Cross-over to quasi-condensation
a non-gaussian challenge to mean-field theories

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Abstract

We discuss in a low-dimensional Bose gas the cross-over from a dilute, degenerate system to a quasi-condensate where density fluctuations are suppressed. A few variants of mean-field theories are discussed who predict a critical point in a homogeneous system, as a condensate-related parameter is lowered: condensate, quasi-condensate, or anomalous density. We compare to numerical simulations within a stochastic Gross-Pitaevskii equation [1], to an interacting classical field theory [2] and to solutions of the Yang-Yang equations [3].

[1] S. Cockburn, A. Negretti, N. Proukakis, and C. Henkel, *Phys Rev A* **83** (2011) 043619
→ C. Gardiner, H. Stoof, M. Gajda, K. Rzążewski, W. H. Zurek ...

[2] L. Gruenberg and L. Gunther, *Phys Lett A* **38** (1972) 463;
D. Scalapino, M. Sears, and R. Ferrell, *Phys Rev B* **6** (1972) 3409
→ Y. Castin, *J Phys IV (France)* **116** (2004) 89

[3] C. N. Yang and C. P. Yang, *J Math Phys* **10** (1969) 1115
→ K. Kheruntsyan, (R. Walser) ...

Motivation & Outline

low-dimensional Bose gas: fluctuations significant

— no condensation, but density fluctuations suppressed

— this talk: cross-over dilute (degenerate) $\rightarrow$ quasi-condensate

problems with mean-field theories

— critical point when "condensate" parameter lowered

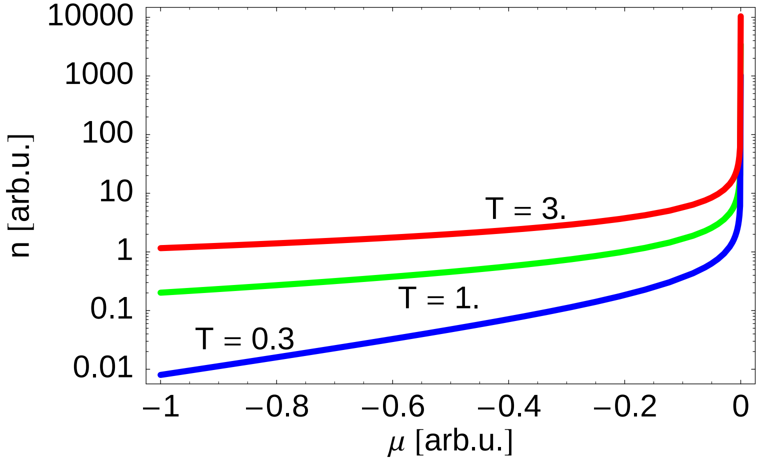
(quasi)condensate, anomalous density

benchmarks

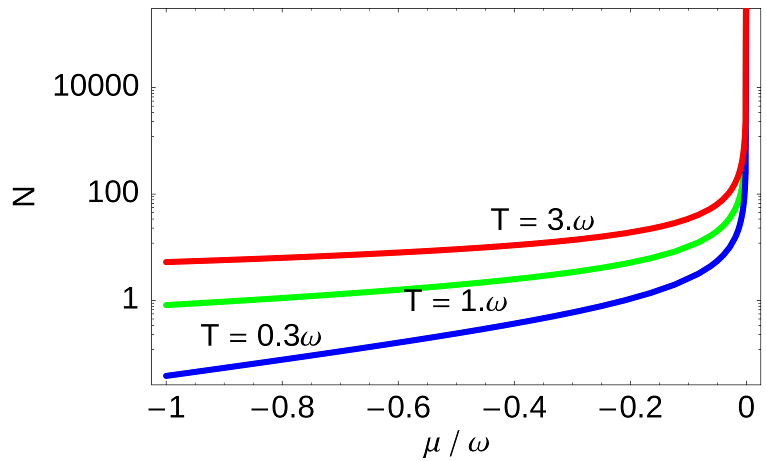
— stochastic GP, classical $|\phi|^4$ theory, Yang-Yang thermodynamics

Cross-over to quasi-condensation (1D)

