Spontaneous pattern formation in exciton-polariton condensates

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Outline

- Introduction: Exciton–polariton condensates
- Model $\rightarrow$ Experimental realisation
- Experiment $\rightarrow$ Model
- Nonequilibrium condensate in a parabolic trap
- Controllable half-vortex lattices
- Turbulence

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Acknowledgements

Theory:

Magnus Borgh
Southampton University

Jonathan Keeling
St Andrews University

Experiment: NanoPhotonics group

Jeremy Baumberg

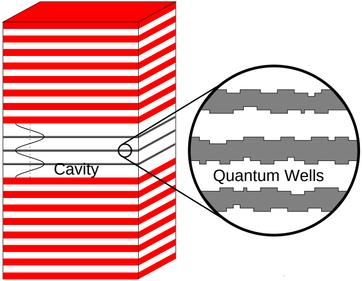
Gabriel Christmann

Guilherme Tosi

CLERMONT4: Exciton-polaritons: Physics and Applications

Absorption of photon by semiconductor ⇒ exciton ⇒ emitting photon ⇒ mirrors ⇒ exciton photon superposition ⇒ polariton $m_{\mathrm{pol}} = 10^{-4} m_e$ ⇒

BEC expected at "high" temperature!



Absorption of photon by semiconductor $\Rightarrow$ exciton $\Rightarrow$ emitting photon $\Rightarrow$ mirrors $\Rightarrow$ exciton photon superposition $\Rightarrow$ polariton $m_{\text{pol}} = 10^{-4} m_e$ $\Rightarrow$

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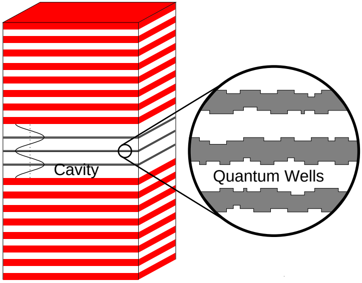
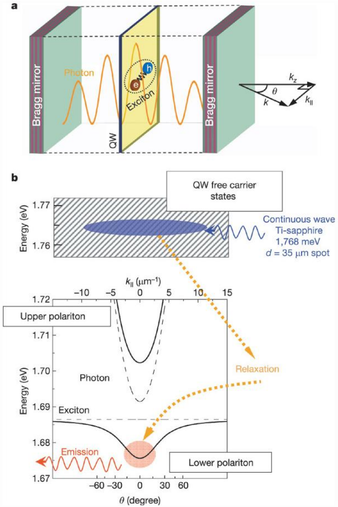




Properties of exciton-polaritons

- **polariton-polariton interactions:**
  interactions between charged particles, saturation of the exciton-photon interactions, electron-electron exchange;
  for low densities pseudo-potential $U(\mathbf{r}) \to U\delta(\mathbf{r})$;
  typical scale of U is 10^{-3} meVμm^2.
- **short lifetime (5-10 ps):**
  (i) non-equilibrium condensate (ii) helps image the properties.
  $ck = E_{\vec{k}}^{\mathrm{LP,UP}}\,\sin(\theta)$, therefore, refer to polariton momentum, wavevector or emission angle θ interchangeably.
- **two polarisation states:**
  left- and right-circularly polarised photon states;
- **coupling** between mechanical strain in the sample and the energy of electron and hole breaks symmetry and favours a particular linear polarisation.

Superfluidity checklist

Table 1 | Superfluidity checklist

	Quantized vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydro-dynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✓	✓	✓	✓	✓	✓
Non-interacting Bose-Einstein condensate	✓	X	X	X	✓	X
Classical irrotational fluid	X	✓	X	✓	✓	✓
Incoherently pumped polariton condensates	✓	X	?	?	X	?

