Constructing a *U(1)* gauge symmetry in electronic circuits

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Lattice gauge fields

Gauge theories are fundamental to the Standard Model of High-Energy Physics.

Fermionic and bosonic particles represent matter field and force carriers.

Requirements for simulating a high-energy process:

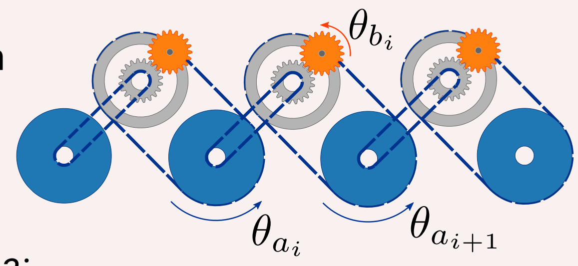
- Local implementation of gauge field,
- Include both fermions and bosons,
- Interactions preserve local gauge invariance → *Gauss's law*

A typical high-energy process is *Schwinger pair production:* The vacuum becomes unstable at very high static electric fields leading to electron-positron pair creation.

Local *U(1)* symmetry

By discretization, a 1D field theory becomes a chain of sites and links with associated phases θ: $a \sim e^{i\,\theta}$

Local gauge freedom:



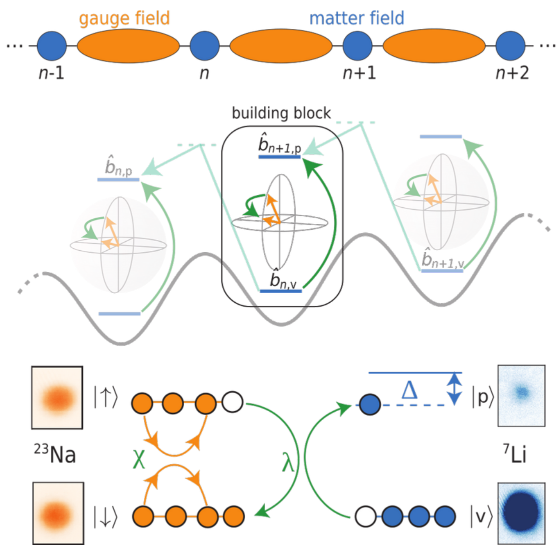
The link b absorbs the local phase transformation $\delta\theta$ of neighboring sites a:

$$H \supset a_i\, b_i\, a_{i+1}^\dagger \sim e^{i\left(\theta_{a_i} + \theta_{b_i} - \theta_{a_{i+1}}\right)}$$

Cold atoms & ions

There are efforts to simulate increasingly complicated lattice field theories in *trapped ion* and in *cold atom experiments,* aimed at supplementing high-energy experiments for Standard Model theories.

Schwinger process	Martinez et al., Nature 534, 2016
Z2	Schweizer et al., Nat Phys 15, 2019
U(1)	Mil et al., Sciene 367, 2020



(Fig 1 from Mil et al., Science 367, 2020)

Electronic circuits

Non-quantum but **fast** and **low-cost** experiment to to reach for larger lattices and more complicated gauge theories.

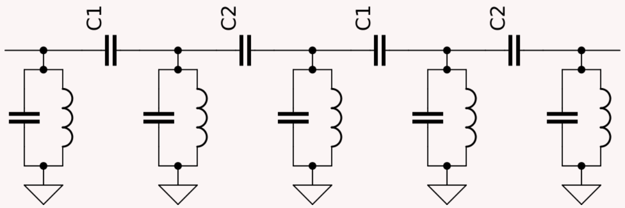
Topological circuit: SSH model

In the past electronic circuits have been used to engineer metamaterials. The *Su-Schrieffer-Heger* model is the simplest one with topological properties.

