Bose-Einstein Condensation of Photons and Periodic Potentials for Light

Martin Weitz

Institut für Angewandte Physik der Universität Bonn
Ground State of Bosonic Ensembles (3D-Regime)

atoms

Bose-Einstein condensate of gaseous rubidium atoms @ 180nK
Earlier Work related towards a Photon BEC

- Proposal for a photon BEC in Compton scattering off a thermal electron gas

Zel’dovich and Levich, 1969
... Earlier Work

- Exciton-polariton condensates
  strong coupling ('half matter, half light'); in equilibrium for condensed part

  Kasprzak et al., 2006

- Proposal for photon fluid in nonlinear resonator

atom

\[ \text{photon-photon scattering} \]

(four-wave mixing)

R. Chiao
Outline of Talk

- thermodynamics of a two-dimensional photon gas in a dye-filled optical microcavity

- Bose-Einstein condensation of photons

- condensate intensity correlations, grand canonical BEC

- periodic potentials for light
Bonn 2D-Photon Gas Experimental Scheme

- use curved-mirror microresonator to modify photon dispersion

\[ d = n \cdot \lambda / 2 \]  
(for \( n=1 \))

- thermal equilibrium of photon gas by scattering off dye molecules…

\[ \omega_{\text{cutoff}} \]

\[ \omega \]

energy

min. (‘cutoff’) energy of light in resonator

photon in free space

(transversal) wavevector

dye molecule
Spectrum of Perylene-Dimide Dye Molecule (PDI)

Kennard-Stepanov theory:

\[
\frac{f(\omega)}{\alpha(\omega)} \propto \exp\left(-\frac{\hbar \omega}{k_B T}\right)
\]

T: (internal rovibrational) temperature of dye solution

Collisionally induced thermalization in dye medium
Model for Photon Gas Thermalization

thermalization of photon gas by repeated absorption-emission processes in dye molecules

2D-photon gas

heat bath (300 K):

dye solution
Photon Number Variation during Thermalization?

Planck Blackbody Radiation

- Thermal excitation
- Photon emission
- \( \Delta E \sim k_B T \)
- \( =1/40 \text{ eV} \)

New Scheme

- Photon absorption
- Photon emission
- \( \Delta E \sim \hbar \omega_{\text{cutoff}} \)
- \( =2.1 \text{ eV} \)

- Thermal excitation suppressed by\( \sim e^{-\frac{\hbar \omega_{\text{cutoff}}}{k_B T}} \approx 10^{-36} \)

\( \rightarrow \) Photon average number conserved

',white-wall box' for photons
Photon Trapping versus Atom Trapping

System formally equivalent to 2D-gas of massive bosons with \( m_{\text{eff}} = \frac{\hbar \omega_{\text{cutoff}}}{c^2} \)

\[
E = m_{\text{eff}} c^2 + \frac{1}{2} m_{\text{eff}} \Omega^2 r^2
\]

→ BEC expected for \( N > N_c \approx 77000 \)  
\[ (T=300 \text{K}, \Omega = 2\pi \cdot 4 \cdot 10^{10} \text{Hz}, \quad m_{\text{eff}} \approx 6.7 \cdot 10^{-36} \text{kg} \approx 10^{-10} \text{m}_{\text{Rb}}) \]
Spectrum of Thermal Photon Gas in Cavity

residual pump radiation

$\mu = -6.76(17) k_B T$

$\mu = -7.16(17) k_B T$

→ evidence for thermalized two-dimensional photon gas with $\mu \approx 0!$

Photon Bose-Einstein Condensate in Dye Microcavity

Images of micocavity emission

$N \ll N_c$

$N > N_c$

Spectrum

see also recent Imperial College experiment: J. Marelic and R. Nyman, PRA 91, 033813 (2015)
Bose-Einstein Condensation versus Lasing

Equilibrium versus out of equilibrium

Ideal photon box (with number-conserving thermalization & low-frequency cutoff) → BEC

Pumping and losses dominate → laser, requires inverted active medium

See also: lasing a nonequilibrium phase transition (Haken,...), polariton BEC ↔ polariton lasing.
Experimental Data: Laser to BEC Crossover

Low thermal contact

Thermalization of photons

mode-locked laser state

Bose-Einstein condensate

pump beam (27 μm focus)
Towards Calometric Properties of 2D Photon Gas

main aim: heat capacity of photon gas
- early work on heat capacity of Bose gases: \( \lambda \)-transition in liquid He
- difficult to measure in cold bosonic atomic gases, see however work with fermion pairs (Zwierlein)

Experimental Spectra (with new 4f-grating spectrometer)

\[ \int n(\lambda)hc \left( \frac{1}{\lambda} - \frac{1}{\lambda_c} \right) d\lambda \]

\( \frac{N}{N_c} = \left( \frac{T}{T_c} \right)^2 \)

Internal Energy \( U \)
Determination of Heat Capacity of the Photon Gas

\[ C = \frac{\partial U}{\partial T} = k_B \frac{\partial (U / k_B T_c)}{\partial (T/T_c)} \]