[J. Keeling and NGB, N & V, Nature (2009)]

[Keeling & NGB, PRL, **100**, 250401 (2008)]
Equation for the macroscopically occupied polariton state $\Psi(\mathbf{r}, t)$:

$$i\hbar\partial_t\Psi = \left[E(i\nabla) + U|\Psi|^2 + V(\mathbf{r})\right]\Psi + i\left[P_{\text{coh}}(\mathbf{r}, t) + \left(P_{\text{inc}}(\mathbf{r}) - \kappa - \sigma|\Psi|^2\right)\Psi\right.$$

Polariton dispersion, $E(k)$ (eg. a quadratic dispersion $E(k) \simeq \hbar^2 k^2/2m_{\text{pol}}$);
Strength of the $\delta-$function interaction (pseudo)potential U;
External potential $V(\mathbf{r})$;
Coherent pump field $P_{\text{coh}}(\mathbf{r})e^{i\omega_p t}$;
Incoherent pump field $P_{\text{inc}}(\mathbf{r})$;
κ and σ describe linear and nonlinear losses respectively.

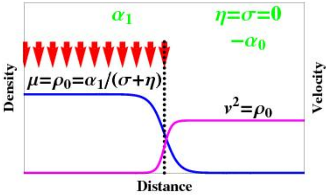
cf. "generic laser model" of Wouters and Carusotto PRA (2007)

Vortex formation in equilibrium condensates:

- interactions of finite amplitude sound waves;
- existence of critical velocities of the flow;
- modulational instabilities.

In addition in nonequilibrium condensates – pattern forming, interaction of fluxes with a disorder etc.

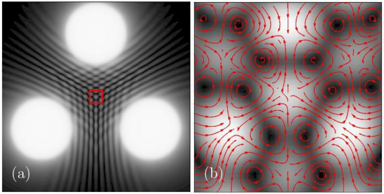
Vortex formation due to interference of supercurrents



Analytical solution for the velocity $u(r)$ on $\infty < r < \infty$.

Theory:

[Keeling and NGB, arXiv:1102.5302]

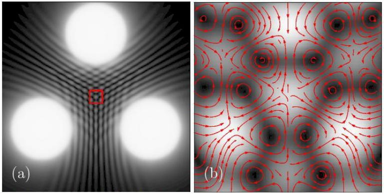


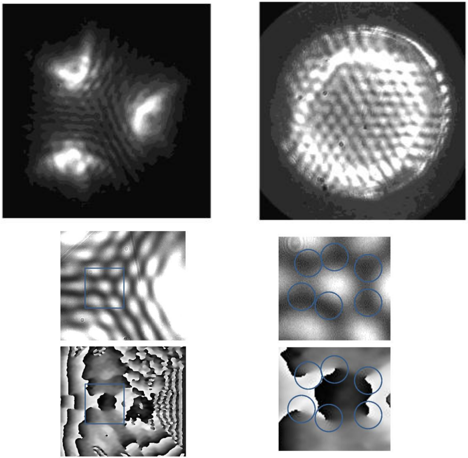
Pumping in three equidistant spots

Theory:

Experiment:

[Keeling and NGB, arXiv:1102.5302]





Quantum fluid pendulum

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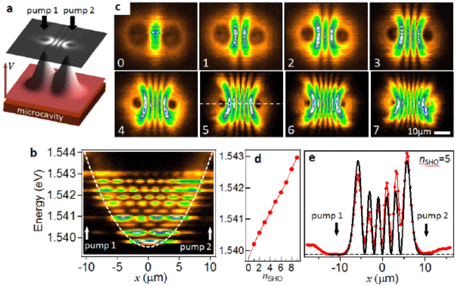

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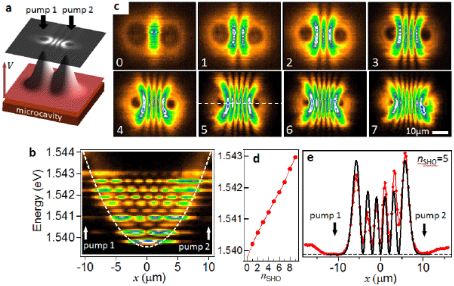

