Experimental Observation of Oscillating and Interacting Matter Wave Dark Solitons

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(Received 7 April 2008; published 22 September 2008)

We report on the generation, subsequent oscillation and interaction of a pair of matter-wave dark solitons. These are created by releasing a Bose-Einstein condensate from a double well potential into a harmonic trap in the crossover regime between one dimension and three dimensions. Multiple oscillations and collisions of the solitons are observed, in quantitative agreement with simulations of the Gross-Pitaevskii equation. An effective particle picture is developed and confirms that the deviation of the observed oscillation frequencies from the asymptotic prediction $\nu_z/\sqrt{2}$, where ν_z is the longitudinal trapping frequency, results from the dimensionality of the system and the soliton interactions.

DOI: 10.1103/PhysRevLett.101.130401 PACS numbers: 03.75.Lm, 05.45.Yv

Solitons are one of the most prominent features of nonlinear dynamics emerging in diverse fields extending from hydrodynamics to solid state physics and from nonlinear optics to biophysics. Dark solitons are the fundamental excitations of the defocusing nonlinear Schrödinger equation [1], and have the form of a localized "dip" on a background wave, accompanied by a phase jump [2]. These localized waveforms have been demonstrated experimentally in different contexts, including liquids [3], discrete mechanical systems [4], thin magnetic films [5], optical media [6–8], and, more recently, Bose-Einstein condensates (BECs) [9–15]. The possibility of creating pairs of dark solitons [7] has stimulated considerable interest in the repulsive [16] short-range interactions between them [17,18]. The resulting collisions, during which the solitons approach within a distance comparable to their width, have a universal character and thus, e.g., optical solitons interact essentially the same way as matter-wave solitons.

In this Letter we report on the systematic generation of a pair of matter-wave dark solitons, which is subsequently oscillating and colliding in a harmonic trap. Our experiment is performed in the crossover regime between one and three dimensions [19], where dark solitons exist and are robust [20]. This allows us to monitor multiple oscillations and collisions of dark solitons, permitting the precise measurement of their oscillation frequency and their mutual repulsive interactions. Previous experiments have been performed in a genuine 3D regime where dark solitons are unstable due to the so-called snaking instability and eventually decay into vortex rings [11,20]. In these experiments, solely their translation in the trap has been shown [9–11]. Only very recently dark solitons have been reported to undergo a single oscillation period in a harmonic trap [15].

Different methods have been explored to create dark solitons in Bose-Einstein condensates [9–15]. In our experiment, the solitons are generated by merging two coherent condensates initially prepared in a double well potential. This formation process can be regarded as a consequence of matter-wave interference of the two condensates [21–24]. The further evolution of the created solitons in the trap is shown in Fig. 1(a). Our procedure is very similar to the recently reported generation of vortices out of a triple well potential [25].

Since the two dominant solitons are created with a distance of a few healing lengths ξ (ξ is on the order of 250 to 400 nm), which defines the range of the repulsive soliton interaction, the collisions between them lead to a significant modification of the oscillation frequency. The measured frequencies deviate up to 16% from the single soliton asymptotic Thomas-Fermi 1D (TF1D) prediction of $\nu_z/\sqrt{2}$ [26], where ν_z is the longitudinal trapping frequency. Our experimental results are in quantitative agreement with numerical simulations of the Gross-Pitaevskii equation (GPE). They reveal that dark solitons can behave very similar to particles. This is confirmed by explaining the essential features of the dynamics within a simple physical picture regarding the dark solitons as particles in an effective potential due to the external trap and their mutually repulsive interactions. Being in the crossover regime, the role of the transverse degrees of freedom has to be included in the effective potential [27].