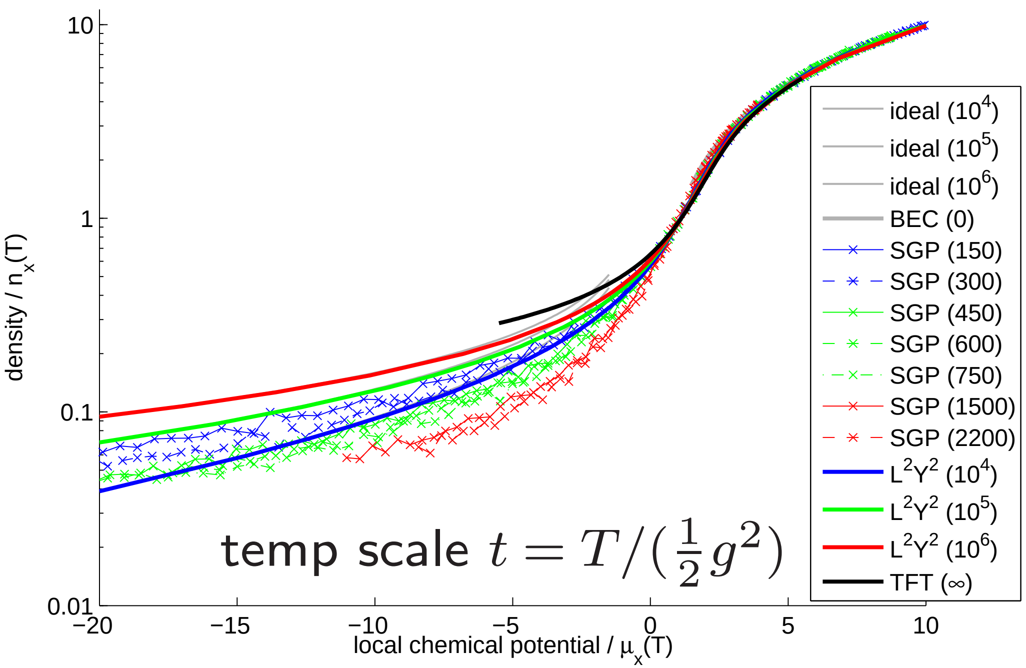
ideal gas, homogeneous



ideal gas, harmonic trap



eqn of state: interacting gas



temp scale $t = T/(\frac{1}{2}g^2)$

dense phase $\mu > 0$

(quasi) condensate $\mu \approx gn$

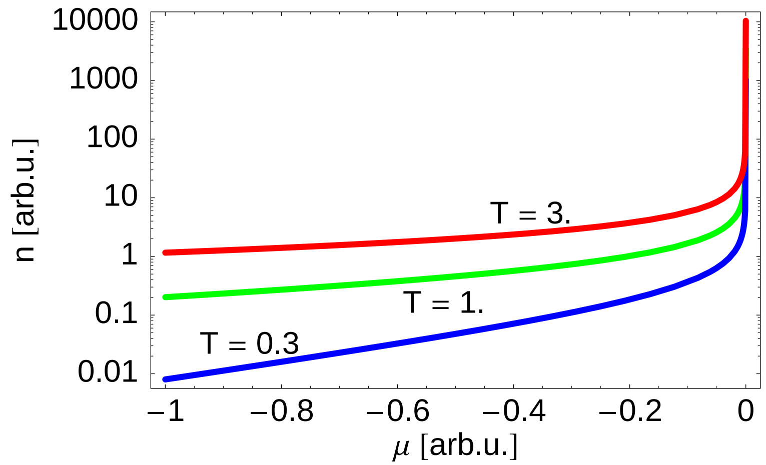
Bogoliubov sound $mc^2 \approx \mu$ or $gn_{\rm qc}$

no long-range coherence

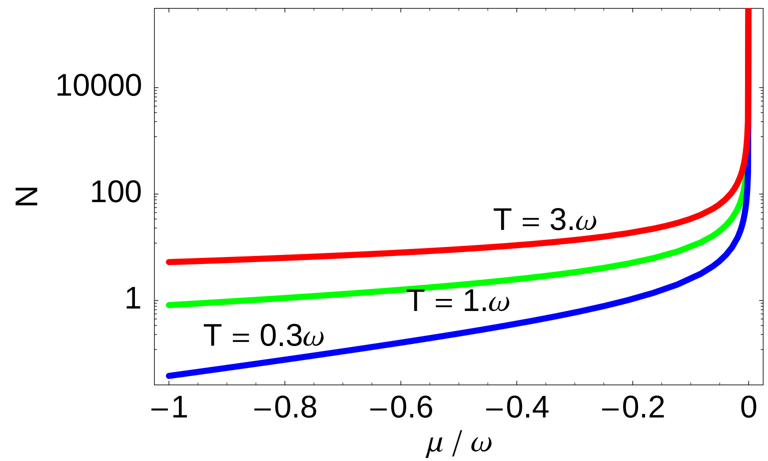
$|x - x'| \to \infty : \quad \langle \phi^\dagger(x)\phi(x') \rangle \to 0$

Cross-over to quasi-condensation (1D)

