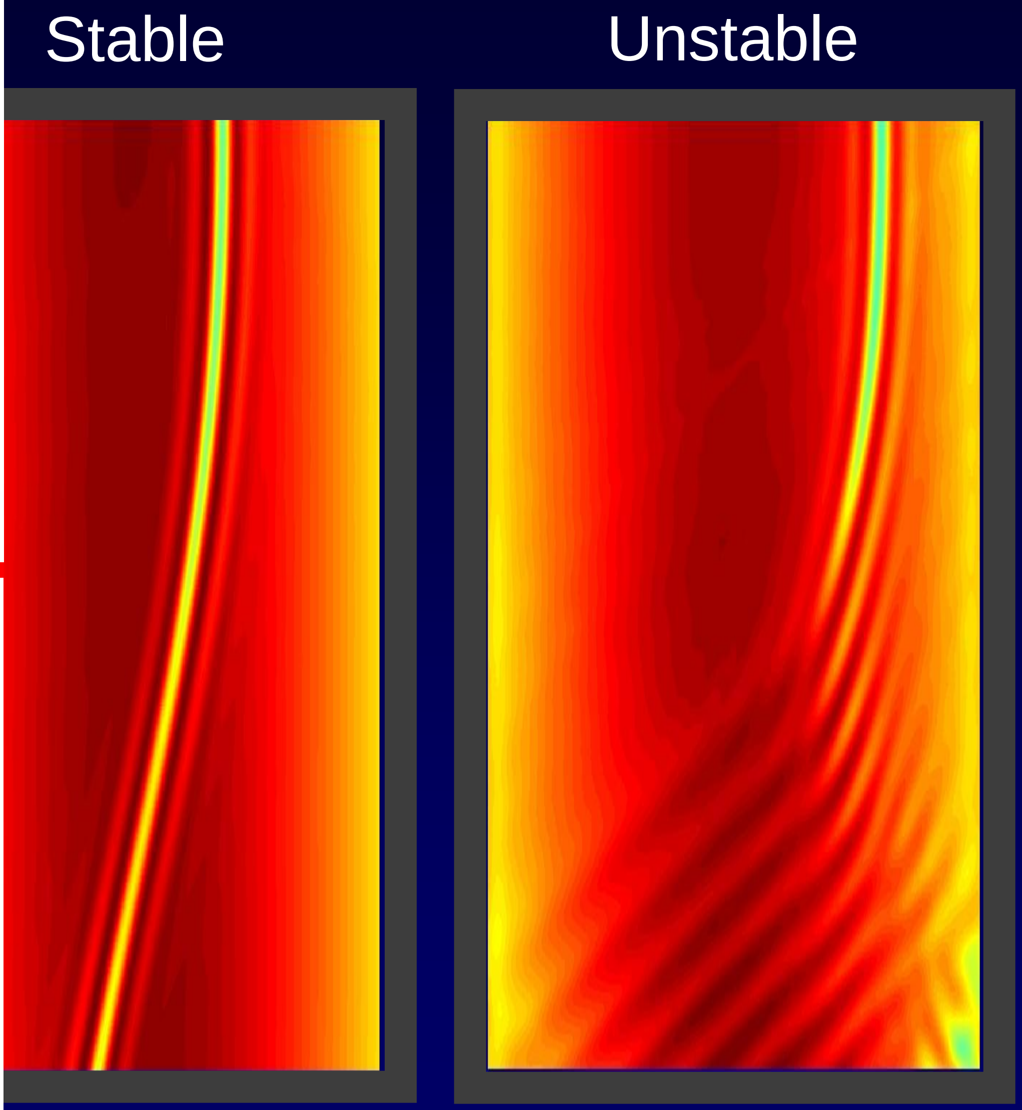

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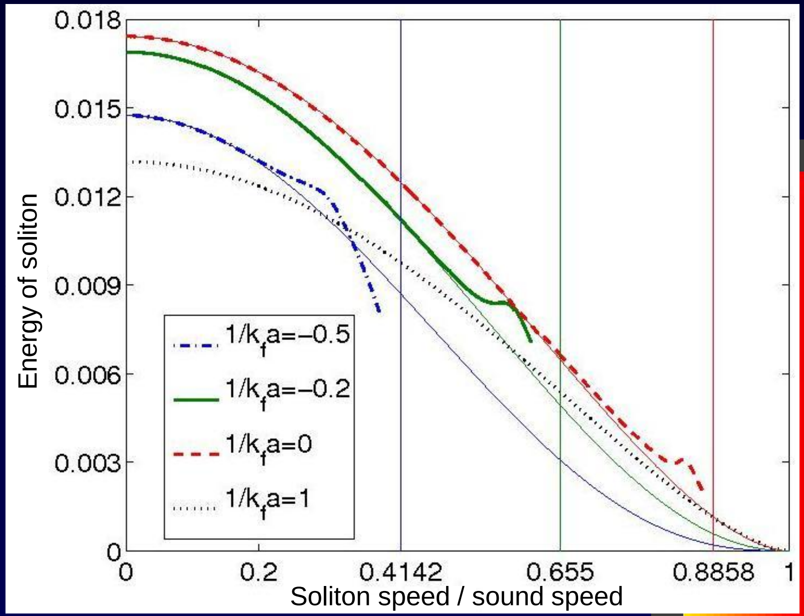
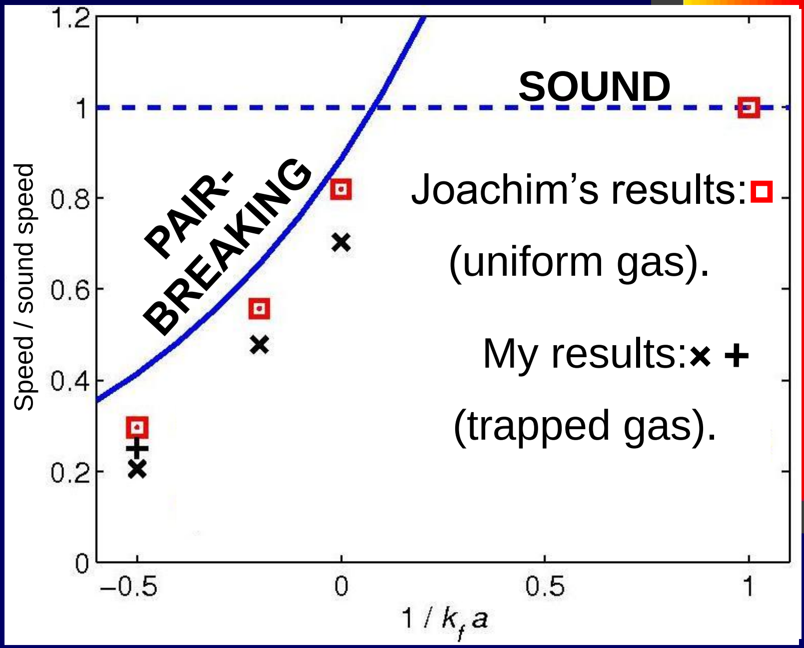
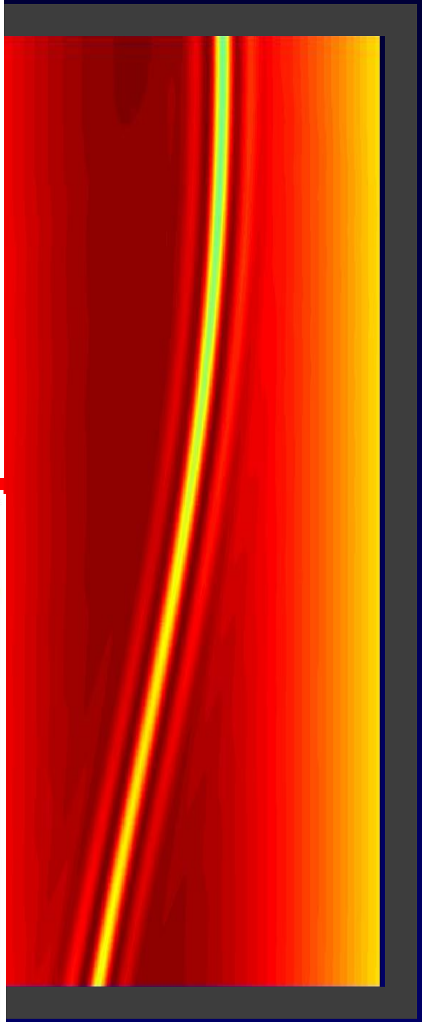
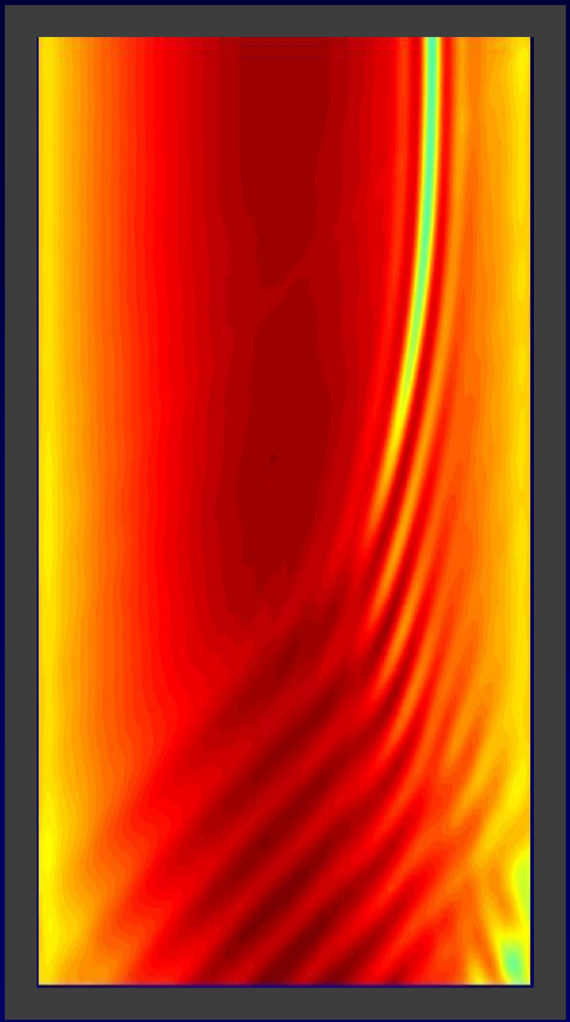




