Sauter-Schwinger like tunneling in the Bose-Hubbard model

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Dirac Sea

Schrödinger equation
(non-relativistic)

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi + V\psi \quad \rightsquigarrow \quad \mathcal{E} = \frac{p^2}{2m} + V$$

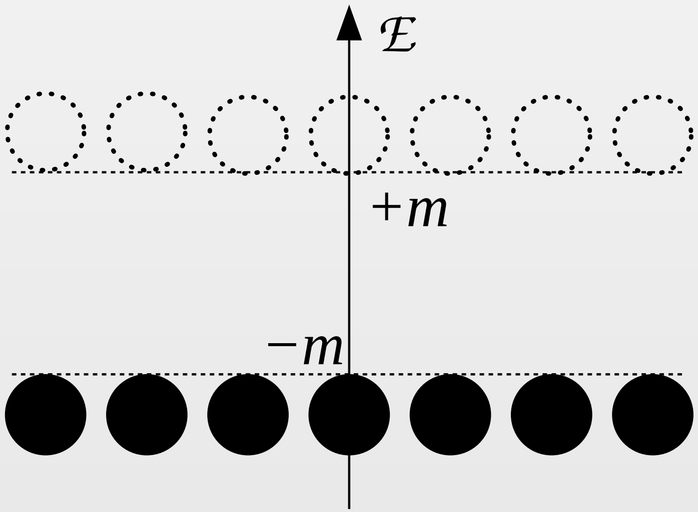
Dirac equation (relativistic)

$$\gamma^\mu\left(i\hbar\partial_\mu + qA_\mu\right)\Psi = mc\Psi \;\rightsquigarrow\; \mathcal{E} = V \pm \sqrt{c^2p^2 + m^2c^4}$$

$\longrightarrow$ positive and negative
energy levels!

$\longrightarrow$ filled up in vacuum
(Pauli principle)

$\longrightarrow$ holes: positrons
(prediction!)



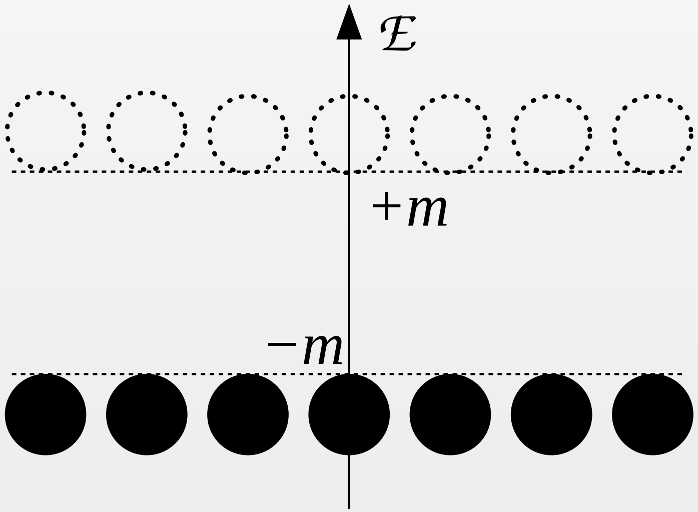
Sauter-Schwinger Effect

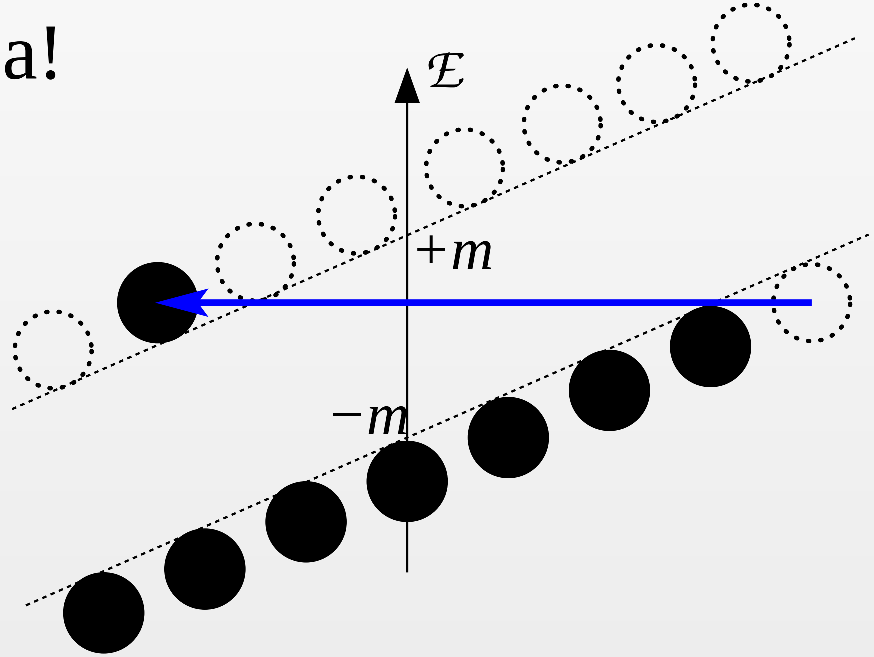
Constant electric field E

$\rightarrow$ potential $V(x) = qEx$

$\rightarrow$ tilt of level spectrum

$\rightarrow$ tunneling from Dirac sea!