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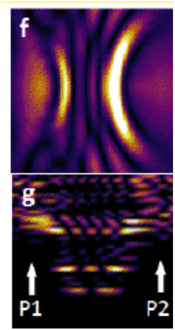
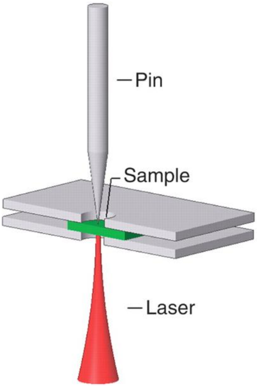
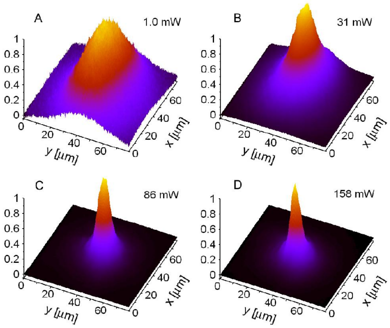


[Balili et al Science **316**,(2007)]:
A harmonic trapping potential is created by squeezing the sample by a sharp pin.





Signatures of BEC:
spatial and spectral narrowing; coherence

Mean-field model of a non-equilibrium BEC of exciton-polaritons

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}} + U|\psi|^2 + i(\gamma_{\text{net}} - \Gamma|\psi|^2)\right]\psi,$$

V_{ext} is an external trapping potential, $= \frac{1}{2}m\omega^2 r^2$, γ_{net}– net gain, Γ – effective loss, U – effective (pseudo-) interaction potential.

Length in units of oscillator length $\sqrt{\hbar/m\omega}$, energies in units of $\hbar\omega$, and $\psi \to \sqrt{\hbar\omega/2U}\psi$, yields:

$$i\partial_t\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha - \sigma|\psi|^2\right)\right]\psi.$$

Two parameters: $\alpha = 2\gamma_{\text{net}}/\hbar\omega$ (gain), and $\sigma = \Gamma/U$ (loss).
Estimate from experiments: $0 \le \alpha \le 10$ and $\sigma \sim 0.3$

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α not too large, Thomas-Fermi solution $|\psi|^2 = (\mu - r^2)$ for $r < r_{TF} = \sqrt{\mu}$

$\int d^2r\,(\alpha - \sigma|\psi|^2)\,|\psi|^2 = 0 \Rightarrow \mu = 3\alpha/2\sigma$.

Madelung transformation, $\psi = \sqrt{\rho}e^{i\varphi}$:

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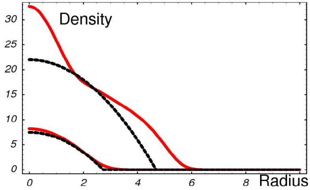

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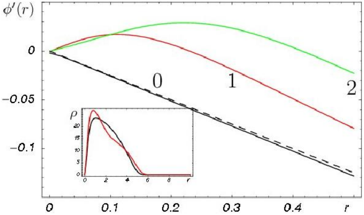

$$\psi = f(r)\exp[is\theta + i\phi(r)]$$

Leading order
$\phi'(r) \sim \alpha/2(s+1)r$.

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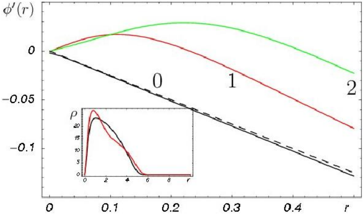

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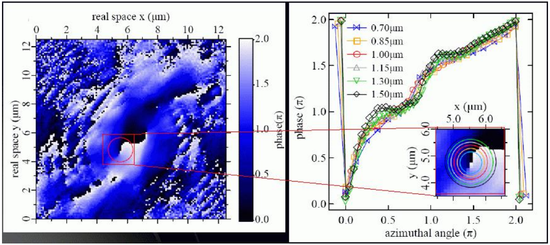
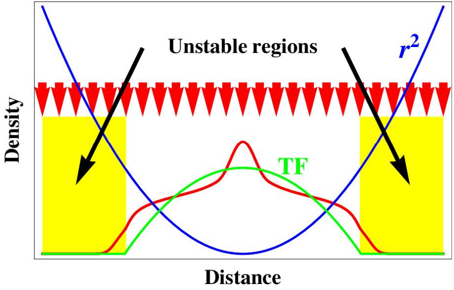
Experiment:

[Lagoudakis et al. Nature Physics (2008)]





$$\frac{1}{2}\partial_t \rho + \nabla \cdot [\rho \mathbf{v}] = (\alpha - \sigma\rho)\,\rho, \qquad \partial_t \mathbf{v} + \nabla(\rho + r^2 + |\mathbf{v}|^2) = 0$$

If α, σ small, find normal modes in 2D trap: $\delta\rho_{n,m} = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$
$\omega_{n,m} = 2\sqrt{m(1+2n) + 2n(n+1)}$.
Add weak pumping and decay

$$\omega_{n,m} \to \omega_{n,m} + i\alpha\left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2}\right]$$

Finite Spot Size

In experiments: finite spot, of size comparable to observed cloud, is used.

Model this as $\alpha = \alpha(r) = \alpha\,\Theta(r_0 - r)$

For small r_0 ($r_0 < r_{TF} \sim \sqrt{3\alpha/2\sigma}$), this stabilises the radially symmetric modes and vortex modes:

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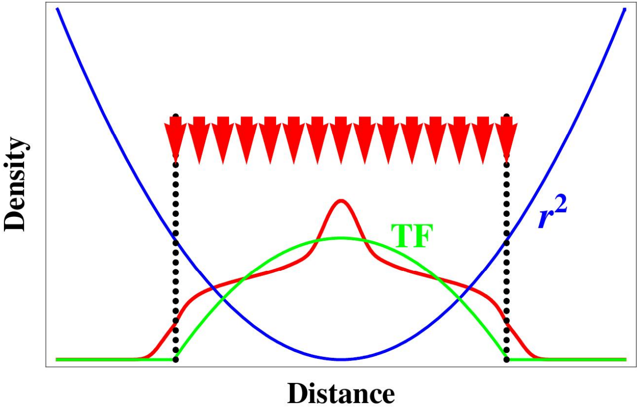
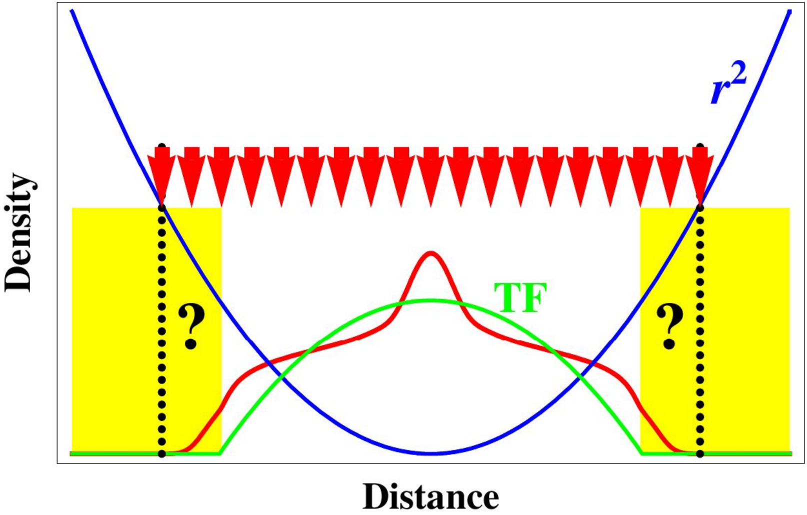
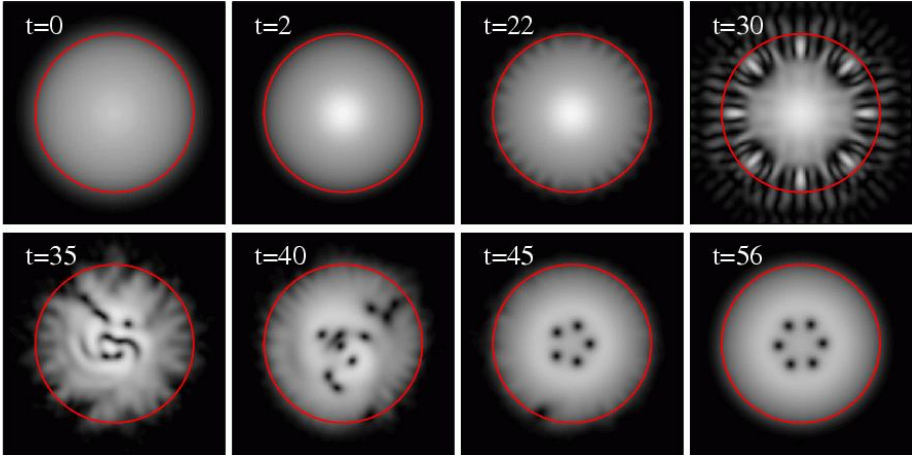
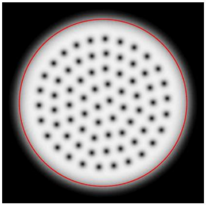
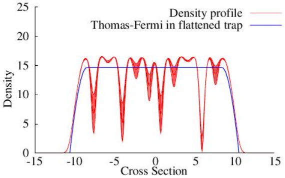