Stable



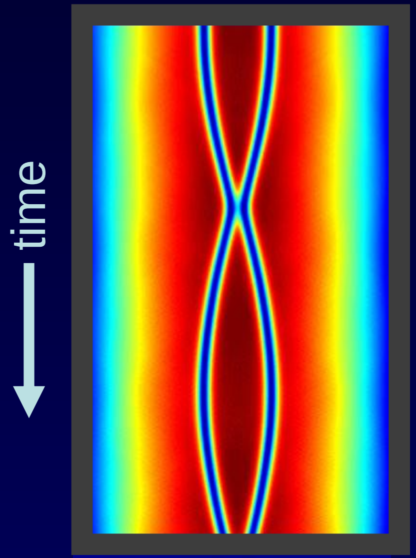
Unstable

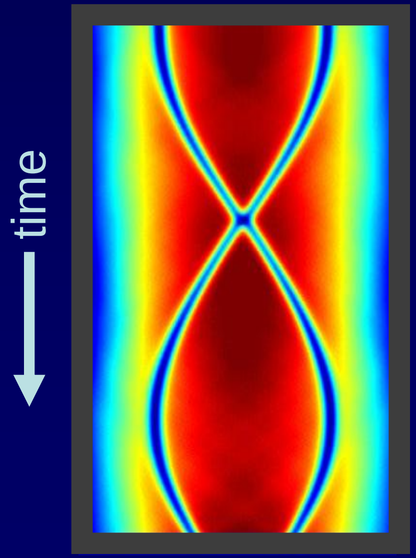


Soliton collisions in a trap across the crossover

$1/k_f a = 1.0$

Solitons collisions are elastic in the BEC limit.