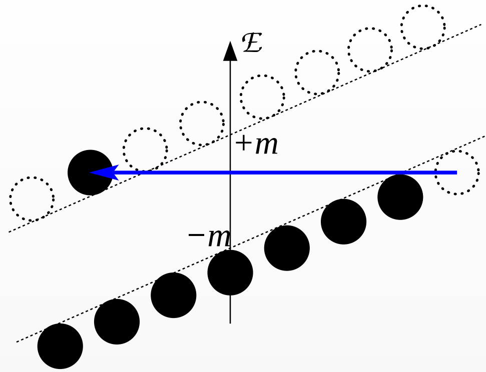




Creation of e^+e^- pairs out of the vacuum

$$P_{e^+e^-} \propto \exp\left\{-\frac{L\Delta V}{\hbar c}\right\} \propto \exp\left\{-\frac{2mc^2}{qE}\frac{\mathcal{O}(mc^2)}{\hbar c}\right\}$$

Critical Field Strength



Exact calculation yields tunneling exponent

$$P_{e^+e^-} \propto \exp\left\{-\pi \frac{c^3}{\hbar}\frac{m^2}{qE}\right\} = \exp\left\{-\pi \frac{E_S}{E}\right\}$$

Non-perturbative QED vacuum effect (→ difficult…)

Critical field strength (QED birefringence etc.)

$$E_S = \frac{c^3}{\hbar}\frac{m^2}{q} \approx 1.3 \times 10^{18}\,\mathrm{V/m}$$

Corresponds to intensity $I_S = \mathcal{O}(10^{29}\mathrm{W/cm^2})$

Planned ultra-strong lasers $I = \mathcal{O}(10^{26}\mathrm{W/cm^2})$

Enhancement by time-dependent fields, e.g.,

R. S., H. Gies, G. Dunne, Phys. Rev. Lett. **101**, 130404 (2008);

G. Dunne, H. Gies, R. S., Phys. Rev. D **80**, (rapid comm.) 111301 (2009)

Bose-Hubbard Model

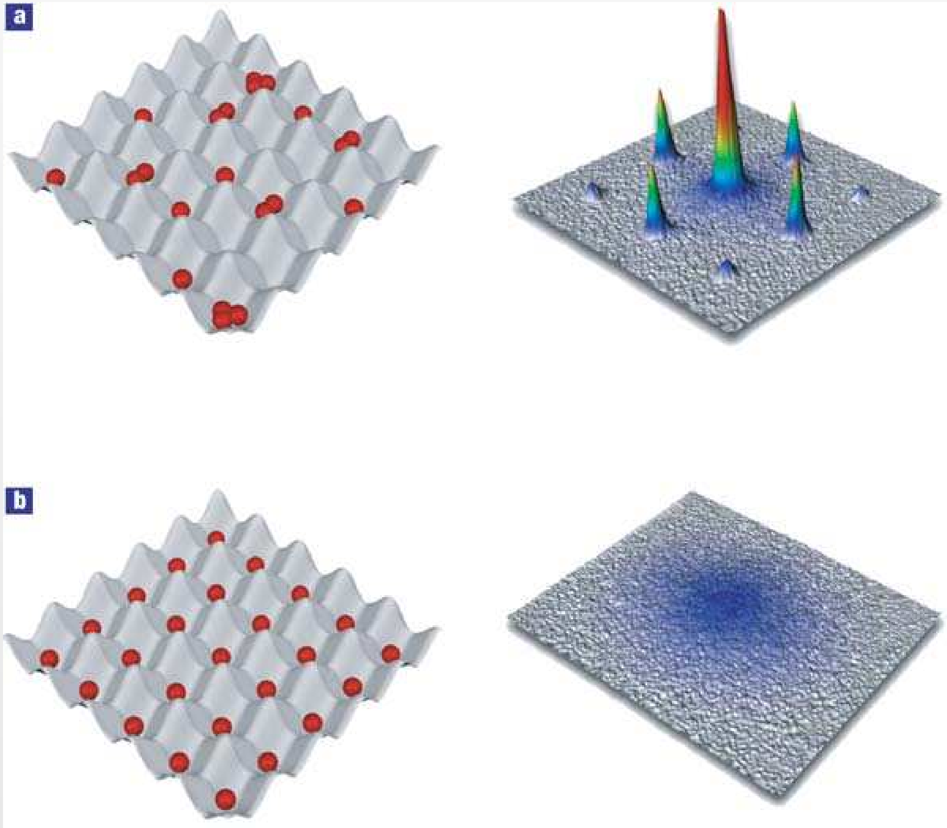
Atoms in optical lattice (lattice sites μ, ν) $\hbar = 1$