Density
r^2
?
?
TF
Distance

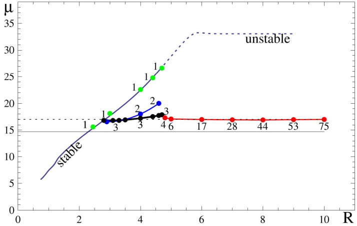
Vortex Lattices

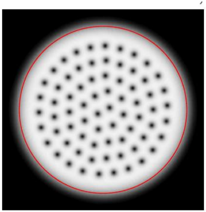


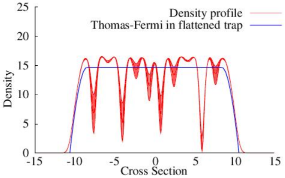




Stationary $\mu \sim 3\alpha/2\sigma$; Vortex lattice $\mu \sim \alpha/\sigma$







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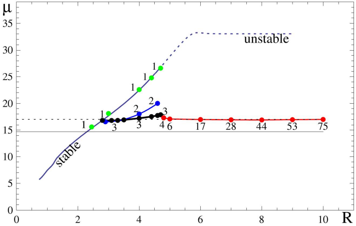

In rotating frame

$$\nabla \cdot [\rho(\nabla\phi - \Omega \times \mathbf{r})] = (\alpha\Theta(r_0 - r) - \sigma\rho)\,\rho,$$

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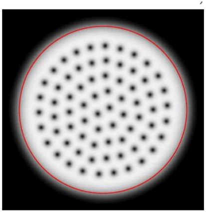
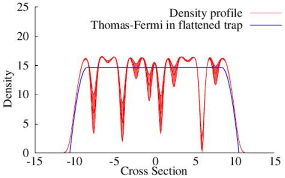
In TF regime away from boundaries solution is
$\nabla\phi = \Omega \times \mathbf{r} + v.c., \rho = \alpha/\sigma = \mu, \Omega^2 = 1.$





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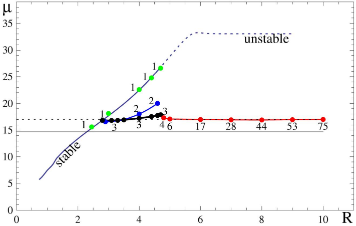

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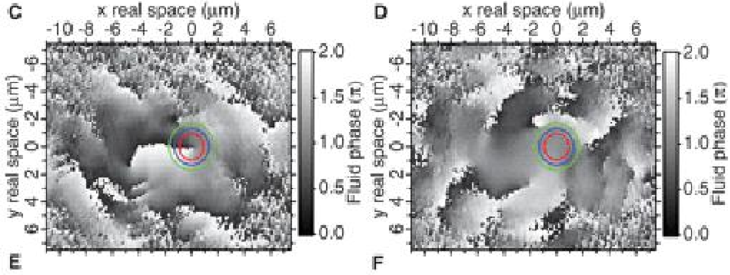
Results so far do not involve polariton spin:
[Lagoudakis et al, Science, November 2009]:
Phase maps of left- and right-circular polarized polariton states



Observed all possible $(\pm 1, \pm 1)$ vortex states.