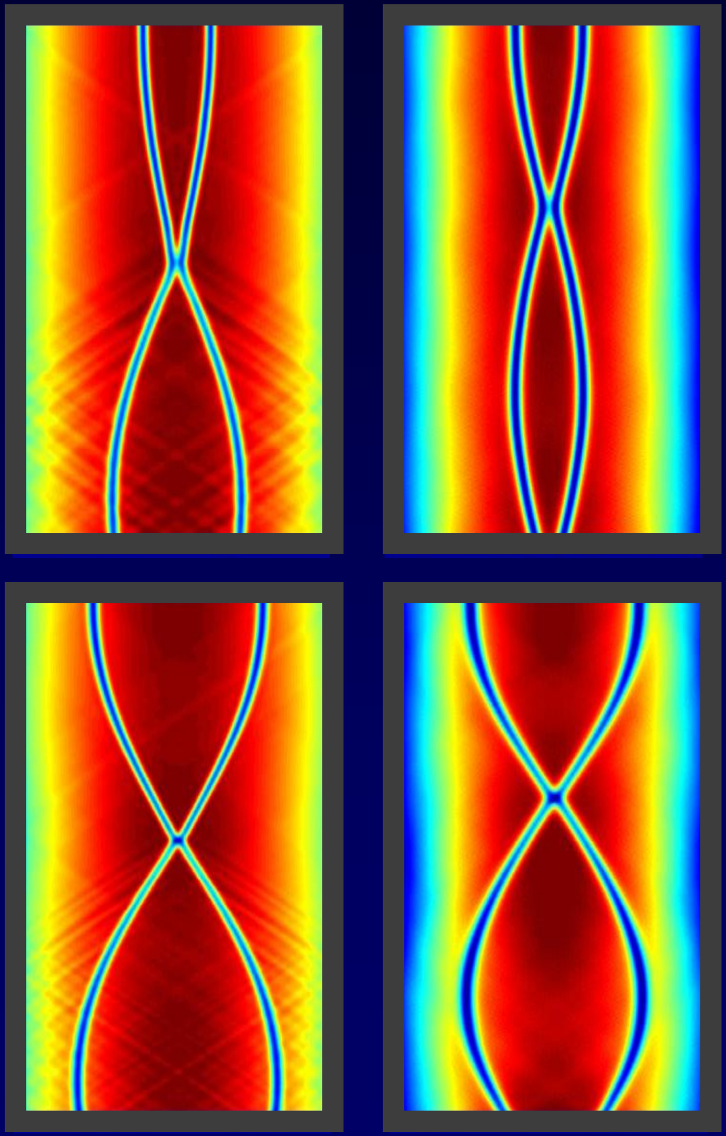




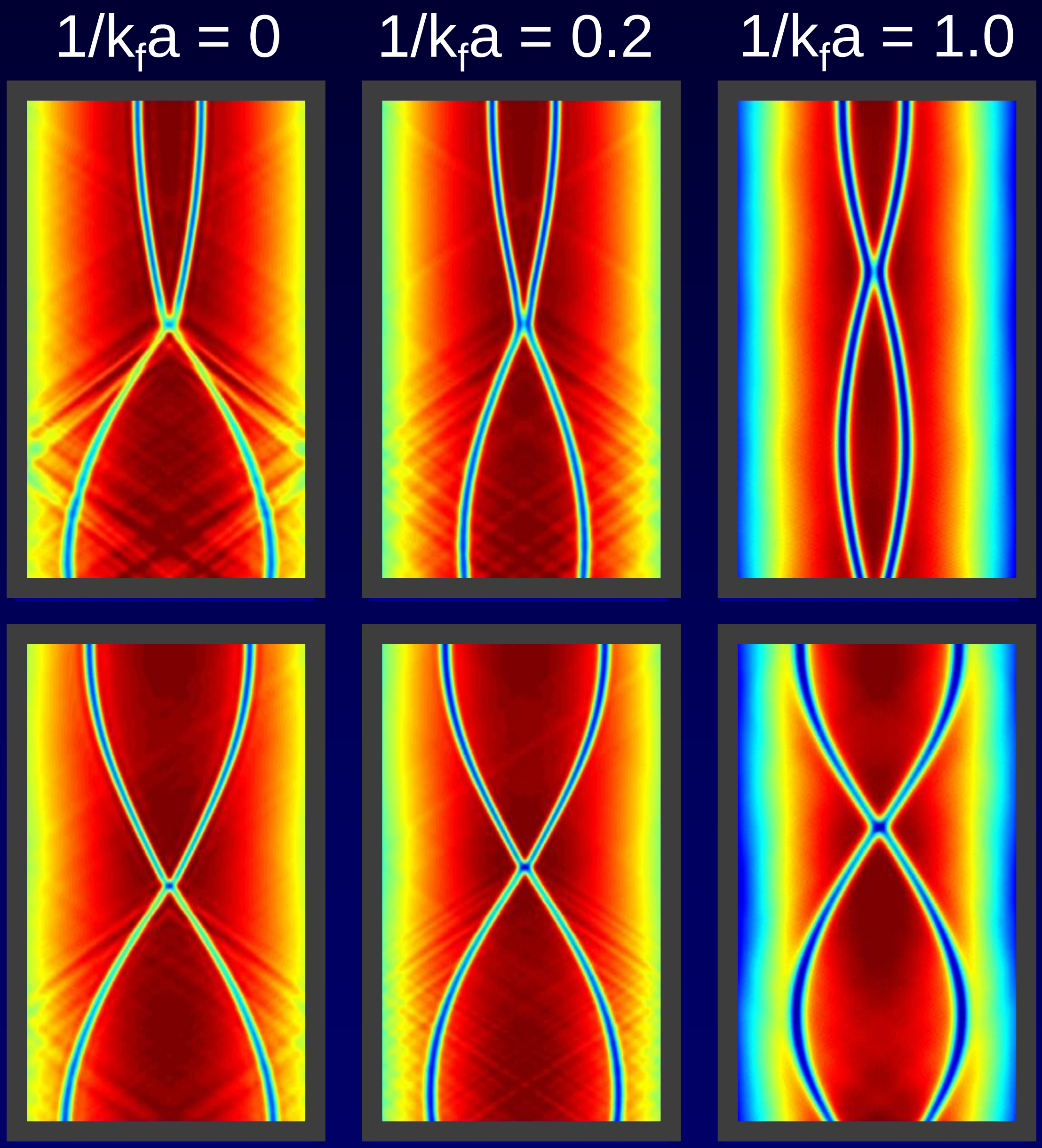
Soliton collisions in a trap across the crossover

Solitons collisions become inelastic for small $1/k_f a$, causing the solitons (counter-intuitively) to speed up. Slow collisions are more inelastic than fast collisions.