$$\hat{H} = -\frac{J}{Z}\sum_{\mu\nu} T_{\mu\nu}\hat{a}^\dagger_\mu \hat{a}_\nu + \frac{U}{2}\sum_\mu (\hat{a}^\dagger_\mu)^2 \hat{a}^2_\mu$$

Hopping J dominates: super-fluid phase

Tunneling matrix $T_{\mu\nu}$ coordination number $Z = \sum_\nu T_{\mu\nu}$

Interaction U stronger: Mott insulator state $\rightarrow$ energy gap $\Delta\mathcal{E}$

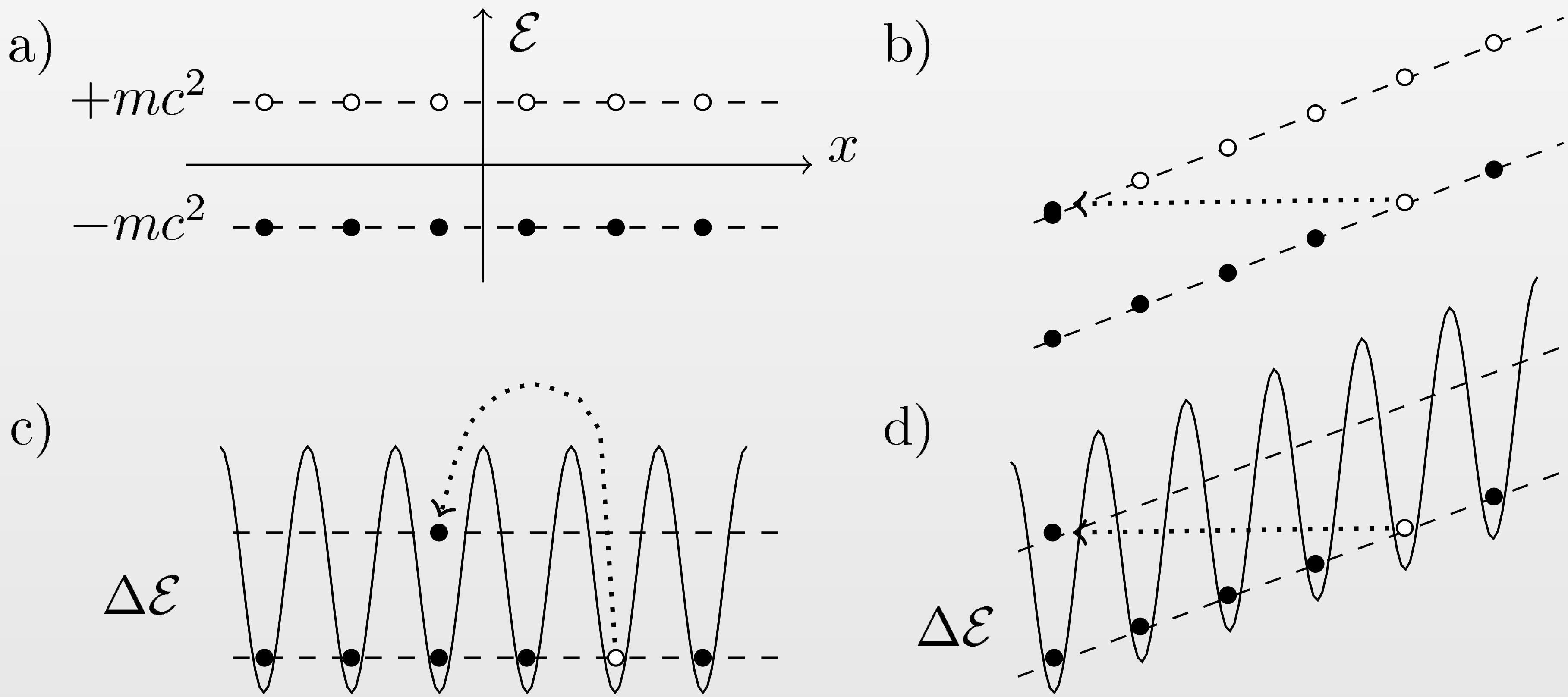


I. Bloch, Nature Physics **1**, 23 (2005)

Lattice with Small Tilt $\to V_\mu$

$$\hat{H} = -\frac{J}{Z}\sum_{\mu\nu} T_{\mu\nu}\hat{a}^\dagger_\mu \hat{a}_\nu + \frac{U}{2}\sum_{\mu}(\hat{a}^\dagger_\mu)^2\hat{a}^2_\mu + \sum_\mu V_\mu \hat{n}_\mu$$