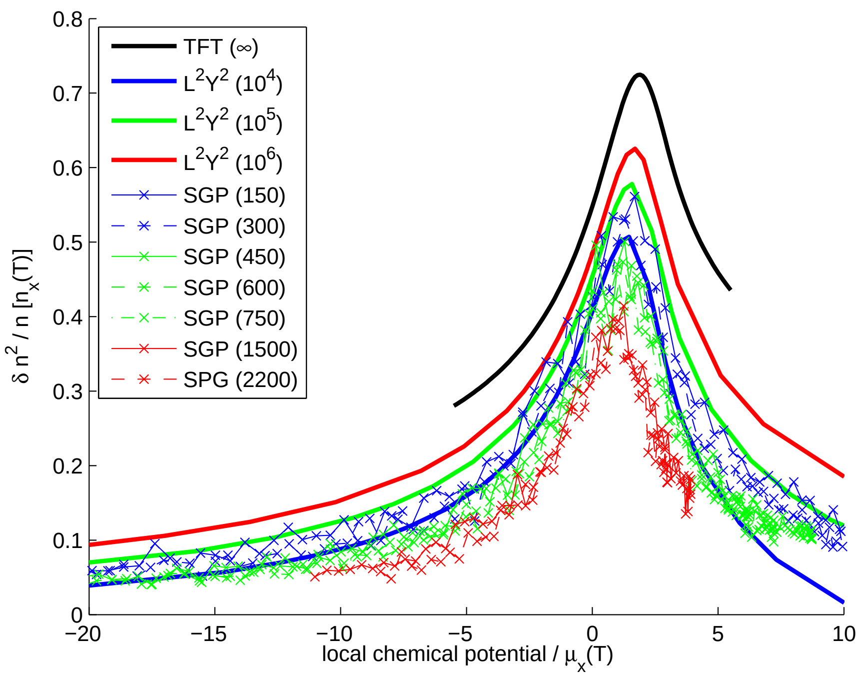
ideal gas, homogeneous



ideal gas, harmonic trap



density fluctuations $\delta n^2/n$



temp scale $t = T/(\frac{1}{2}g^2)$

dense phase $\mu > 0$

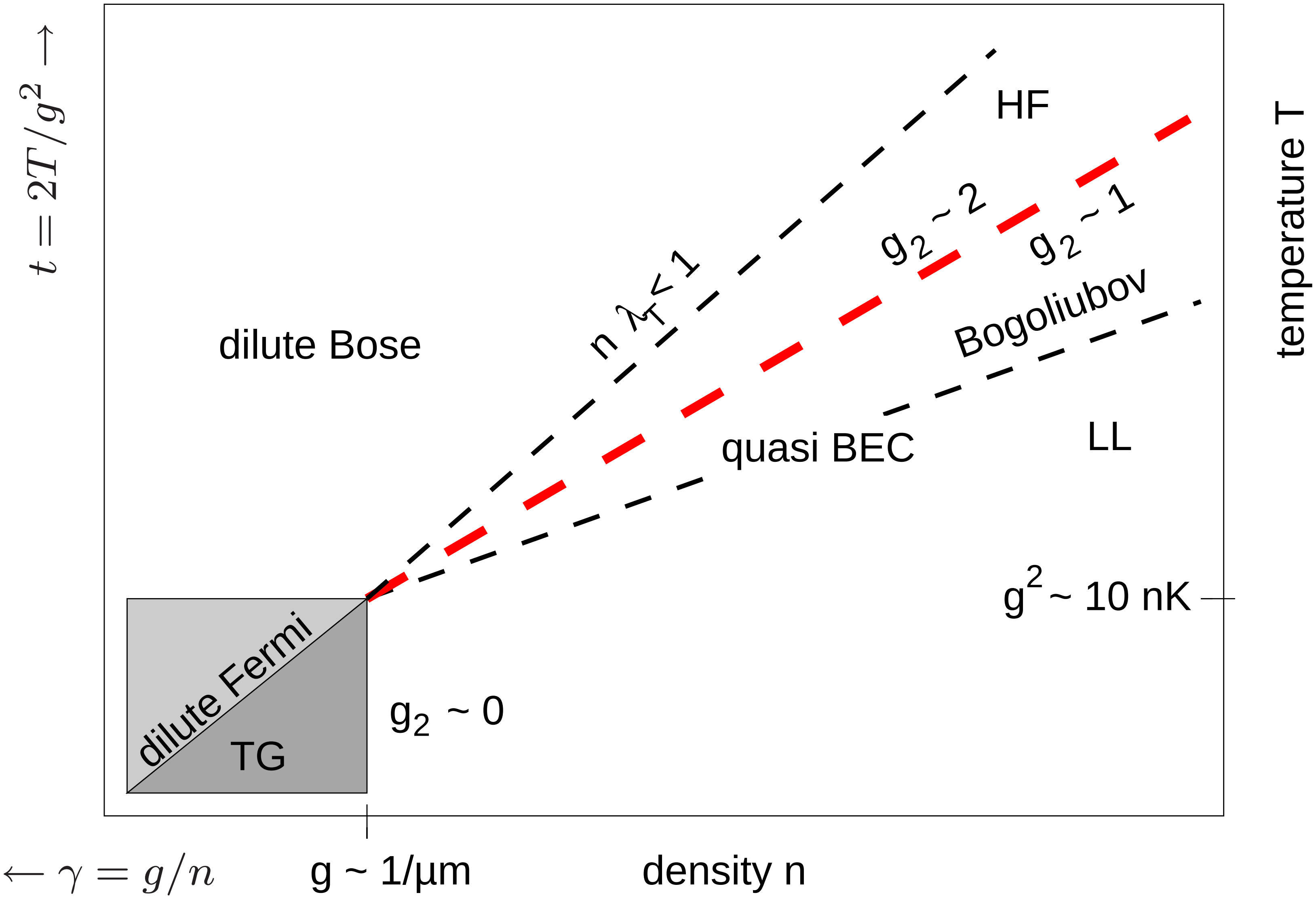
"quiet" density $\delta n \ll n$

"critical fluctuations"

dilute phase $\mu < 0$

chaotic complex field $\delta n^2/n \approx n$

Phase diagram



second-order correlation $g_2(x) = \dfrac{\langle: \hat{n}(x)^2 :\rangle}{\langle: \hat{n}(x) :\rangle^2}$ interaction $g = 2\,\hbar\omega_\perp a_s$