$1/k_f a = 0.2$ $1/k_f a = 1.0$



Soliton collisions in a trap across the crossover



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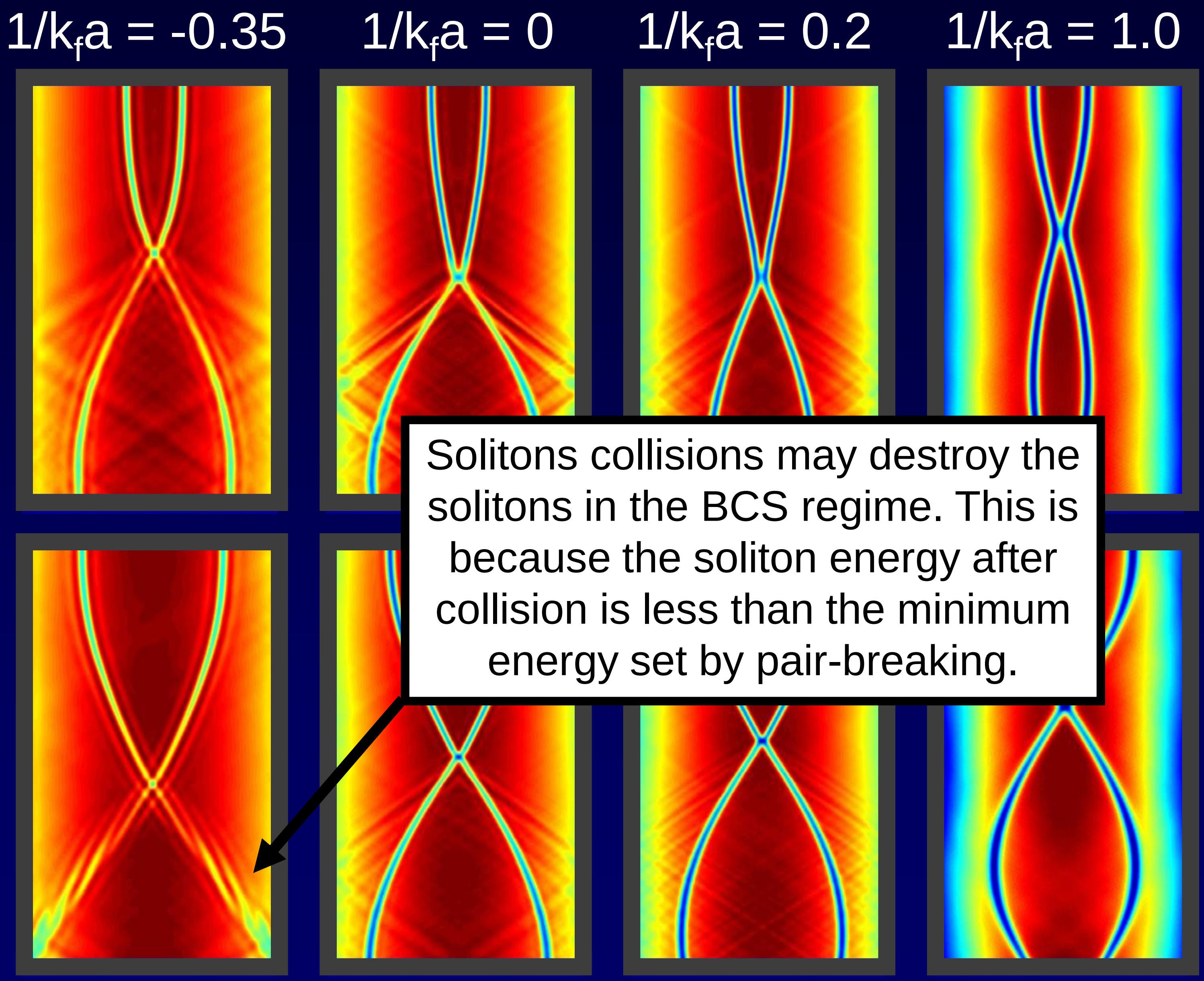

