Constructing a *U(1)* gauge symmetry in electronic circuits

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Lattice gauge fields

Gauge theories are fundamental to the Standard Model of High-Energy Physics. They are built up of fermionic and bosonic particles, which represent matter field and force carries.

To simulate a high-energy process the requirements are:

- Work with finite dimensional Hilbert space
  → local implementation of gauge field
- Include both fermions and bosons, and
- Interactions preserve local gauge invariance
  → *Gauss's law* and associated conserved quantities.

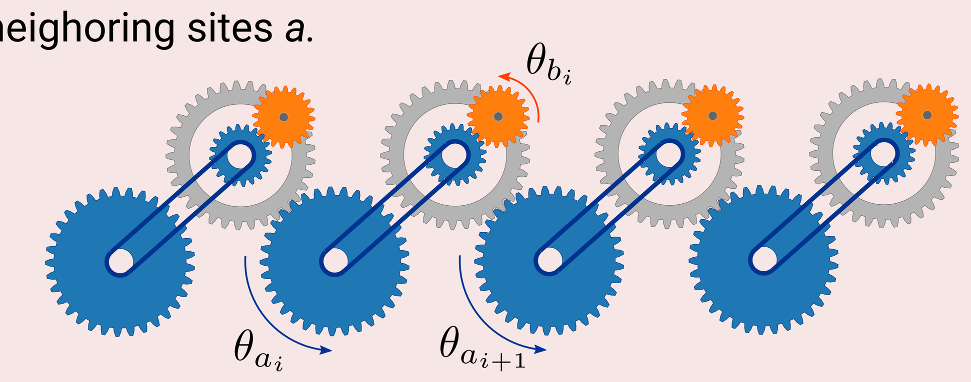
A typical high-energy process is *Schwinger pair production:* The vacuum becomes unstable at very high static electric fields leading to electron-positron pair creation.

Local *U(1)* symmetry

By discretization, a 1D field theory becomes a chain of sites and links with associated phases θ: $a \sim e^{i\,\theta}$

$$H \supset a_i\, b_i\, a^\dagger_{i+1} \sim e^{i\left(\theta_{a_i}+\theta_{b_i}-\theta_{a_{i+1}}\right)}$$

The link *b* absorbs the local phase transformation $\delta\theta$ of neighoring sites *a*.



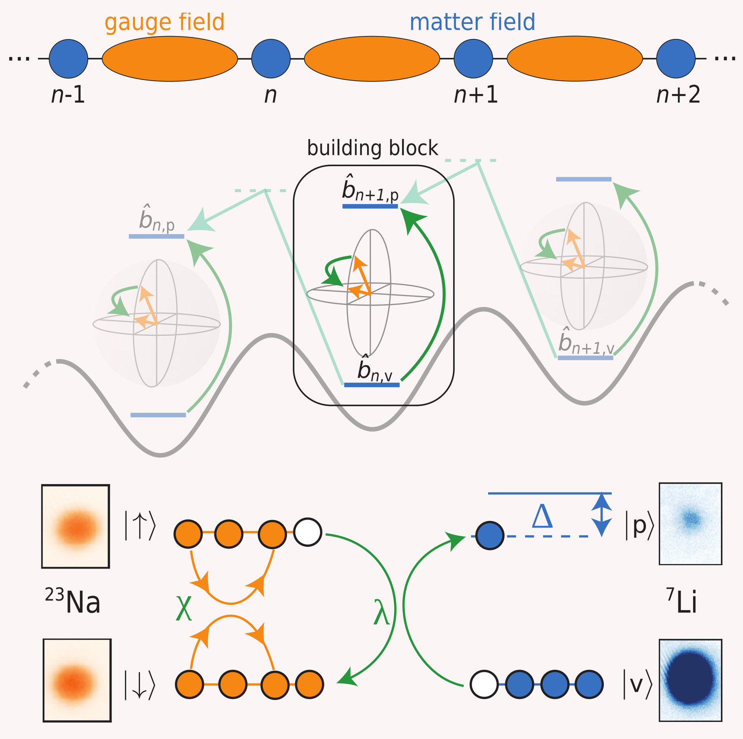
Cold Atoms & Ions

There are efforts to simulate increasingly complicated lattice field theories *trapped ion experiments* and in *cold atom experiments,* aimed at supplementing high-energy experiments for Standard Model theories.