Particle-hole pair creation via tunneling (Mott state)



Hierarchy of Correlations

P. Navez and R. S., Phys. Rev. A **82**, 063603 (2010)

Reduced density matrices (for lattice sites μ, ν)

$$\hat{\rho}_\mu = \mathrm{Tr}_{\not\mu}\{\hat{\rho}_{\mathrm{total}}\}\,,\ \hat{\rho}_{\mu\nu} = \mathrm{Tr}_{\not\mu\not\nu}\{\hat{\rho}_{\mathrm{total}}\}\,,\ \text{etc.}$$

Correlations

$$\begin{aligned}\hat{\rho}_{\mu\nu} &= \hat{\rho}^{\mathrm{corr}}_{\mu\nu} + \hat{\rho}_\mu\hat{\rho}_\nu \\ \hat{\rho}_{\mu\nu\lambda} &= \hat{\rho}^{\mathrm{corr}}_{\mu\nu\lambda} + \hat{\rho}^{\mathrm{corr}}_{\mu\nu}\hat{\rho}_\lambda + \hat{\rho}^{\mathrm{corr}}_{\mu\lambda}\hat{\rho}_\nu + \hat{\rho}^{\mathrm{corr}}_{\nu\lambda}\hat{\rho}_\mu + \hat{\rho}_\mu\hat{\rho}_\nu\hat{\rho}_\lambda\end{aligned}$$

Hierarchy of correlations for $Z \gg 1$ (coordination #)

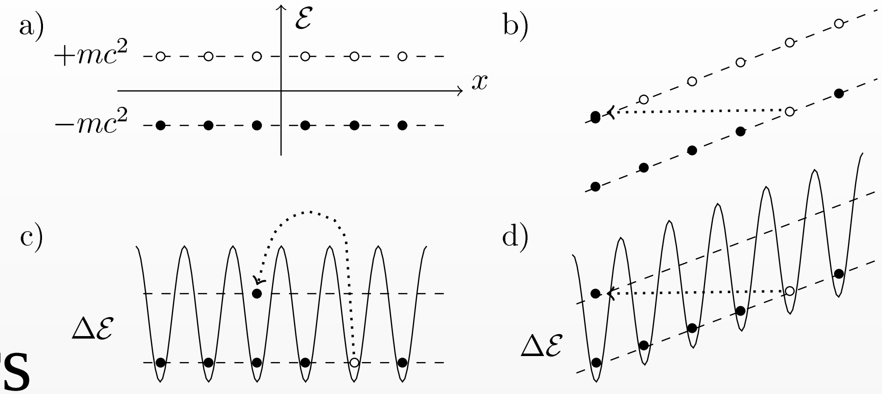
$$\hat{\rho}_\mu = \mathcal{O}(1)\,,\ \hat{\rho}^{\mathrm{corr}}_{\mu\nu} = \mathcal{O}(1/Z)\,,\ \hat{\rho}^{\mathrm{corr}}_{\mu\nu\lambda} = \mathcal{O}(1/Z^2)\,,\ \text{etc.}$$

Time-evolution

$$\begin{aligned}\partial_t\hat{\rho}_\mu &= f(\hat{\rho}_\mu, \hat{\rho}^{\mathrm{corr}}_{\mu\nu}) \to \tilde{f}(\hat{\rho}_\mu) + \mathcal{O}(1/Z) \\ \partial_t\hat{\rho}^{\mathrm{corr}}_{\mu\nu} &= g(\hat{\rho}_\mu, \hat{\rho}^{\mathrm{corr}}_{\mu\nu}, \hat{\rho}^{\mathrm{corr}}_{\mu\nu\lambda}) \to \tilde{g}(\hat{\rho}^0_\mu, \hat{\rho}^{\mathrm{corr}}_{\mu\nu}) + \mathcal{O}(1/Z^2)\end{aligned}$$

$\to$ iterative solution: first $\hat{\rho}^0_\mu$, then $\hat{\rho}^{\mathrm{corr}}_{\mu\nu}$, etc.

Analogy



Particle $\hat{p}_\mu = |1\rangle_\mu\langle 2|$ and hole $\hat{h}_\mu = |0\rangle_\mu\langle 1|$ operators

$\rightarrow$ effective scalar field $\hat{\phi}(t, \boldsymbol{r}_\mu) = \hat{h}_\mu(t) + \hat{p}_\mu(t)$

$$\left[i\partial_t - V(t, \boldsymbol{r})\right]^2 \hat{\phi} = \left[m_{\text{eff}}^2 c_{\text{eff}}^4 - c_{\text{eff}}^2 \boldsymbol{\nabla}^2 + \mathcal{O}(\boldsymbol{\nabla}^4)\right] \hat{\phi}$$

Energy gap $\Delta\mathcal{E} = \sqrt{J^2 - 6JU + U^2} = 2m_{\text{eff}} c_{\text{eff}}^2$

Propagation speed $c_{\text{eff}}^2 \propto (3JU - J^2)$

Klein-Fock-Gordon equation in continuum limit!