Before elaborating on the theoretical models and systematic studies we will briefly describe the details of the experimental setup. We prepare a BEC of ^{87}Rb in the $|F = 2, m_F = 2\rangle$ state, containing about $N = 1500$ atoms in a double well potential. This potential is realized by superimposing a far detuned crossed optical dipole trap

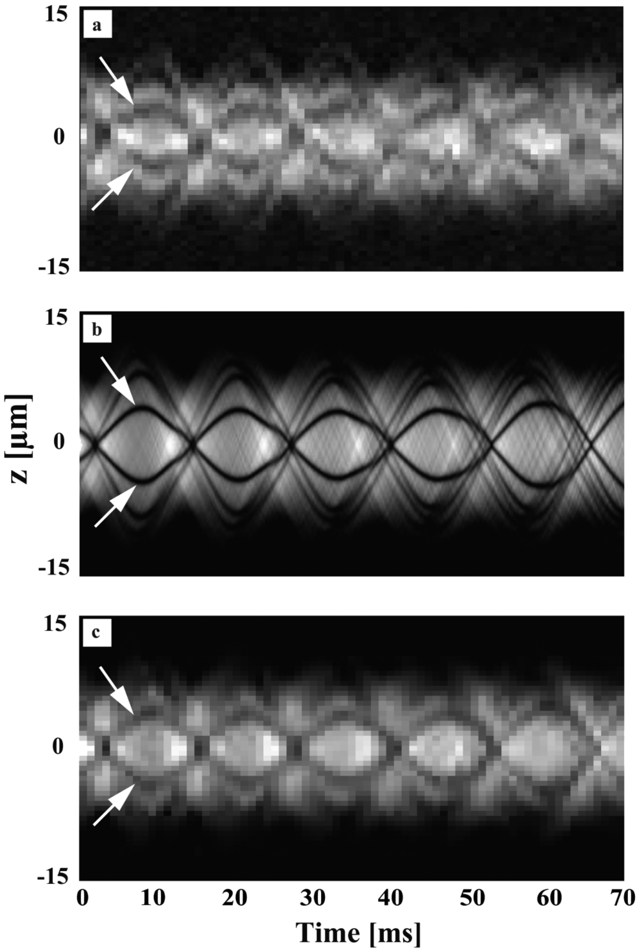

FIG. 1. Observation of the time evolution of dark solitons in a harmonic trap. The dominant soliton pair is indicated by arrows. (a) Experimental observation of the dynamics of the longitudinal atomic density. Each longitudinal density profile (vertical lines), corresponding to a given evolution time, is deduced from typically 10 experimental realizations. The obtained absorption images of the condensate at each time step are averaged and integrated over their transverse direction. The number of atoms in the shown case is $N = 1700$ and the trapping frequencies are $(\nu_z, \nu_\perp) = (53\text{ Hz}, 890\text{ Hz})$. (b) Result of the numerical integration of the 3D GPE taking into account the full preparation process of the solitons. (c) Same as (b), taking into account the finite spatial (1 μm) as well as temporal resolution (1 ms) of the experiment. The loss of contrast due to the convolution process explains the experimentally observed fading out of the solitons with time.

($\lambda = 1064$ nm) and a one dimensional optical lattice ($\lambda = 843$ nm). The first beam of the dipole trap has a Gaussian waist of 5 μm and results in a strong transverse and weak longitudinal confinement. The second beam orthogonally crosses the first one and has an elliptic shape (60 μm $\times$ 230 μm waist) leading to an extra adjustable confinement only in the longitudinal direction of the trap. We start our experiments with a transverse frequency of the total harmonic trap of $\nu_\perp = 408$ Hz and a longitudinal one of $\nu_z = 63$ Hz. The barrier height of the optical lattice is chosen to be approximately 1 kHz and the lattice spacing is 5.7 μm. This results in a double well potential with a well distance of 5.4 μm.