Trapped ion experiments realized Schwinger pair production using quantum computation and variational quantum solvers. Martinez et al., Nature 534, 2016; Kokail et al., Nature 569, 2019

Cold atom experiments have proceeded to simulate $\mathbb{Z}_2$ lattice gauge theories. Schweizer et al., Nat Phys 15, 2019; Görg et al., Nat Phys 15, 2019

A building block for *U(1)* lattice gauge theories was implemented recently in a cold atom mixture experiment via spin-changing collisions. Mil et al., Sciene 367, 2020



(Reprinting Fig 1 from Mil et al., Science 367, 2020)

Both species are trapped in same harmonic potential. Two-level system of lithium represents neighboring sites.

Total sodium spin represents gauge field between neighboring sites.

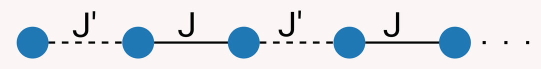
Building blocks are coupled by tunneling to form a chain.

SSH model as a circuit

In the past electronic circuits have been used to engineer metamaterials with topological boundary modes. Ningyuan et al., PRX 5, 2015; Imhof et al., Nat Phys14, 2018

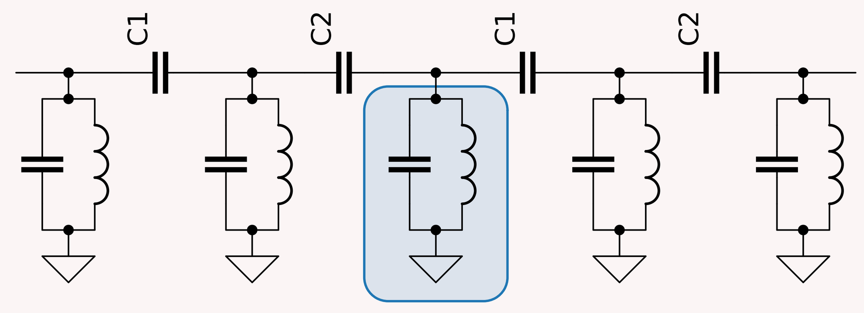
The *Su-Schrieffer-Heger (SSH)* model is the simplest Hamiltonian with topological properties.

Next-neighbor hopping with alternating coupling:



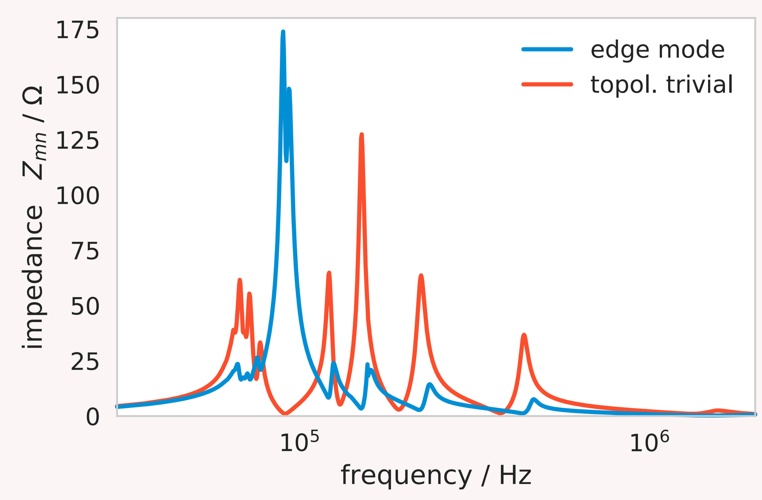
It is *topological*, because there is a one-to-one relation between *bulk* and *boundary properties.*

The circuit implementation for this model uses coupled LC oscillators: Lee, Commun Phys 1, 2018



Measure impedance across chain

→ edge-mode for correct configuration



The SSH model is also a *U(1) gauge theory* in the limit of highly occupied or driven links.