[Borgh, Keeling, NGB, PRB, **81**, 235302 (2010)]

- Include spin degree of freedom: left- and right-circular polariton states ψ_L and ψ_R.
- For weakly-interacting dilute Bose gas model:

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- Tendency to biexciton formation $\rightarrow U_1$. Magnetic field: Ω_B
- J_2 Circular symmetry broken – two equivalent axes.
  J_1 preferred direction – inequivalent axes.

Polariton spin degree of freedom

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&+ J_1\left(\psi_L^\dagger\psi_R + H.c\right) + J_2\left(\psi_L^\dagger\psi_R + H.c.\right)^2
\end{aligned}
$$

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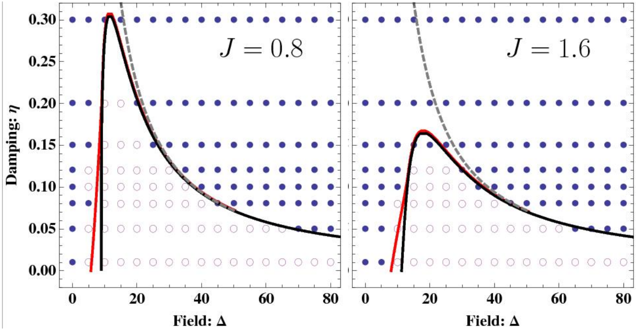
[J. Keeling and NGB, arXiv:1102.5302 (2011)]
Vortex patterns generated by superposition of fluxes.
Spinor complex Ginzburg-Landau equation:

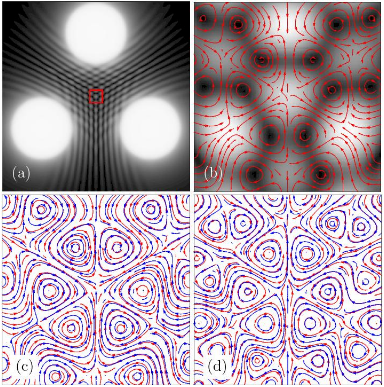
$$2i\partial_t\psi_{l,r} = \left[\pm\frac{\Delta}{2} - \nabla^2 + v(r) + |\psi_{l,r}|^2 + (1-u_a)|\psi_{r,l}|^2 \right. \\ \left. + i\left(\alpha - 2i\eta\partial_t - \sigma|\psi_{l,r}|^2 - \tau|\psi_{r,l}|^2\right)\right]\psi_{l,r} + J\psi_{r,l}.$$

η – energy relaxation [Wouters and Savona arXiv:1007.5431 (2010)];
τ – cross-spin nonlinear dissipation;
Δ – effect of the magnetic fied;
J – electric field, stress or due to asymmetry of quantum well interfaces;
Parameters estimated from [Larionov et al, PRL, **105**, 256401 (2010)]

For nonzero η there is a second transition at Δ_{c2} back to synchronized state, $\Delta_{c2} \simeq (2\alpha/\eta)(\sigma - \tau + \eta u_a)/(\sigma + \tau + \eta(2 - u_a))$ (dashed line)

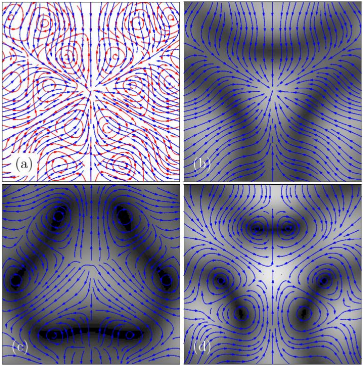


• –synchronized states (vortex-free states or synchronized vortices);
○– desynchronized states (vortices of opposite sign for l and r).
Conclude: homogeneous model gives good prediction of spatially varying system.