In order to start with a well-defined phase between the two condensates, the barrier height is chosen to be low enough such that thermal phase fluctuations are negligible for the measured temperature of $T \approx 10$ nK [28] (the critical temperature for condensation is $T_c \approx 110$ nK) and high enough so that high contrast solitons are formed. The solitons are created by switching off the optical lattice and merging the two condensates in the remaining harmonic potential. After the switching off, the trap frequencies are ramped to the parameters of interest $(\nu_z, \nu_\perp)$. The distance between the formed solitons is adjusted by choosing different sets of final frequencies and different atom numbers. For each parameter set, the ramping time is empirically optimized to minimize the excitation of the quadrupole mode [e.g., from $(\nu_z, \nu_\perp) = (63\text{ Hz}, 408\text{ Hz})$ to (53 Hz, 890 Hz) within 10 ms for $N = 1700$ atoms, or to (58 Hz, 408 Hz) within 3 ms for $N = 950$]. The atomic density after a certain evolution time in the harmonic trap is obtained using standard absorption imaging with an optical resolution of approximately 1 μm. We use a short time of flight between 0.6 and 0.9 ms to enhance the contrast.

In our experiment, the initial distance $D = 5.4$ μm between the two colliding condensates is well within the regime where the interaction energy exceeds the kinetic energy and thus the formation of dark solitons is expected due to nonlinear interference. This regime is reached if D is smaller than the critical distance $D_c = \pi(6\frac{N\hbar a_s}{\nu_z m})^{1/3} = 25.8$ μm with a_s being the s-wave scattering length, ν_z the longitudinal trap frequency and m the atomic mass [22]. The formation of dark solitons for our experimental parameters is confirmed by 3D GPE simulations as shown in Fig. 1. Including the optical and time resolution, the experimentally observed density profile evolution is well reproduced. A dominant pair of solitons oscillates close to the center of the cloud and we can also distinguish additional pairs of solitons with much lower contrast. In the following, we focus on the dynamics of the dominant central pair and show that its oscillation frequency is well described within a two soliton approximation.

We experimentally investigate the oscillation frequency of the dominant soliton pair for different trap parameters and different intersoliton distances. A typical data set consists of 50 time steps and 10 pictures per time step. The numerical simulations predict that the solitons do not cross each other at the collision points [see inset of Fig. 3(c)], but our finite resolution does not allow us to distinguish whether this is actually the case in the experiment. In order to extract the oscillation frequency of the solitons, we fit the time evolution of the intersoliton distance as shown in the inset of Fig. 2. The obtained frequency is divided by two in order to compare it to the oscillation frequency expected for a single trapped soliton. The shot to shot reproducibility of the soliton dynamics up to 100 ms allows the observation of up to 7 oscillation periods. The typical statistical experimental error in the frequency measurement is $\pm 1.5\%$. Figure 2 shows the results of our frequency measurements and their comparison with numerical simulations for the motion of two trapped solitons using the nonpolynomial Schrödinger equation (NPSE) [29], which is an excellent approximation to the 3D GPE in the dimensionality crossover regime [27].

In order to capture the essentials of the dynamics of the experimentally realized soliton pairs in the simulations, we initialize the condensate with two solitons such that the rms amplitude of their oscillating motion matches the one observed experimentally. The good agreement between numerics and experiments shows that the dynamics produced by our experimental method is well described within a two soliton approximation even though extra solitons are produced. From our experiment and the NPSE simulations, we observe an upshift up to 16% from the $\nu_z/\sqrt{2}$ prediction which was the first value theoretically derived for the oscillation frequency of a single trapped soliton [26]. It is expected to be valid in a 1D trap in the asymptotic Thomas-Fermi limit ($N\Omega a_s/a_\perp \ll 1$ and $((N/\sqrt{\Omega})a_s/a_\perp)^{1/3} \gg 1$) [19], where $\Omega = \nu_z/\nu_\perp \ll 1$ is the aspect ratio of the trap and $a_\perp$ the transverse harmonic oscillator length. Our experimental parameter range is $\Omega \approx 0.06$–0.14, $N\Omega a_s/a_\perp \approx 1.2$–$1.8$ and $((N/\sqrt{\Omega})a_s/a_\perp)^{1/3} \approx 2.8$–$4.4$, which sets us out of the validity domain of the $\nu_z/\sqrt{2}$ prediction.