Cross-over units

density	$n_x(T) = (T^2/g)^{1/3}$	healing length
temperature	$T_x(n) = (gn^3)^{1/2}$	$=$ phase correlation length
chemical potential	$\mu_x(T) = (gT)^{2/3}$	density $\dfrac{\mu_x(T)}{g} = \dfrac{1/\lambda_T}{\sqrt{\mu_x(T)/T}}$
"quantum scale"	$\beta = (g^2/T)^{1/3} = \dfrac{\mu_x(T)}{T}$	classical limit $\beta \to 0$

Typical numbers

radial confinement	$\omega_\perp/2\pi \approx 5 \ldots 100\,\mathrm{kHz}$	atom chip . . . 2D opt latt
coupling constant	$g \sim (1 \ldots 20)\,\mu\mathrm{m}^{-1}$	weakly interacting: $g \ll n$
$g = 2\omega_\perp a_\mathrm{s}$	$\approx (10 \ldots 4000\,\mathrm{nK})^{1/2}$	high temp: $g^2 \ll T \leftrightarrow \beta \ll 1$

units $\hbar = m = 1$ $\longrightarrow$ review Bouchoule, van Druten & Westbrook (in *Atom Chips*)

Mean-field theories and beyond

Gross-Pitaevskii	$T = 0$ and $n \gg n_x$	artefact: long-range order
Popov-Bogoliubov [mP][exB]	$0 \le T \ll T_x$ and $n \gg n_x$	variants fail at cross-over
Hartree-Fock [HF]	dilute phase $\mu \ll -\mu_x$	perturbative in g
[HFc]	dense phase $\mu_x \ll \mu$	artefact: critical point
quantum kinetics [QKT]	any T, n	(thermo)dynamics is coupled
Lieb-Liniger-Yang-Yang	any T, n	higher moments difficult
thermal field theory [TFT]	high T, classical n	no quantum effects
c-field simulations [SGP]	high T, in trap	full thermal cloud dynamics?

[mP] Andersen, Al-Khawaja, Proukakis, Stoof & al $\geq$ 2002

[exB] Mora & Castin 2003

[HF] Kheruntsyan, Drummond, Shlyapnikov, Deuar & al $\geq$ 2003

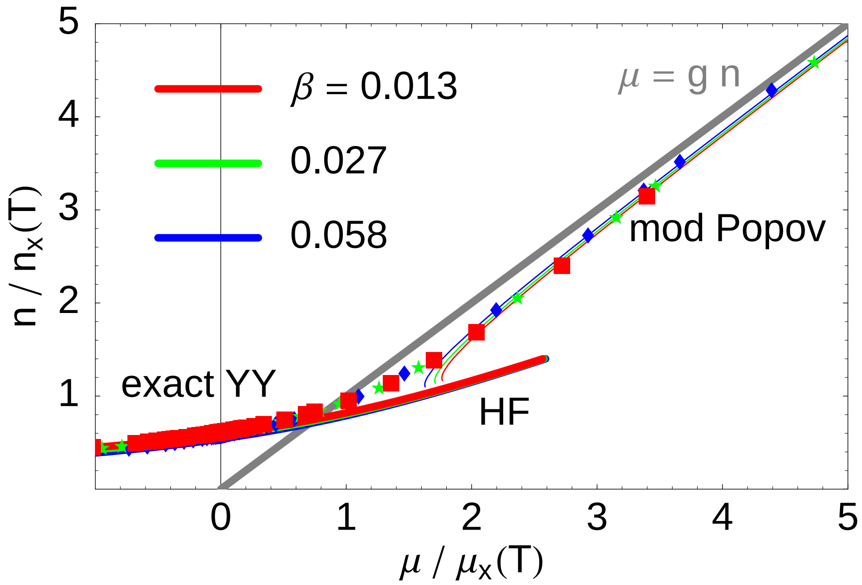
[TFT] Gruenberg, Scalapino & al $\geq$ 1972

[QKT] Drummond, Griffin, Holland, Walser, Zaremba, Zoller . . . $\geq$ 1998/2002

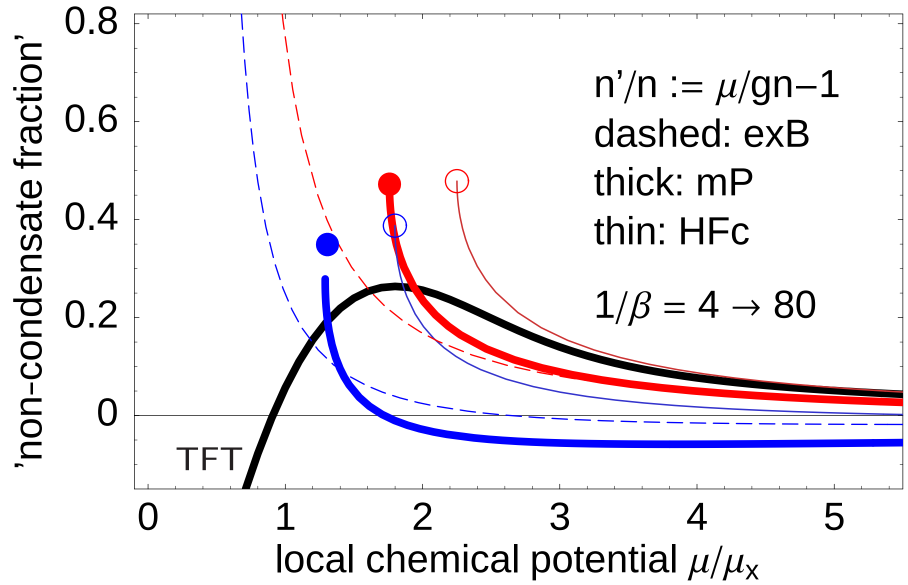
[SGP] Drummond, Gajda, Gardiner, Proukakis, Rzążewski, Stoof, Zurek . . . $\geq$ 1998/2002