(a) $\Delta = 0$ showing geometry of pumping;
(b) Desynchronized: steady majority density with streamlines;
(c) Lower synchronized: steamlines of both polarizations;
(d) Upper synchronized: steamlines of both polarizations.

"Half-vortices" have been seen in experiments:
[Lagoudakis et al Nature Phys. (2008)]
Are "half-vortices" pinned and stabilized by disorder?



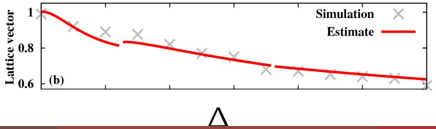
(a) Desyncronized: half-vortex lattice;
(b) -(c) -(d) evolution of minority component in desyncronized regime.

Majority component is stationary in both regimes;
Minority component is stationary in syncronized regime only.
In desyncronized regime averages to vortex-free state.

Currents are neglible at the pumping centre, $\mu(\rho_{l,r})$;
away from pumping spot – densities are neglible.

Synchronized regime: away from the pump
$\mu - |\vec{u}|^2 \mp \Delta/2 = J\,(\rho_l/\rho_r)^{\mp 1/2}\cos(\theta)$ and
$\nabla\cdot(\rho_{l,r}\vec{u}) + \alpha_1\rho_{l,r} = \mp J\sqrt{\rho_l\rho_r}\sin(\theta)$.
These are solved by $\sin(\theta) = 0$ and $\nabla(\rho_l/\rho_r) = 0$,
so $|\vec{u}|^2 = \mu + \sqrt{J^2 + \Delta^2/4}$.

Desynchronized regime: θ and ρ_l/ρ_r are not time independent, so one calculates averages. If $\rho_r \gg \rho_l$, then for majority component
$\langle|\vec{u}_r|^2\rangle = \langle\mu_r\rangle + \Delta/2$.
Superposition of such currents results in hexagonal vortex lattice with spacing $l = (2\pi/|\vec{u}|)\times 2/3\sqrt{3}$.



Classical Turbulence

In 50th Batchelor wrote to his friend and close colleague, Alan Townsend, who remained in Australia:

You will come to Cambridge, study turbulence, and work with G. I. Taylor.

The answer came immediately: *I agree, but I have two questions:*

who is G. I. Taylor and ... **what is turbulence?**

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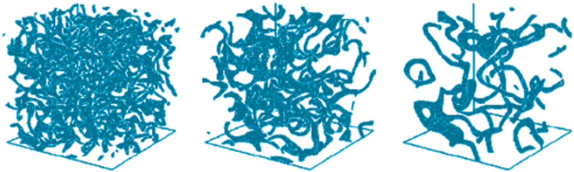
Turbulence

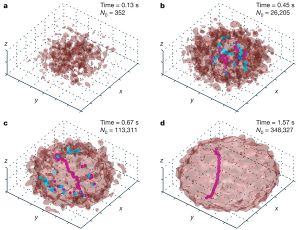
Classical turbulence – cascading vorticity;
Superfluid turbulence – quantisation of velocity circulation – differences with classical turbulence;
Strong turbulence– unstructured vortices (distance between vortices of the order of their core);
Weak turbulence regime – almost independent motion of weakly interacting dispersive waves.

Stages in condensate formation from a nonequilibrium state: [Berloff & Svistunov Phys Rev A (2002)]

weak turbulence → strong turbulence → superfluid turbulence → condensate



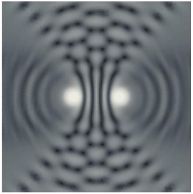
Vortex formed during nonequilibrium kinetics of BEC
[Weiler et al. Nature (2008)]



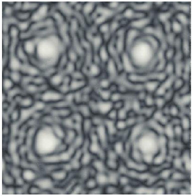
Reverse the process going from condensate to weak turbulent state?
[Henn at el PRL (2009)]: applied an external oscillatory perturbation to produce vortices.