Quantitative analogy – R. Feynman:

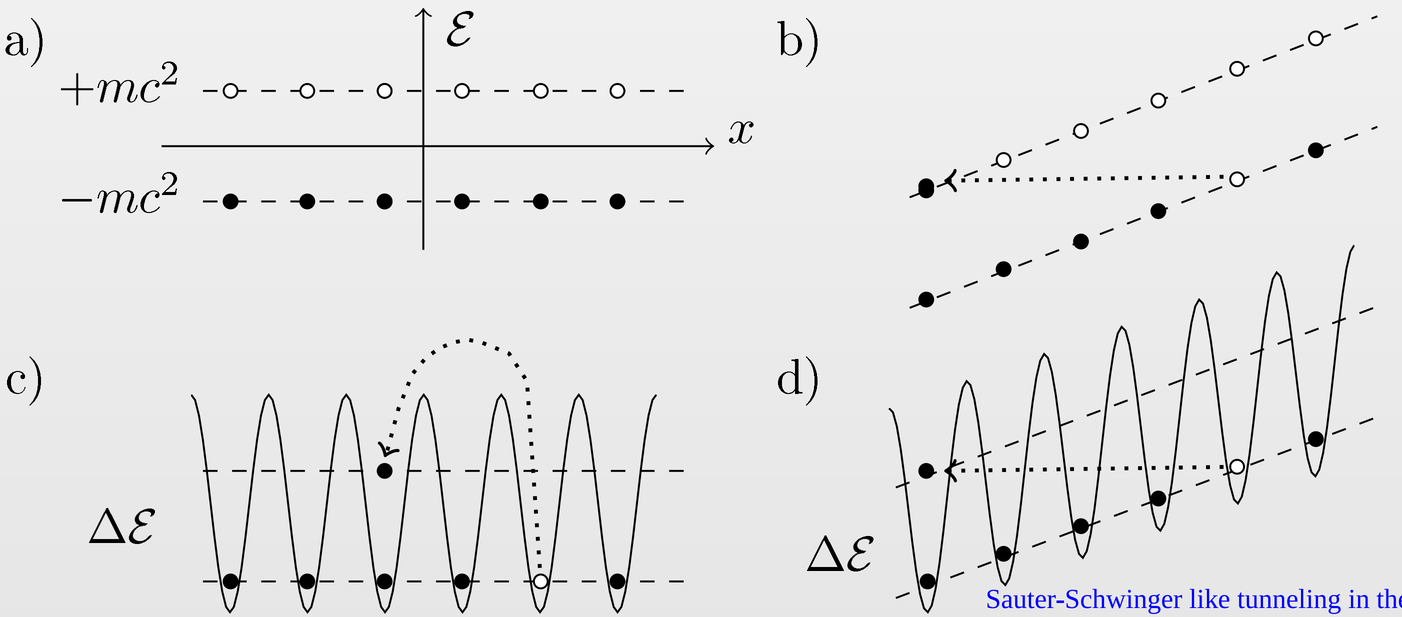
“*The same equations have the same solutions.*”

F. Queisser, P. Navez, R. S., `arXiv:1107.3730`

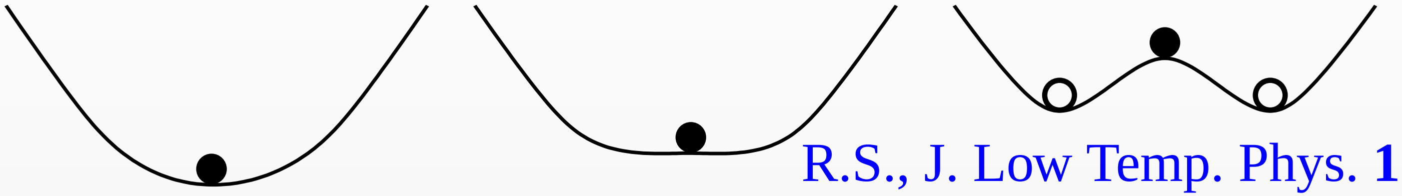
Summary

F. Queisser, P. Navez, R. S., `arXiv:1107.3730`

Sauter-Schwinger effect	Bose-Hubbard model
electrons & positrons	particles & holes
Dirac sea	Mott state
mass of electron/positron	energy gap $\Delta\mathcal{E}$
electric field $\boldsymbol{E}(t)$	lattice tilt $V_\mu(t)$
speed of light c	velocity $c_{\rm eff}$