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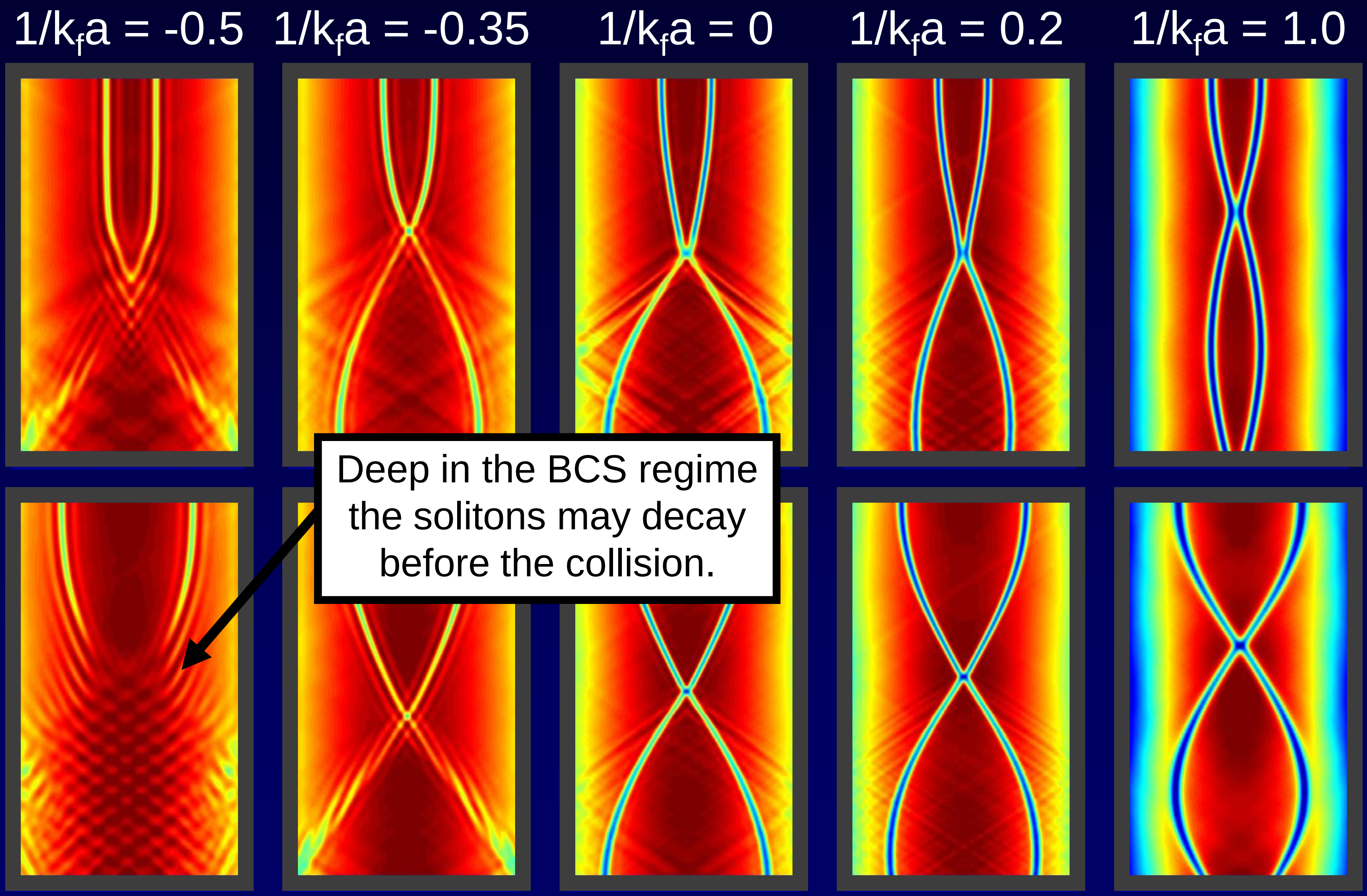

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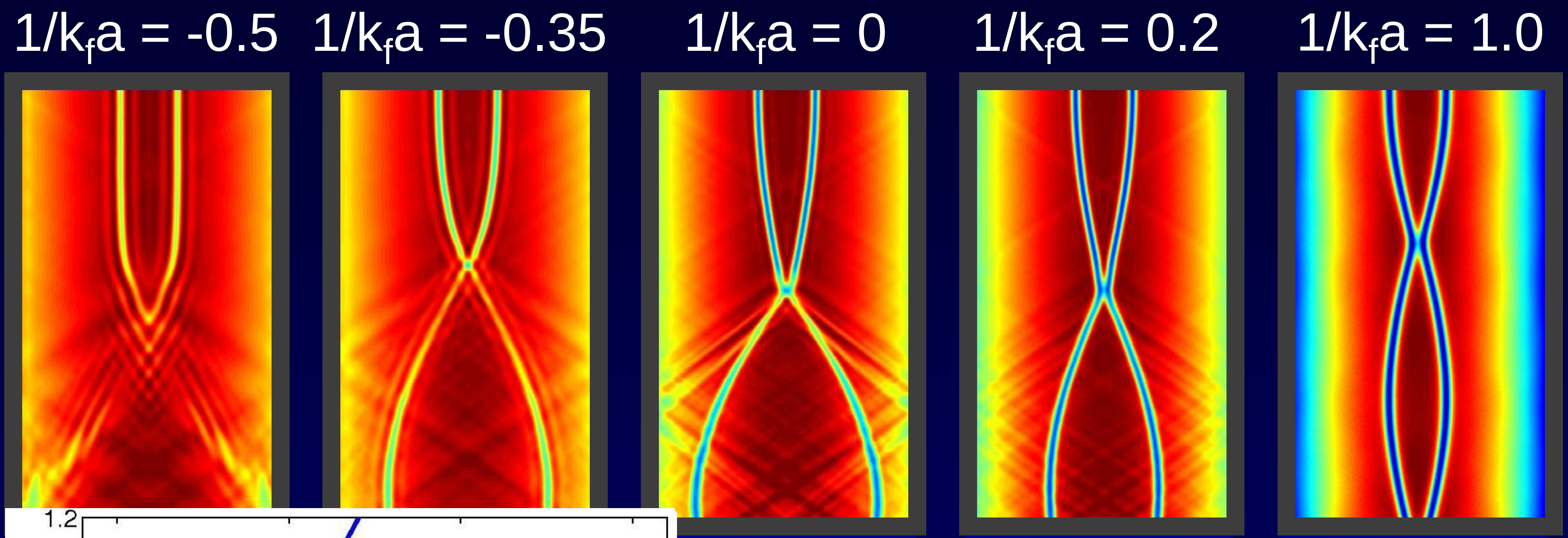
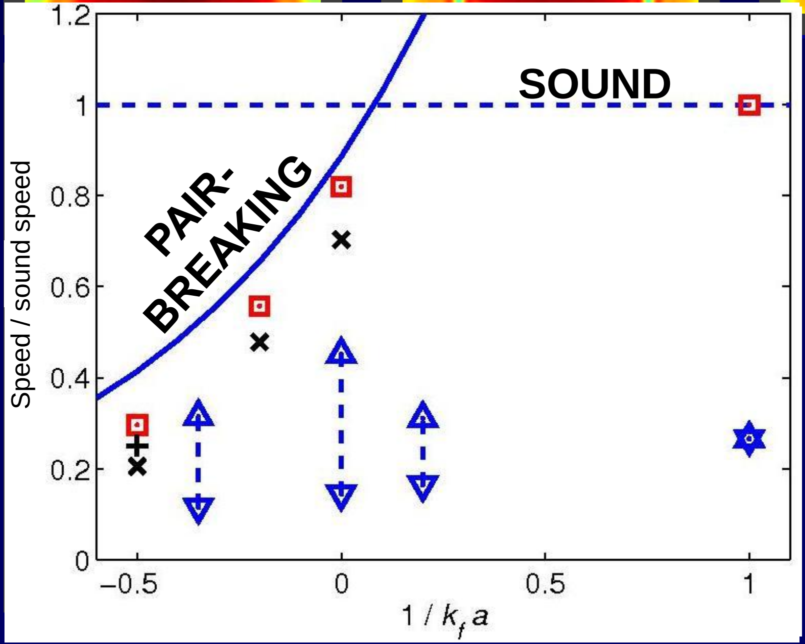