[N.G.Berloff, arXiv:1010.5225 (2010)]

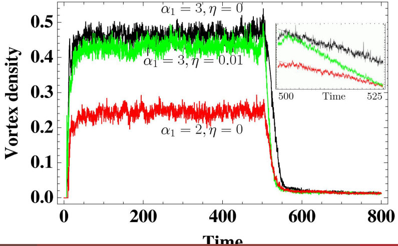
Regular emission of vortices



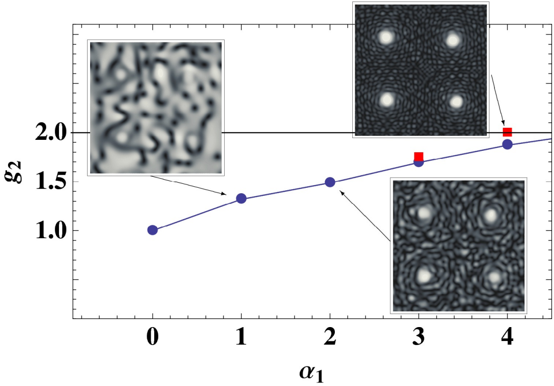
Many irregular spots: turbulence



Two regimes: forced turbulence and turbulence decay.



In forced turbulence it is possible to reach a weak turbulence state: $g_2 = \langle|\psi|^4\rangle/\langle|\psi|^2\rangle^2$. Weak turbulence implies $g_2 \sim 2$.



Red Squares – nonzero η facilitates the transition to weak turbulence.

- Nonequilibrium condensates: condensates made of light
  - Gross-Pitaevskii equation with loss and gain

$$\mathrm{i}\partial_t\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi|^2\right)\right]\psi.$$

  - Radially symmetric stationary states: narrowing of density profile
  - Spiral vortex states
  - Vortex lattices

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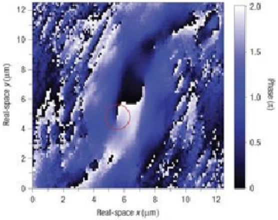

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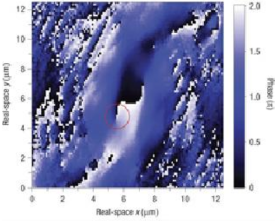

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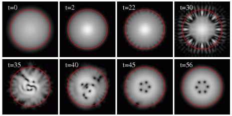
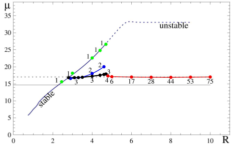




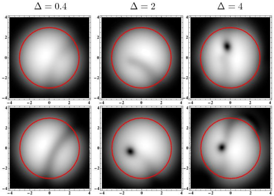
Conclusions-2

- Non-equilibrium spinor system

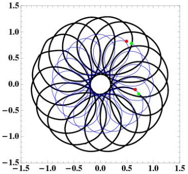
$$i\partial_t\psi_L = \Big[-\nabla^2 + V(r) + \frac{\Delta}{2} + |\psi_L|^2 + (1-u_a)|\psi_R|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi_L|^2\right)\Big]\psi_L + J\psi_R$$

- Effect of Δ and J on vortices.

Densities of L and R components for $J = 1$



Trajectories for $\Delta = 4$



Spirographs (epitrochoids/hypotrochoid)

- Synchronization/desynchronization with the region of bistability.

Conclusions-3

- Turbulence in exciton-polariton condensates may lead to novel regimes of turbulence of classical matter field.
  - The regimes can be distinguished by finding second order correlation function.
  - What are the stages in transition from strong turbululence to weak turbulence and back?
- Spinor condensates: predictions of homogeneous model (syncronization/desynchronization) are not significantly modified by spatial inhomogeneity.
  - Observation of the experimental behaviour in an applied field can thus be used to distinguish the loss nonlinearities σ, τ and η.
  - Vortices, vortex lattices and half-vortex latices in spinor condensates. Being stationary these textures can be studied experimentally.
- Turbulence in spinor condensates.

Scaling laws? Cross-overs of different regimes? Interplay between turbulent regimes and the effects of magnetic field?...