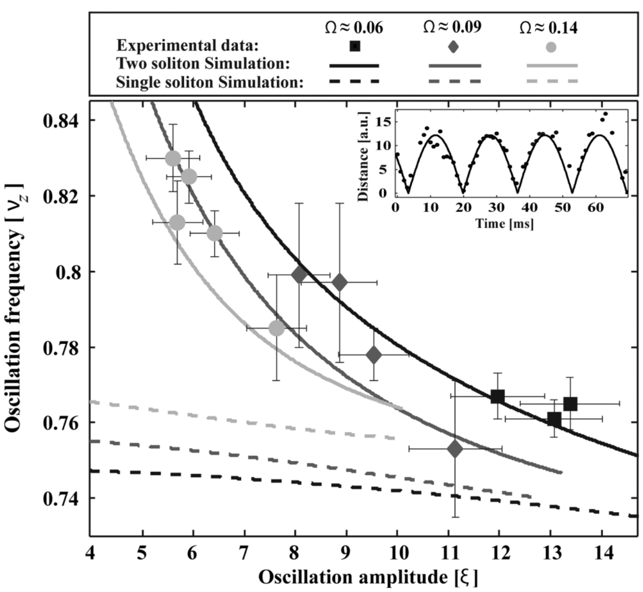

FIG. 2. Comparison between experimentally obtained soliton oscillation frequencies and NPSE simulation for one and two solitons. Each frequency point is deduced from the temporal evolution of the soliton distance as shown in the inset. Different symbols correspond to different aspect ratios Ω of the trap. For each aspect ratio the oscillation amplitude is varied as explained in the text. NPSE simulations are represented by solid lines for the two soliton case, and by dashed lines for the respective single soliton oscillations. The error bars on the measured frequencies account for statistical errors on the measured soliton and trap frequencies and systematic errors on the atom number used to calculate the healing length.

We now give a theoretical description of the different effects leading to the observed upshift. We consider the two solitons as particles moving in an effective potential which arises from the combination of a harmonic potential due to the trap [26] [see Fig. 3(a)] and a repulsive potential due to the interaction between the solitons [30]. Because of the spatially symmetric preparation, the effective potential is a symmetric double well potential which is depicted in Fig. 3(b). This potential can be expressed as a function of the distance z of each of the solitons from the trap center and its time derivative $\dot{z}$:

$$V(z, \dot{z}) = (2\pi\nu_{1s})^2 \frac{z^2}{2} + \frac{\mu B^2}{2m\cosh^2(2Bz/\xi)}, \qquad (1)$$