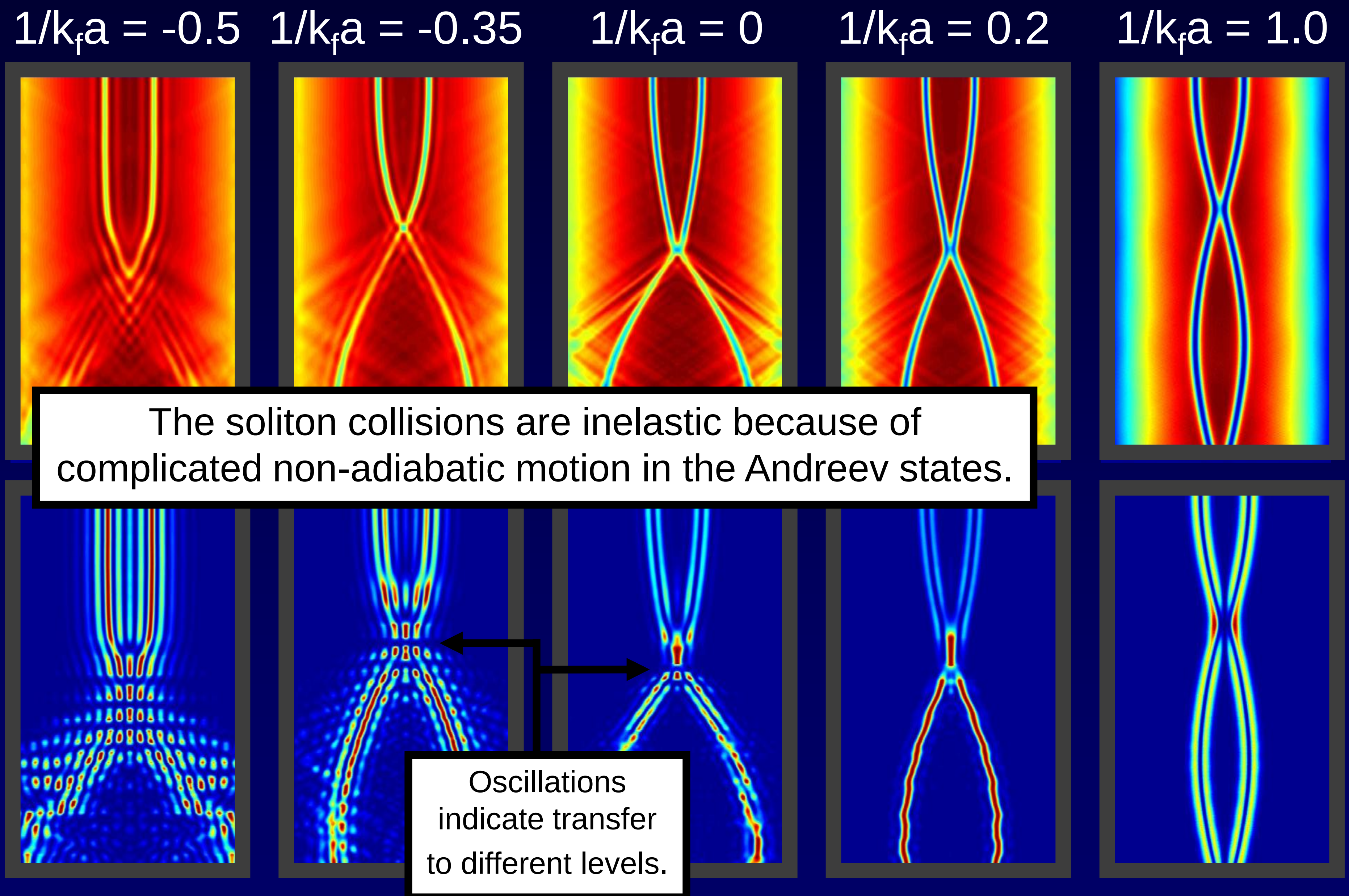


Joachim’s results: ◻ (uniform gas).