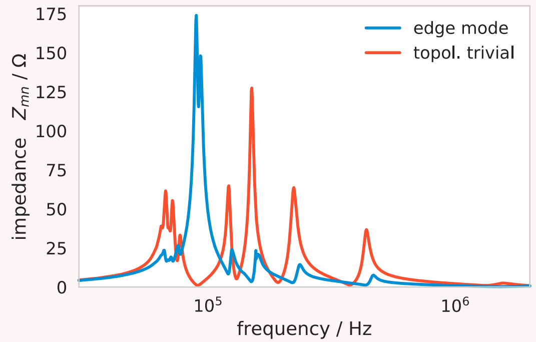
Lee, Commun Phys 1, 2018

Nearest-neighbor hopping with alternating coupling:

It is *topological*, because there is a one-to-one relation between *bulk* and *boundary properties*.



Impedance reveals edge modes



Lattice gauge theory → circuit

Gauge theories appear with very close analogies in *quantum* and *classical mechanics*. We take the SSH Hamiltonian and supplement it with a link operator / variable *b*.

Lattice gauge theory	Circuit
field at position	LC oscillator
SSH hopping term	coupling capacitor
U(1) link: hopping	symmetric multipliers
U(1) link: gauge field	LC oscillator

Flux linkage

$$\Phi = \int V \, \mathrm{d}t$$

Charge

$$Q = \int I \, \mathrm{d}t$$

Energy

$$n = a^* a = \frac{1}{2}\left(\frac{\Phi^2}{L} + \frac{Q^2}{C}\right)$$

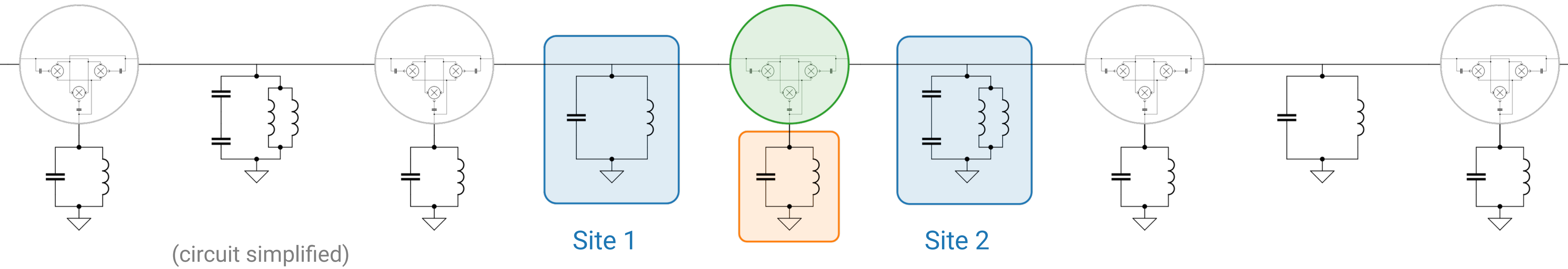
Complex variables:

$$a = \frac{1}{\sqrt{2\omega}}\left(\frac{\Phi}{\sqrt{L}} + i\frac{Q}{\sqrt{C}}\right)$$

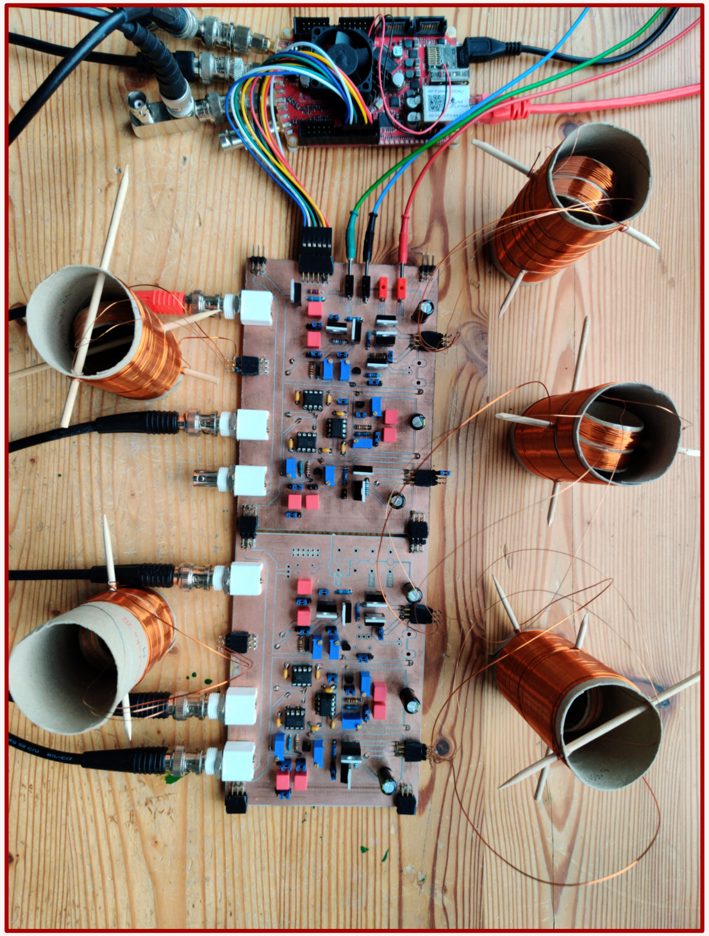
$$a^*_{j+1} a_j + \mathrm{H.c.} \sim Q_j Q_{j+1}$$