Failure of mean field theories

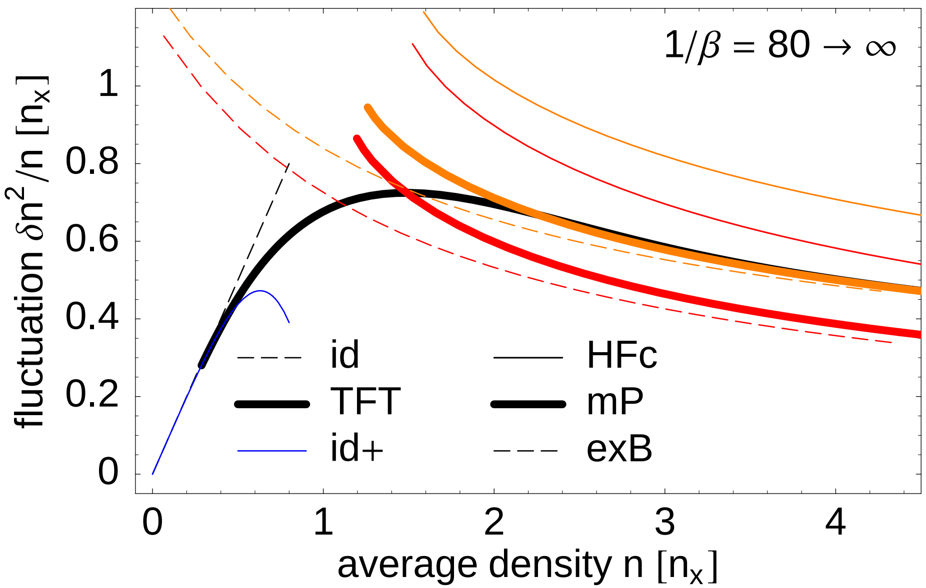
equation of state $n(\mu)$



"thermal fraction" $\mu = g(n + n')$



density fluctuations $\delta n^2/n$



- HF: density too low* $n \to \mu/2g$
- mP: critical point $\dfrac{\mu_c}{\mu_x} \approx 1.89 + \mathcal{O}(\beta^{1/2})$

TFT exact as $1/\beta \to \infty$: benchmark

* Trebbia & al [*Phys Rev Lett* 2006]

Challenges for mean field theories

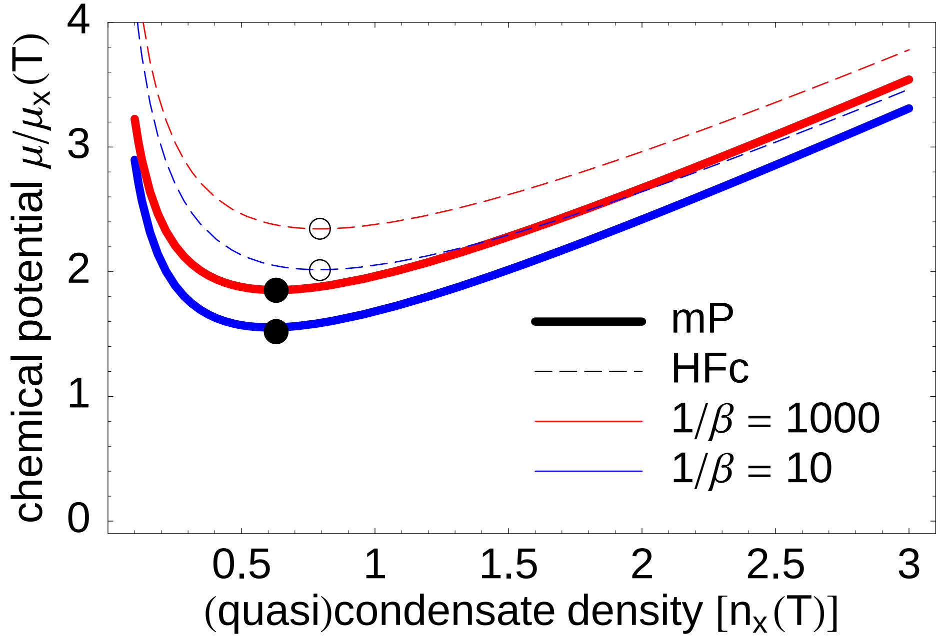
Onset of condensation:

order parameter > 0

(quasi)condensate density $n_{\rm qc}$,

speed of sound c,

anomalous average m



infrared divergence as $c \to 0$

$$n'(\mu, T) \approx \int \frac{{\rm d}k}{2\pi} S(k;c) \frac{T}{\sqrt{(ck)^2 + k^4/4}}$$