Quantum Quenches



R.S., J. Low Temp. Phys. **153**, 228 (2008)

Mott $\longrightarrow$ superfluid (symmetry breaking)

$\longrightarrow$ exponential growth of fluctuations (initially)

$$\langle \hat{a}_\nu^\dagger(t)\hat{a}_\mu(t)\rangle \approx \mathcal{N}(t)\exp\left\{\gamma\sqrt{t^2-(\boldsymbol{r}_\mu-\boldsymbol{r}_\nu)^2/c_{\rm eff}^2}\right\}$$

P. Navez and R. S., Phys. Rev. A **82**, 063603 (2010)

with $c_{\rm eff}^2 \propto (3JU - J^2)$

and $\gamma^2 \propto (J - J_{\rm crit})$

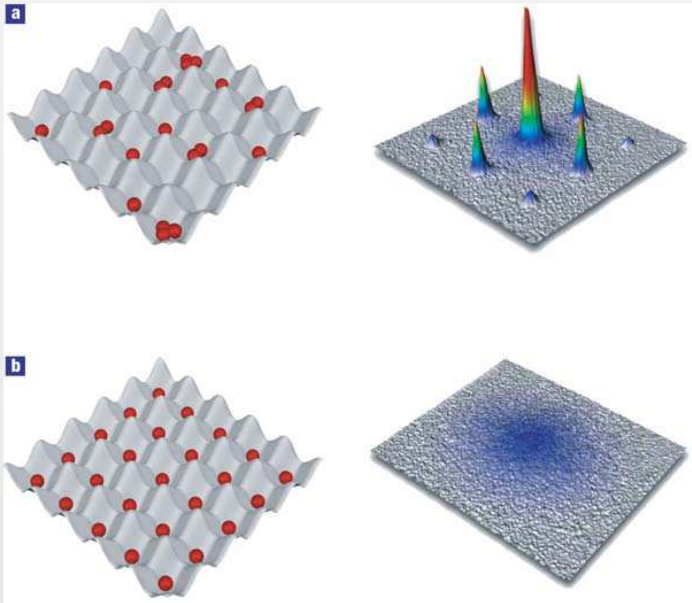
For $J \gg J_{\rm crit}$, there is scale k_*

$\exp\{\gamma_* t\}\,\mathcal{J}_0(k_*|\boldsymbol{r}_\mu - \boldsymbol{r}_\nu|)$

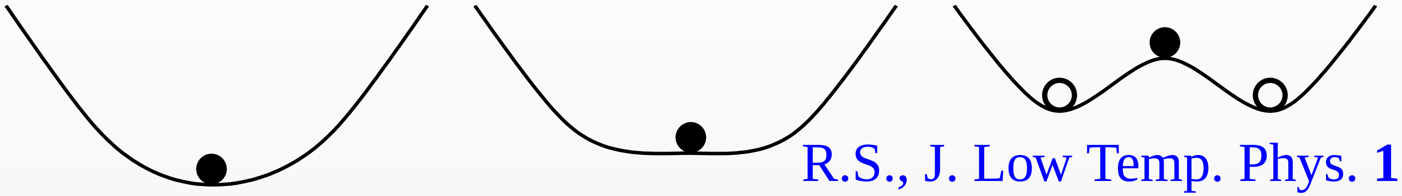
with Bessel function $\mathcal{J}_0$

See also M.Uhlmann, R.S., U.R.Fischer,

Phys. Rev. Lett. **99**, 120407 (2007)