Entropy of Trapped Photon Gas

Further analysis of spectrometer data, using the determined heat capacity $C$, yields entropy:

$$S(T) = \int \frac{dQ}{T} = \int_0^T \frac{C(T'')}{T''}dT''$$

This assumes $S(T \to 0) = 0$

See also earlier work on atomic Bose gases using camera images and the local density approximation: Columbus, Chicago, Paris, ..
Grand Canonical BEC and Condensate Fluctuations

usual BEC
(e.g. cold atoms, polaritons..)
microcanonical ensemble

$N$
particle number fixed

quite condensate
$g^{(2)}(0)=1$

Grand canonical BEC
particle exchange with reservoir

reservoir:
dye electronic excitations

system:
photons

flickering condensate, $\Delta N \approx N$
$g^{(2)}(0)=2$

J. Klaers et al., PRL 108, 160403 (2012), see also: D. Sobyanin, PRE 85, 061120 (2012)
THE IDEAL BOSE–EINSTEIN GAS, REVISITED

Robert M. ZIFF *, George E. UHLENBECK and Mark KAC

The Rockefeller University, New York, N.Y. 10021, U.S.A.

Abstract:

Some questions concerning the ideal Bose–Einstein gas are reviewed and examined further. The bulk behavior including the condensation phenomenon is characterized by the thermodynamical properties, occupations of the states and their fluctuations, and the properties of the density matrices, including the diagonal and off-diagonal long range orders. Particular attention is focused on the difference between the canonical and grand canonical ensembles and a case is made that the latter does not represent any physical system in the condensed region. The properties in a finite region are also examined to study the approach to the bulk limit and secondly to derive the surface properties such as the surface tension (due to the boundary). This is mainly done for the special case of a rectangular parallelepiped (box) for various boundary conditions. The question of the asymptotic behavior of the fluctuations in the occupation of the ground state in the condensed region in the canonical ensemble is examined for these systems. Finally, the local properties near the wall of a half infinite system are calculated and discussed. The surface properties also follow this way and agree with the strictly thermodynamic result. Although it is not intended to be a complete review, it is largely self-contained, with the first section containing the basic formulas and a discussion of some general concepts which will be needed. Especially discussed in detail are the extra considerations that are needed in thermodynamics and statistical mechanics to include the surface properties, and the quantum hierarchy of the density matrices and local conservation laws. In the concluding remarks several problems are mentioned which need further analysis and clarification.
Photon Intensity Correlation in BEC Mode vs. Delay Time

condensate fraction: 56%

system size large: $N > \sqrt{M}/2$

$\approx$ (usual) canonical BEC regime with Poissonian fluctuations

condensate fraction: 4%

system size small: $N < \sqrt{M}/2$

enhanced fluctuations $\rightarrow$ evidence for grand canonical BEC regime!

Photon Intensity Correlation vs. Condensate Fraction

![Graph showing photon intensity correlation vs. condensate fraction with T/Tc on the x-axis and intensity correlation g(2)/(0) on the y-axis. Different curves represent different values of Δ (k_B T) and ρ (mol/l).]
First Order Coherence of Photon BEC: Interference Signals

Condensate Intensity

Beat Signal with Frequency Stable Laser

Periodic Potentials for Light: Motivation

Possible experiments:
- strongly correlated quantum gases: Mott-insulator transition for photons
- artificial magnetic fields, quantum Hall states, ..

Proposals: Plenio, Greentree, Angelakis, Türeci, Carusotto, Hafezi, Hartmann, Stoof ..
See also experimental lattice work in polaritons: Yamamoto, Bloch, Baumberg
One Approach: Use Mirror Structuring to Create Variable Potentials for Light

- Single trap
- Double well

Photon trapping potential from mirror curvature

Potential

$x$
Thermo-Optic Imprinting: Variable Potentials for Trapped Photon Gas

plane mirrors manipulate optical length locally by heating

use thermosensitive polymer solution (PolyNIPAM) admixed to the dye solution to increase temperature dependence of refractive index
Lattice Potentials for Photonic Quantum Gas

Experimental Setup

dye polymer microcavity

heating beam

two-dimensional galvo scanner

405nm Diode Laser

405 nm AOM

absorbing silicon layer

Observed Microcavity Emission

pump beam

D. Dung et al, to be published
Spectral Analysis of the Emission of One Site: Investigating Effective Photon Interactions

 thermo-optic interactions occur temporally delayed → frequency chirp of the emission
we observe tunneling when the sites are tuned into resonance. From the resonance width, the tunnel coupling can be extracted.
Tunnel Coupling Versus Distance Between Sites

D. Dung et al, to be published
Conclusions

- thermalization of 2D-photon gas with nonvanishing chemical potential and Bose-Einstein condensation of photons

- observation of a grandcanonical BEC regime with enhanced intensity fluctuations

- variable potentials for photonic quantum gas. We see tunneling and effective photon interactions in double well system
- photon thermalization: concentration of diffuse sunlight
- photon BEC: new states of light
  (some) future directions:
  • grand canonical BEC regime, superfluidity (?), …
  • study of quantum manybody states in periodic potentials
- light sources in new wavelength regimes, coherent UV sources

possible application: lithography
Quantum optics group, IAP Bonn:

J. Schmitt
T. Damm
H. Brammer
C. Grossert
J. Ulitzsch
M. Leder
D. Dung
C. Wahl
D. Babik
F. Öztürk
S. Christopoulos
H. Alaeian
J. Klaers (→ ETH Zürich)
P. Moroshkin (→ RIKEN)
F. Vewinger
M. Weitz