— need larger set of "relevant moments"

— similar to critical fluctuations ($\to$ shift in T_c)

Non-gaussian field statistics

Thermal field theory (TFT) $\rightarrow$ field distribution function $P(\phi)$

classical path integral

$$n = \int \frac{\mathcal{D}\phi}{Z} \, |\phi(x)|^2 \exp\left[-\frac{1}{T} \int_0^L \mathrm{d}z \left(\frac{1}{2} \left| \frac{\partial\phi}{\partial z} \right|^2 - \mu|\phi^2| + \frac{g}{2}|\phi|^4 \right) \right]$$

imaginary time action $S = \displaystyle\int_0^L \mathrm{d}\tau \left[\frac{1}{2} \left| \frac{\mathrm{d}\phi}{\mathrm{d}\tau} \right|^2 + V_{\mathrm{eff}}(\phi) \right]$

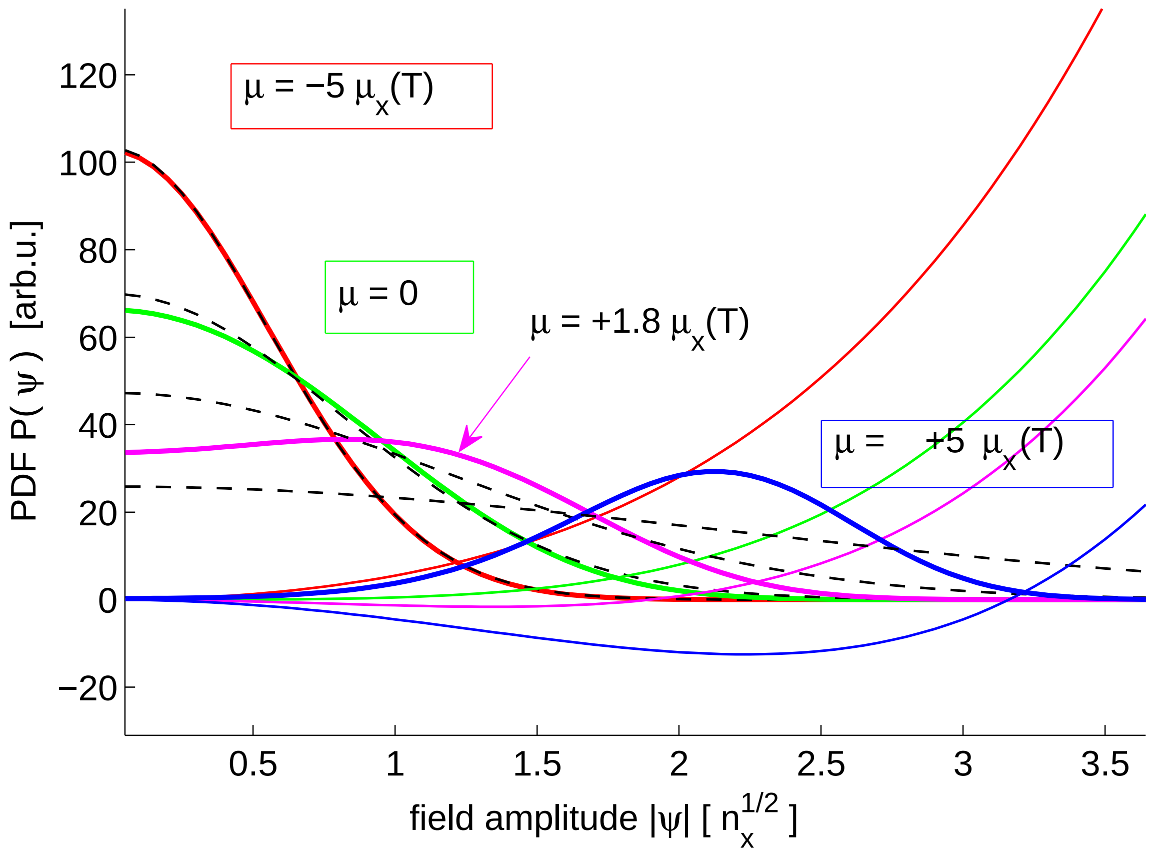
mapping to QM: generate translation operator $U(x,0) = \exp(-xK)$

thermodynamic limit: "ground state"

$$K\sqrt{P} = \left[-\frac{T}{2} \frac{\partial^2}{\partial\phi\partial\phi^*} - \frac{\mu}{T}|\phi|^2 + \frac{g}{2T}|\phi|^4 \right] \sqrt{P} = \kappa_0 \sqrt{P}$$