Translation from lattice theory to circuit

Gauge theories appear with very close analogies in *quantum* and *classical mechanics.*

We take the SSH Hamiltonian and supplement it with a link operator / variable, which is defined to absorb the local gauge transformation.

Lattice gauge theory	Circuit	Complexified classical variables
field at position	LC oscillator	$a = \frac{1}{\sqrt{2\omega}}\left(\frac{\Phi}{\sqrt{L}} + i\frac{Q}{\sqrt{C}}\right)$
SSH hopping term	coupling capacitor	$a^*_{j+1}a_j + \mathrm{H.c.} \sim Q_j Q_{j+1}$
U(1) link: hopping	symmetric multipliers	$a^*_{j+1}b_j\,a_j + \mathrm{H.c.} \sim Q_{a_j}Q_{b_j}Q_{a_{i+1}}$
U(1) link: gauge field	LC oscillator	

Flux linkage $\Phi = \int V\,\mathrm{d}t$

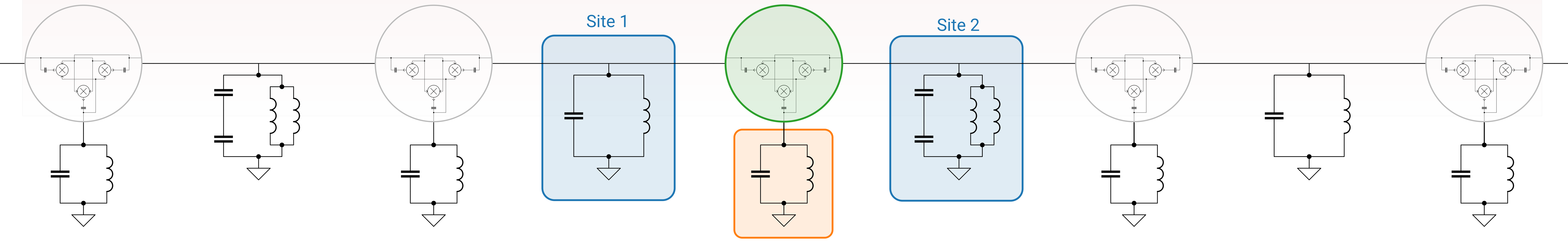
Charge $Q = \int I\,\mathrm{d}t$

Energy $n = a^*a = \frac{1}{2}\left(\frac{\Phi^2}{L} + \frac{Q^2}{C}\right)$

$$H = \omega\, a^*_1\, a_1 + 2\omega\, a^*_2\, a_2 + \omega\, b^*\, b + \lambda\left(a^*_1\, b^*\, a_2 + a^*_2\, b\, a_1\right)$$

Rotating wave approximation:
restrict to state space where symmetry is conserved
→ matching of oscillator resonances

Small coupling approximation:
conj. momenta are local to site (tight binding approximation)
$Q \propto \dot{\Phi}$



Conserved quantities

Local phase trafo: $a_j \to a_j\, e^{i\,\theta_j}$

Absorbed by link: $b_j \to b_j\, e^{-i\,\theta_j + i\,\theta_{j+1}}$

Complex phase corresponds to orthogonal rotation between Φ and Q. In classical mechanics this is a canonical transformation.

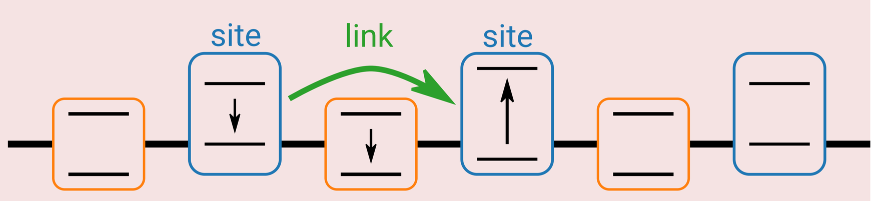
Continuous symmetry is associated to conserved quantities expressed as generating functions *G*:

$$\delta a_j = J\frac{\partial G}{\partial a^*_j}\,\delta\theta_j = \delta\theta_j\,\{a_j, G\}$$