$$a^*_{j+1} b_j \, a_j + \mathrm{H.c.} \sim Q_{a_j} Q_{b_j} Q_{a_{i+1}}$$

$$H = \boxed{\omega \, a_1^* \, a_1 + 2\omega \, a_2^* \, a_2} + \boxed{\omega \, b^* \, b} + \boxed{\lambda\left(a_1^* \, b^* a_2 + a_2^* \, b \, a_1\right)}$$



The prototype



3 sites & 2 links on milled boards
+ Red Pitaya SoC.

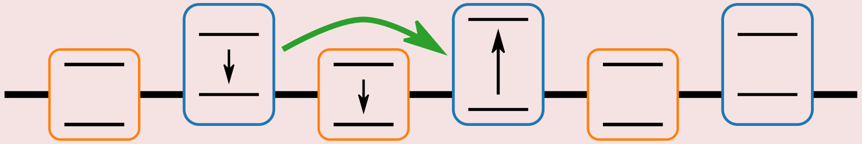
Conserved quantities

Local phase trafo: $a_j \rightarrow a_j\, e^{i\,\theta_j}$

Absorbed by link: $b_j \rightarrow b_j\, e^{-i\,\theta_j + i\,\theta_{j+1}}$

$$G_1 = a_1^*\, a_1 - b^*\, b$$
$$G_2 = a_2^*\, a_2 + b^*\, b$$
$$N = a_1^*\, a_1 + a_2^*\, a_2$$

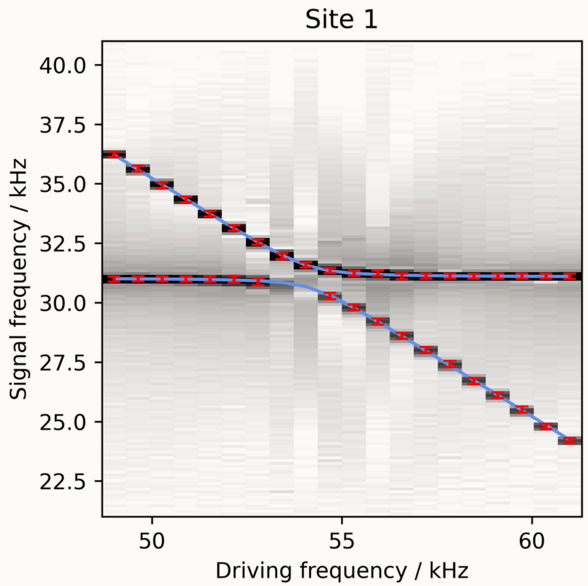
Continuous transformation corresponds to local conserved quantities G
→ *Gauss's law*



"Links track the hopping from site to site."

Rabi oscillations

2 sites with externally driven link
→ Two-level system,
static gauge field model.



Outlook

Transmission along long chain:

Non-linear dynamics are missing simple predictions
→ Can only compare to simulations.

Conservation laws:

Meaningful results despite violation / imperfect implementation?

Are there topological states?

Non-abelian *SU(2)* model?

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