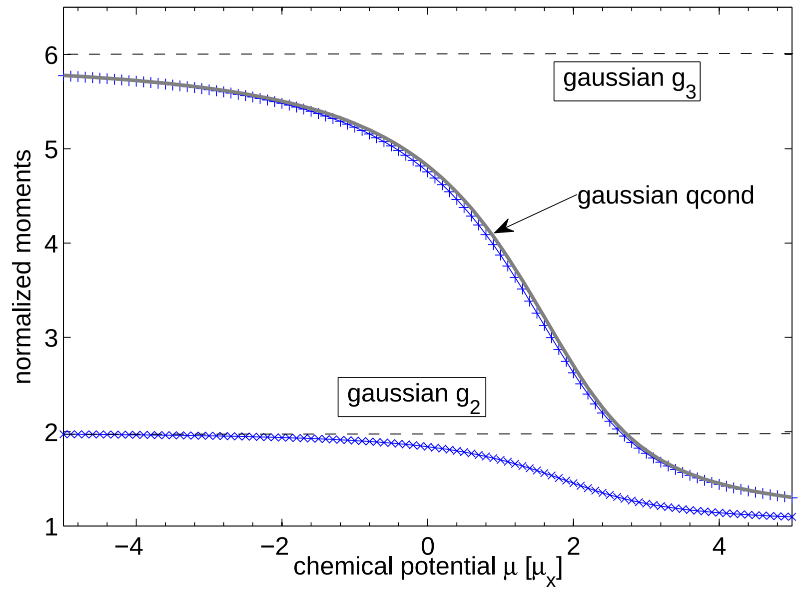
Non-gaussian field statistics

TFT $\to$ field distribution function $P(\phi)$



average $n = \int \mathrm{d}^2\phi\, P(\phi)|\phi|^2$

moments $g_2 \propto \langle|\phi|^4\rangle$, $g_3 \propto \langle|\phi|^6\rangle$



quasi-condensate approach:

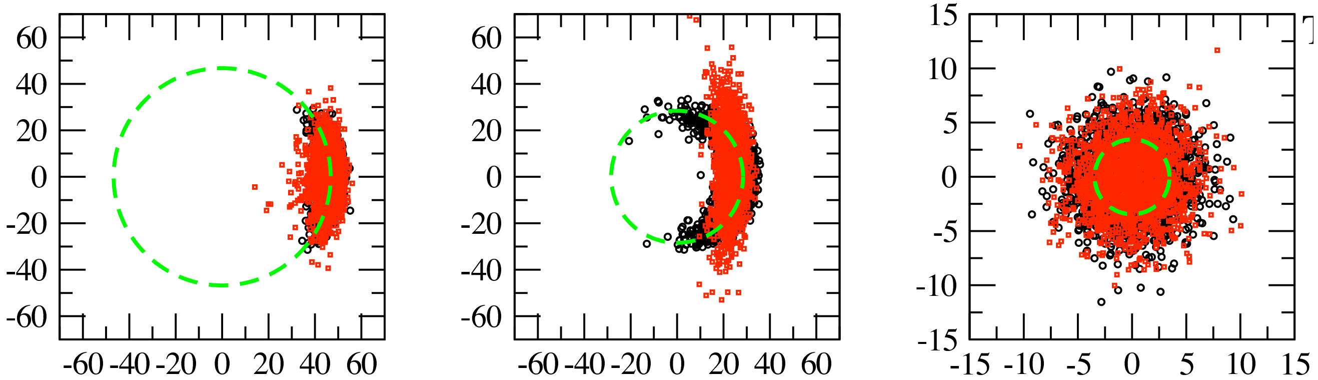
$\phi - \sqrt{n_{\rm qc}} = \chi$ gaussian & normal

mapping to QM: $$\left[-\frac{T}{2}\frac{\partial^2}{\partial\phi\partial\phi^*} - \frac{\mu}{T}|\phi|^2 + \frac{g}{2T}|\phi|^4 \right]\sqrt{P} = \kappa_0\sqrt{P}$$

Non-gaussian field statistics

SGP simulation vs Bogoliubov $1/\beta \approx 110$

$\mu(z) \approx 18\,\mu_x$ $\approx 6\,\mu_x$ $\approx -22\,\mu_x$



- - - average density Cockburn & al *Phys Rev A* 2011

- non-trivial correlation $\langle \hat{n}(\nabla\hat{\theta})^2 \rangle \neq n\langle(\nabla\hat{\theta})^2\rangle$

phase-locked field amplitudes

Penrose-Onsager analysis of $\langle \phi^*(z)\phi(z')\rangle \longrightarrow \phi_c(z),\, N_c$

condensate amplitude $a_c = \int \mathrm{d}z\, \phi_c^*(z)\phi(z) \longrightarrow$ phase $\mathrm{e}^{\mathrm{i}\theta_c}$