In our case the generators are sums of oscillator energies:

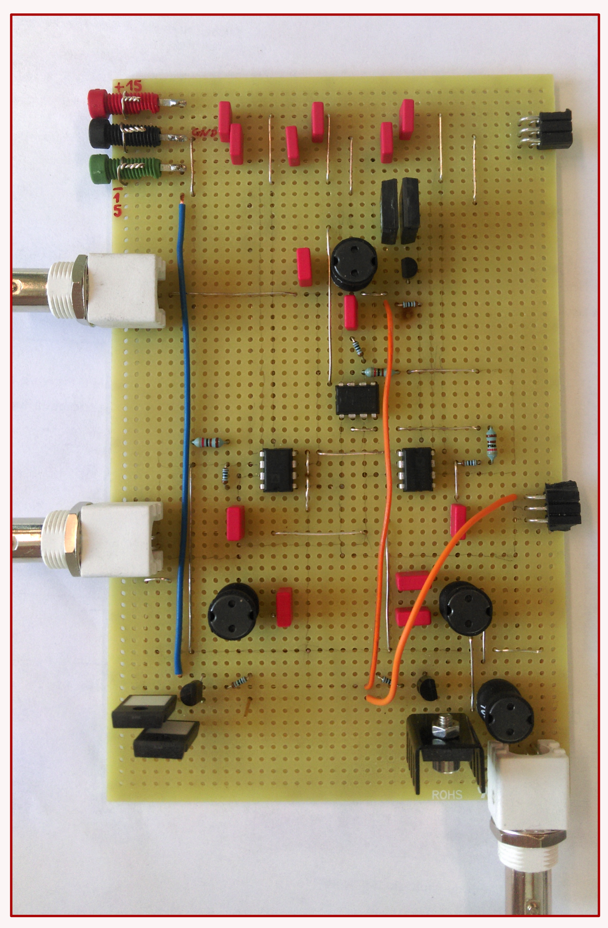
$$G_1 = a^*_1\, a_1 - b^*\, b$$
$$G_2 = a^*_2\, a_2 + b^*\, b$$
$$N = a^*_1\, a_1 + a^*_2\, a_2$$

(Including additional global *U(1)* symmetry.)



Reminiscent of discrete Gauss's law of Jaynes Cummings model.

The prototype

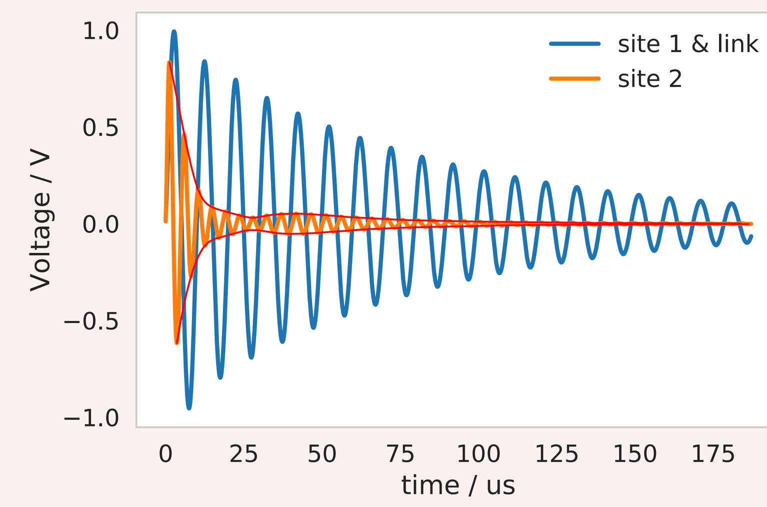


Outlook

Measure Rabi oscillations in building blocks

Detuning of oscillators causes energy sloshing.

High dissipation hides dynamics.



Connect building blocks to a chain

- U(1) link between each site: close to cavity QED
- Alternating U(1) link and static link: Bosonic Wilson formulation of QED.

Zache et al., Quantum Sci Tech 3, 2018

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