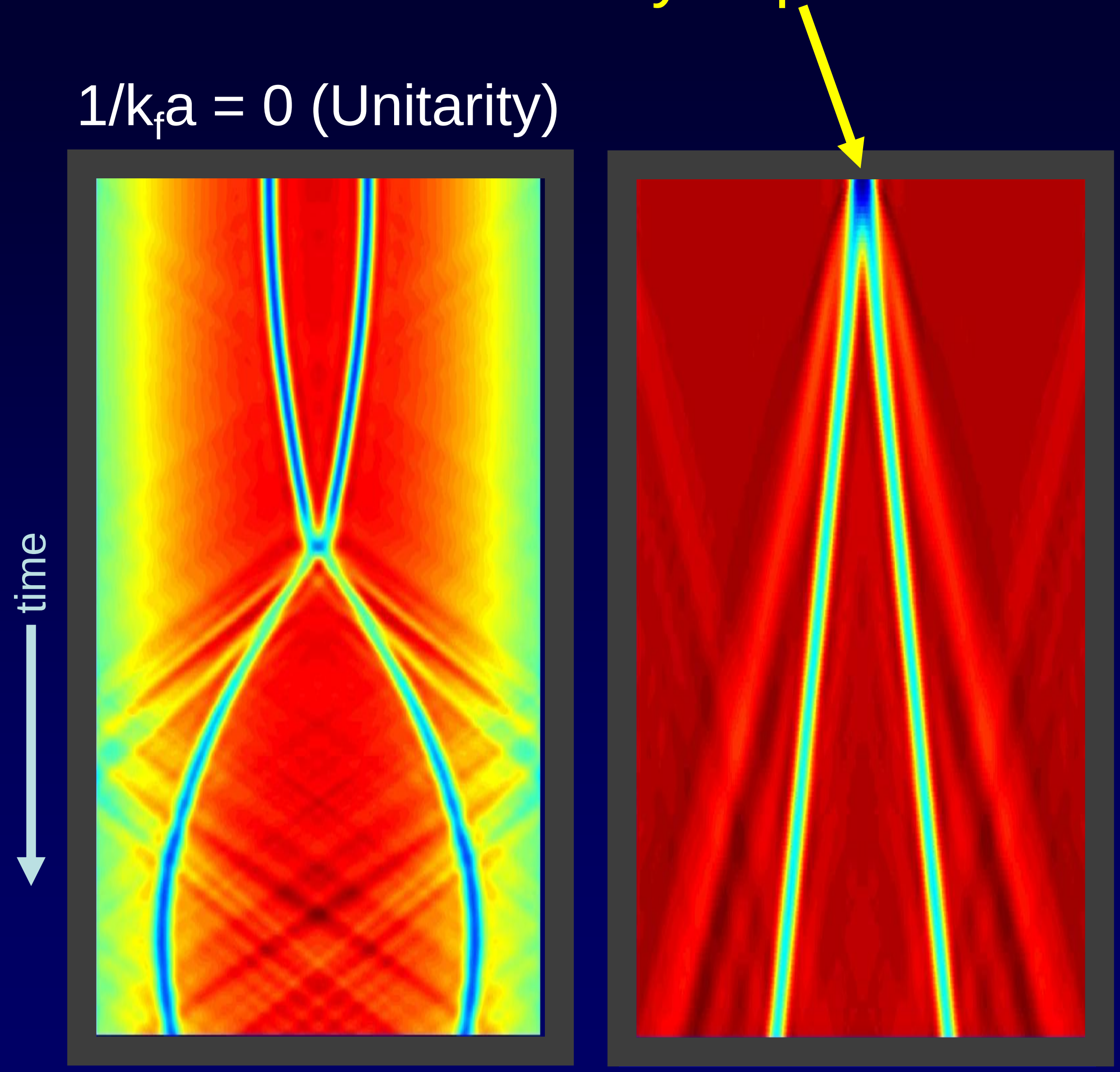
My results: × + (trapped gas).

Speed before and after collision: ▽ △

Soliton collisions in a trap across the crossover



Soliton production from a density imprint



Soliton production from a density imprint

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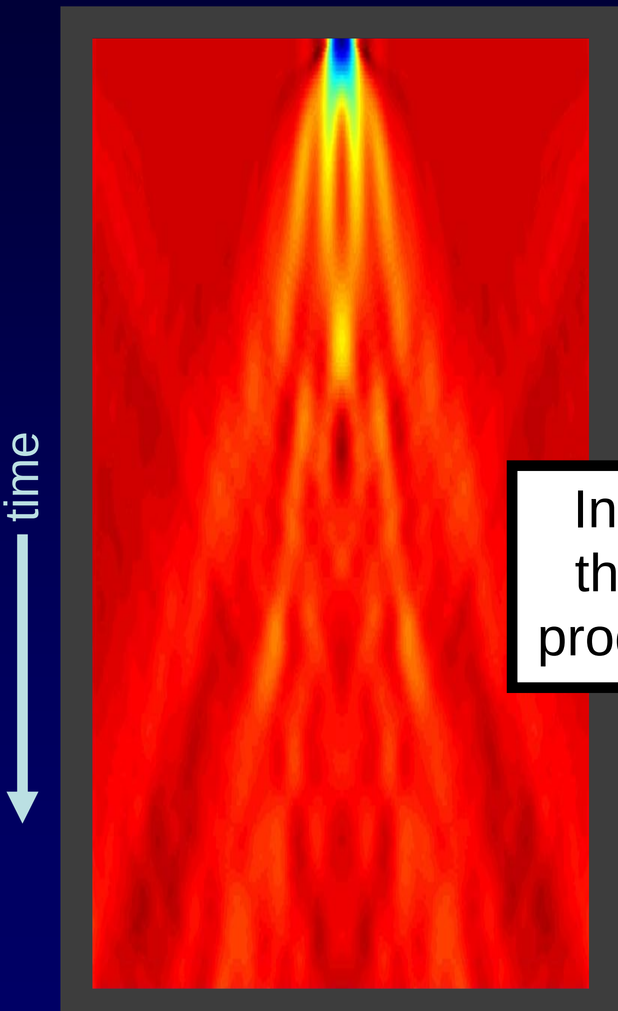
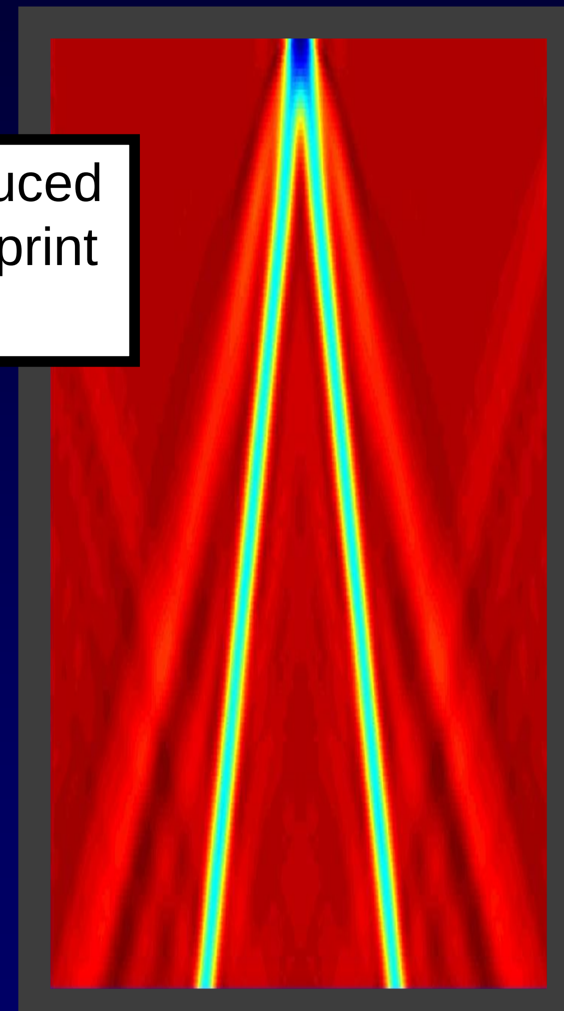