statistical sample of $\mathrm{e}^{-\mathrm{i}\theta_c}\,\phi(z)$ vs. $\mu(z)$

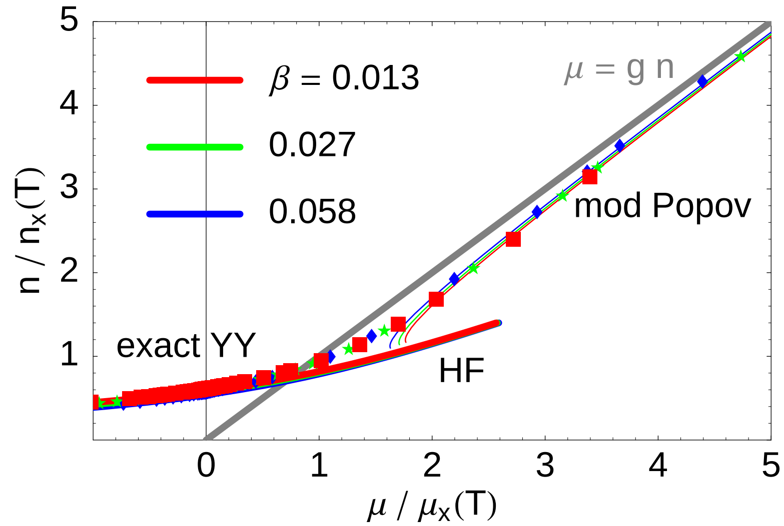
Conclusion & Perspectives

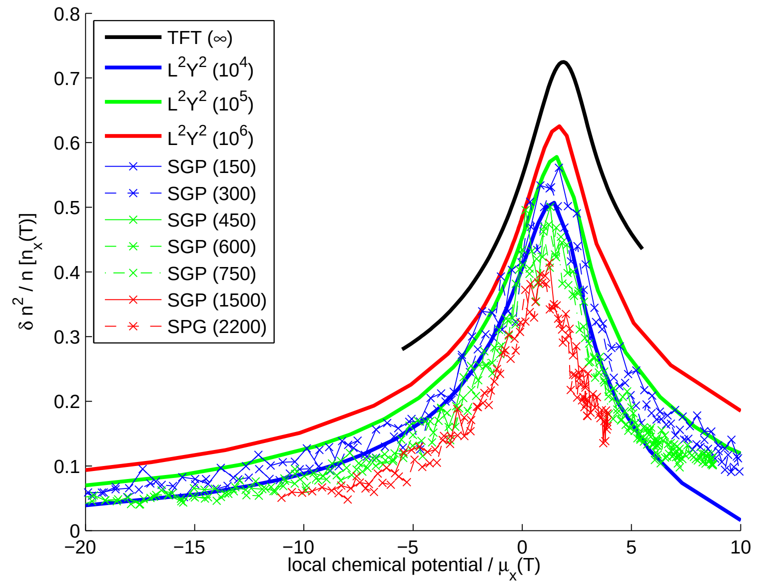
Cross-over to quasi-condensation

- occurs at $\mu \approx 1.8 \ldots 2\,(gT)^{2/3}$
- smooth equation of state $n(\mu, T)$
- "order parameter" from density fluctuations

  quasi-condensate density: $\delta n^2 = n^2 - n_{\rm qc}^2$

  peak in $\delta n^2 / n =$ super-Poisson fluctuations





"No man's land" for mean field theories

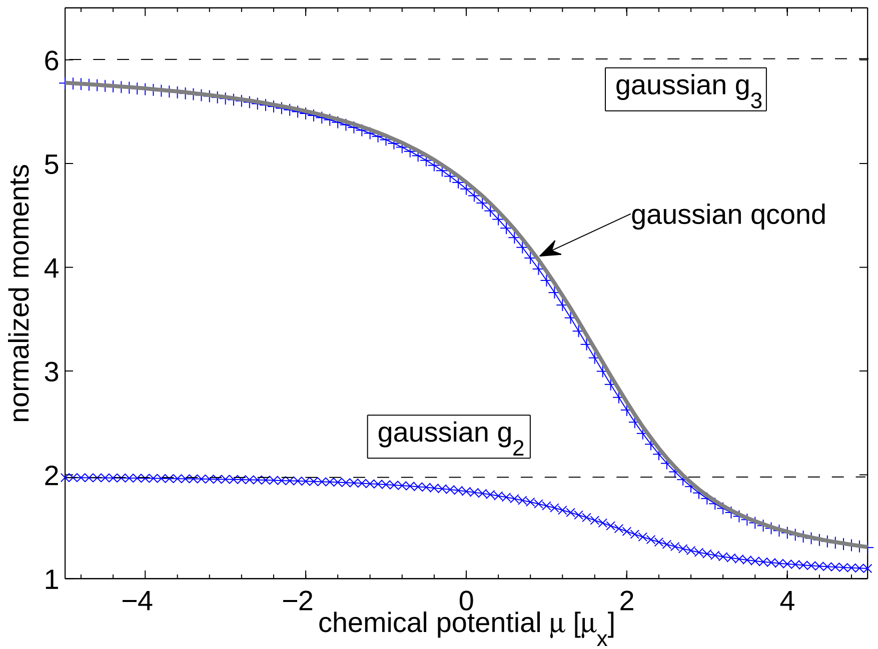
- inconsistent density split $n = n_{\rm qc} + n'$
- self-consistent dispersion relation,

  occupation numbers, structure factor

Conclusion & Perspectives

Thermal field theory

- limited to high T $(t = 2\beta^{-3} = \infty)$
- non-perturbative in interactions
- captures critical fluctuations, counting statistics



exploit full potential

- check against Yang-Yang (difficult at $t \to \infty$!)
- all correlations (density, phase)
- check superfluid response