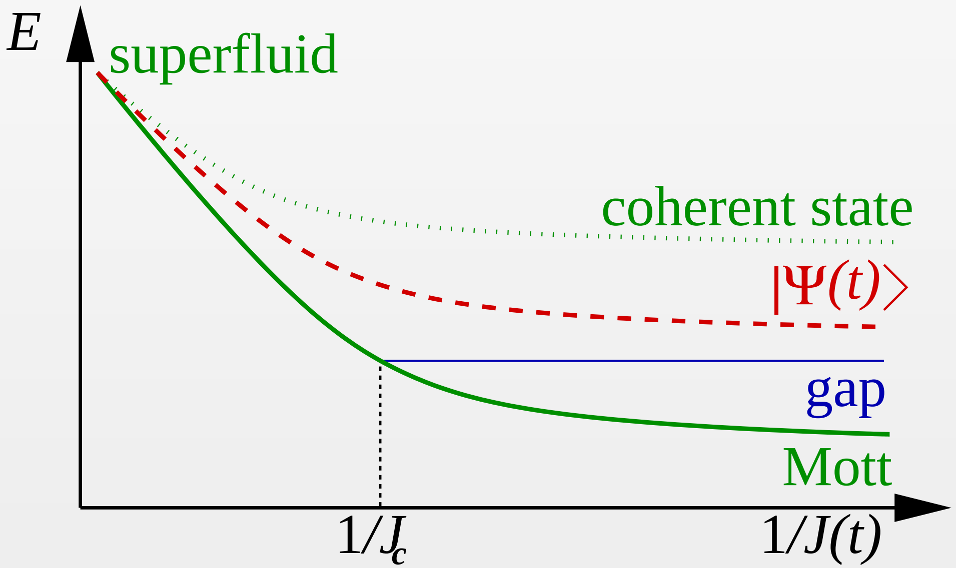


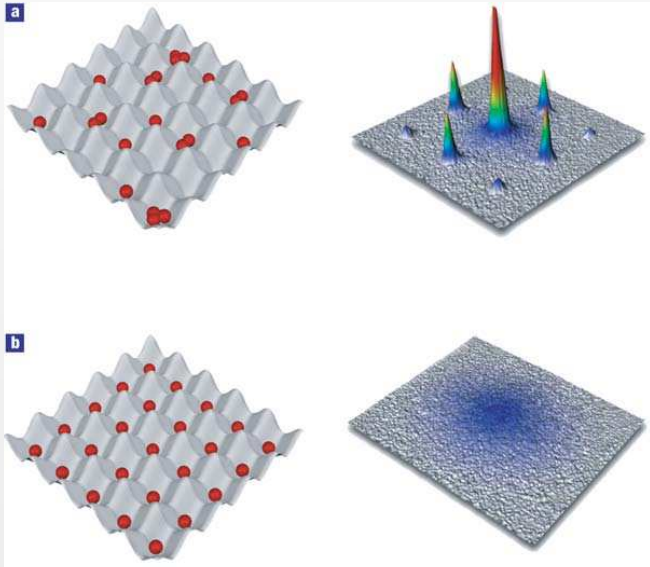
Symmetry Restoring Quench



R.S., J. Low Temp. Phys. **153**, 228 (2008)

Superfluid → Mott (symmetry restoring)





$$\langle \hat{a}^\dagger_\nu(t)\hat{a}_\mu(t)\rangle \approx n\exp\{-t^2U^2\Delta^2 n\}\,,\ \Delta^2 n = n(1-e^{-\eta})/\eta$$

→ freezing of number fluctuations

R.S., M.Uhlmann, Y.Xu, U.R.Fischer, Phys. Rev. Lett. **97**, 200601 (2006)

Revivals: U.R.Fischer, R.S., Phys. Rev. A **78**, (R)061603 (2008)

Conclusions

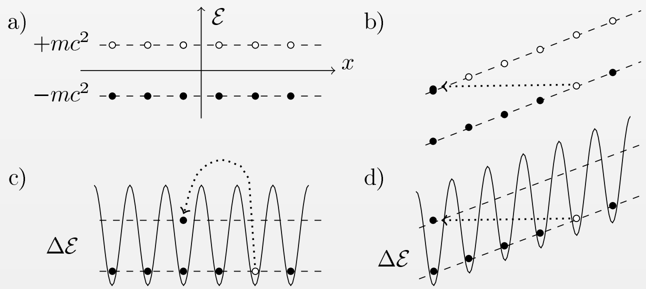
Quantum correlations in the Bose-Hubbard model

- Hierarchy P. Navez and R. S., Phys. Rev. A **82**, 063603 (2010)

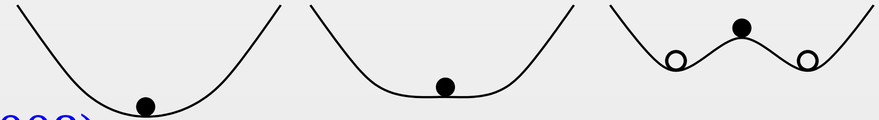
$$\hat{\rho}_\mu = \mathcal{O}(1)\,,\ \hat{\rho}^{\rm corr}_{\mu\nu} = \mathcal{O}(1/Z)\,,\ \hat{\rho}^{\rm corr}_{\mu\nu\lambda} = \mathcal{O}(1/Z^2)\,,\ldots$$

- Sauter-Schwinger tunneling

  F. Queisser, P. Navez, R. S., `arXiv:1107.3730`



- Quenches



  J. Low Temp. Phys. **153**, 228 (2008)

  Phys. Rev. Lett. **97**, 200601 (2006); Phys. Rev. Lett. **99**, 120407 (2007)