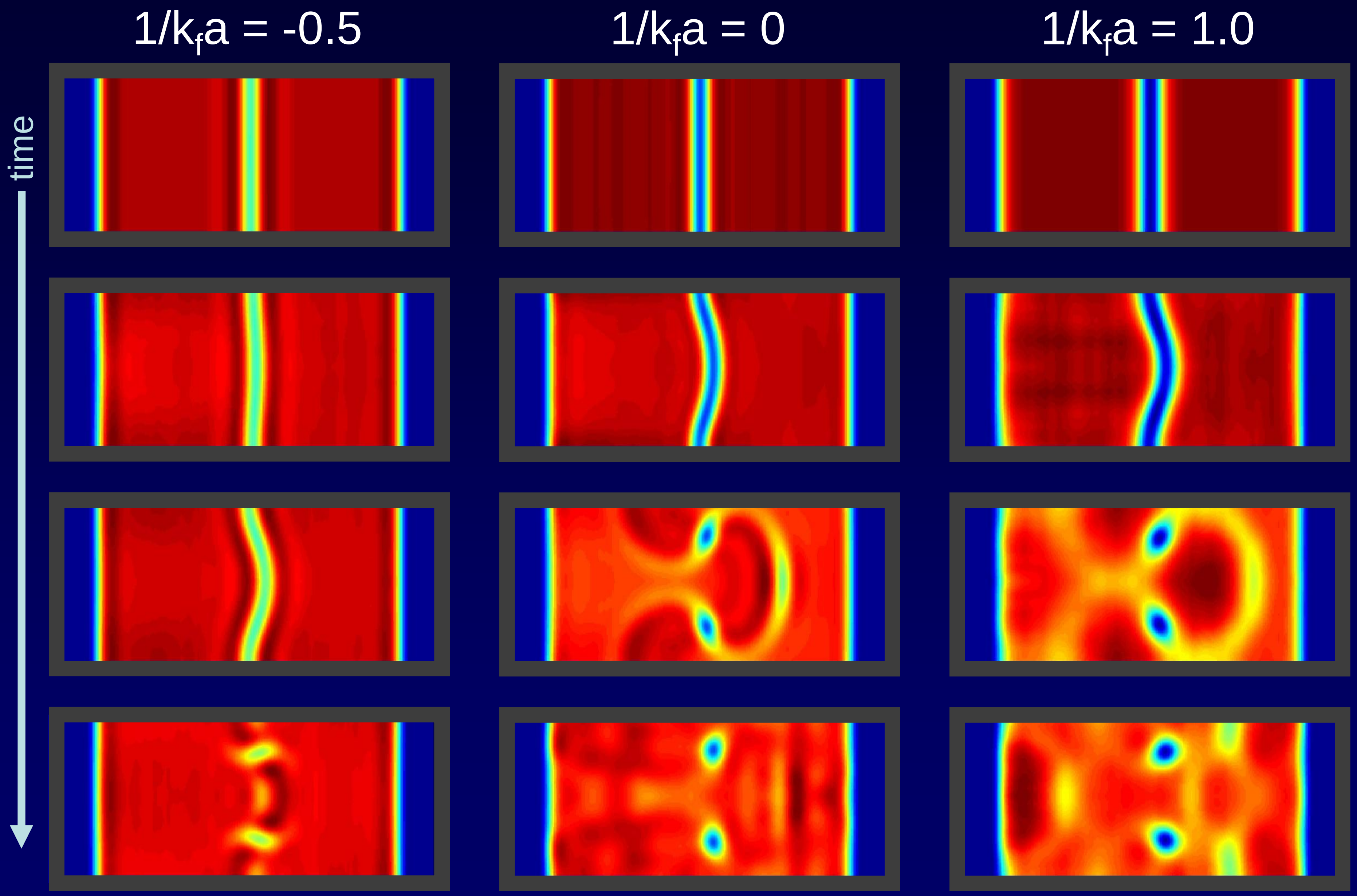
$1/k_f a = 0$ (Unitarity)



Solitons are produced from a density imprint at unitarity.

In the BCS regime the density imprint produces only sound.

2D simulations: the snake instability of the soliton



Conclusions

- Analytic expression - We have derived an general analytic expression for the soliton period. This expression contains only quantities that can be directly measured in experiment.

- Soliton period - This analytic prediction and numerical simulation show that the soliton period increases dramatically as the soliton becomes shallower on the BCS side of the resonance.

- Soliton decay - The soliton decays if it is accelerated above the pair-breaking velocity.

- Soliton collisions - Soliton collisions are only elastic in the BEC limit, and may destroy solitons in the BCS regime. This suggests that solitons will less easily created in the BCS regime, and hence will be less influential in the dynamics.

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