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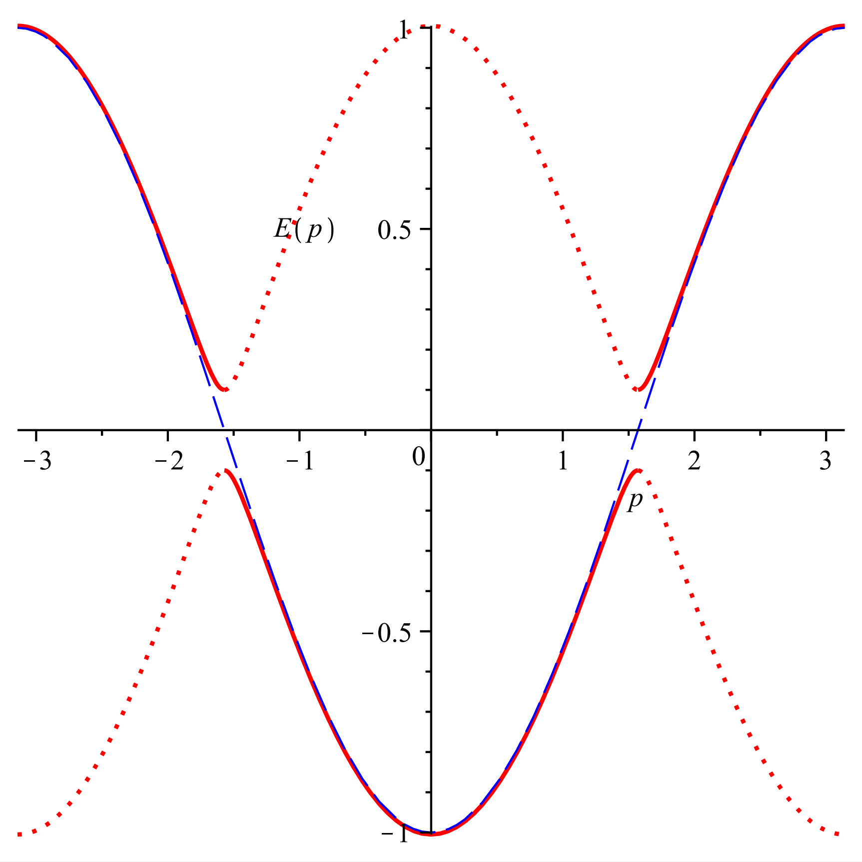
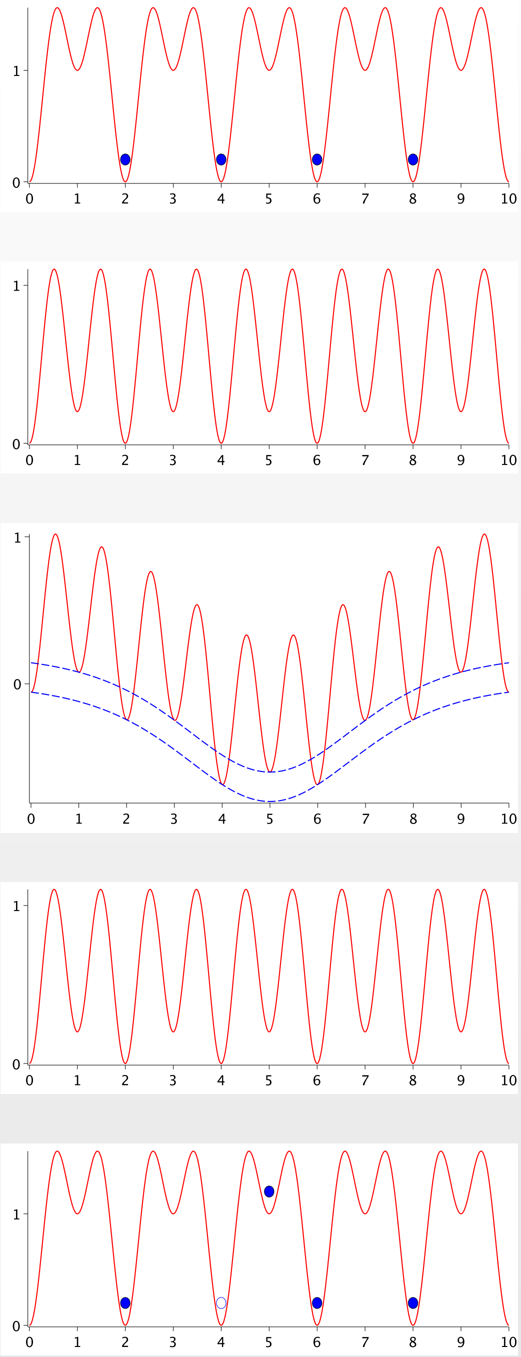
Alternative Set-up

Mott insulator $\longrightarrow$ band insulator





Lower band $\longrightarrow$ Dirac sea

N. Szpak, R. S., arXiv:1103.0541