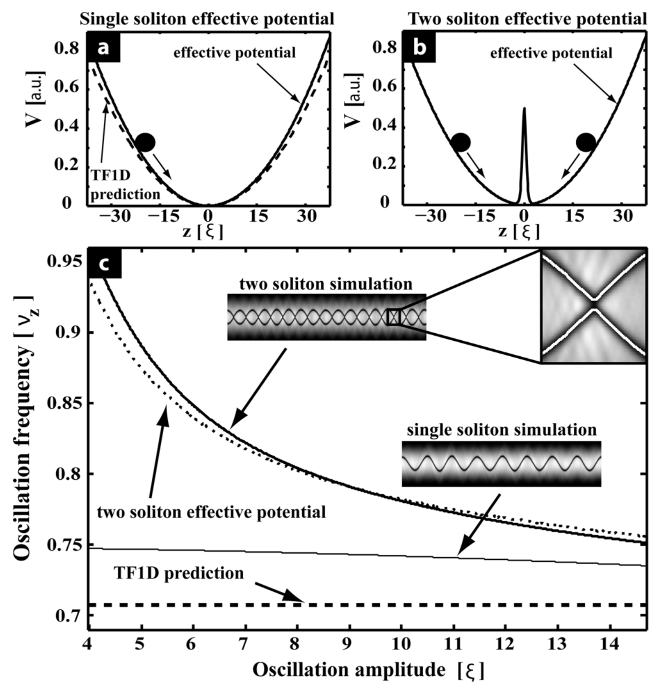

FIG. 3. The oscillation dynamics of dark solitons in a trapped BEC is well captured in an effective particle picture. (a) For one soliton, the particle moves in a harmonic trap. (b) For two solitons, an additional barrier due to the repulsive interaction appears. (c) For the one and two soliton case, the dependence of the oscillation frequencies on the oscillation amplitude from the trap center is shown for one experimental parameter set with $\Omega = 0.06$. The dashed line shows the TF1D prediction ($\nu_z/\sqrt{2}$). The thin solid line indicates the upshift of the single soliton frequency mainly due to dimensionality. For the case of two solitons, the thick solid line also includes the upshift due to the intersoliton interaction deduced from the NPSE. The dotted line represents the result of the simple effective particle model from Eq. (1). Density profile evolutions obtained from the NPSE are shown in the insets. A collision between the two solitons is also shown in detail, demonstrating that they do not cross each other. The white lines correspond to the trajectories of the density minima.

where $B = \sqrt{1 - (\dot{z}/\xi)^2(\hbar/\mu)^2}$ denotes the darkness of the solitons, μ is a typical interaction energy on the order of the chemical potential, $\xi = \sqrt{\hbar/(m\mu)}$ the associated healing length and ν_{1s} the oscillation frequency of a single trapped soliton. The frequency of the motion is obtained by solving the Euler-Lagrange equation associated with the Lagrangian $\mathcal{L}(z, \dot{z}) = \dot{z}^2/2 - V(z, \dot{z})$. To obtain quantitative agreement, the model has to take into account correctly both the free propagation of the solitons in the trap when they are far away from each other ($z \gg \xi$) and the repulsive interaction when they approach.

Good estimates for the single soliton frequency ν_{1s} are obtained by numerical integration of the NPSE describing a single soliton. Because our experimental parameters are both in the crossover regime and slightly out of the Thomas-Fermi limit, corrections to the asymptotic value $\nu_z/\sqrt{2}$ are expected. Therefore the oscillation frequency of a single dark soliton is upshifted by a few percent from the asymptotic value as discussed in detail using the Bogoliubov–de Gennes analysis of the NPSE in [27] [see Fig. 3(a)]. The simulation results for the three different parameter sets used in the experiment are shown in Fig. 2. This upshift for the single soliton case can be decomposed into two contributions. For example, considering one specific parameter set with $\Omega \approx 0.06$, the upshift is 5% [see Fig. 3(c)]. Predictions using the 1D GPE already give a value approximately 2% higher than the asymptotic limit because the Thomas-Fermi limit is not reached [31]. The effect of dimensionality, i.e., the role of the transverse degrees of freedom which is captured only by the NPSE or the 3D GPE, accounts for the remaining 3%. Figure 3(c) shows the comparison between the $\nu_z/\sqrt{2}$ prediction and the single soliton NPSE simulation for the considered parameter set.

As also shown in Fig. 3(c), the repulsive interaction between the solitons results in an additional upshift of the oscillation frequency compared to the single soliton case that strongly depends on the oscillation amplitude. Our effective particle model accurately reproduces the upshift if the interaction parameter μ is set to be the chemical potential of the condensate obtained from the 3D GPE equation. In our experimentally accessible parameter range, the agreement of the model with NPSE simulations is better than 5%. This allows us to clearly identify the significant role of the repulsive interactions and shows that the effective repulsive potential in Eqn. (1) obtained in the 1D homogeneous case is a good approximation to our complex situation.

In conclusion, we controllably create pairs of dark solitons by colliding two atomic clouds released from a double well potential in a harmonic trap. The full dynamics of multiple dark soliton oscillations and collisions is observed, allowing for precise frequency measurements and showing that dark solitons are still stable after several collisions. The experimentally observed total upshifts from the TF1D frequency prediction are up to 16%. A simple effective particle picture confirms that the oscillation frequency of two solitons in a harmonic trap is affected by two effects, namely, the single soliton frequency upshift and the intersoliton interaction. The presented robust method for preparing solitonic excitations will be a starting point for further studies towards multisoliton interactions and perhaps even dark soliton gases.

We thank P. Schmelcher for useful discussions as well as B. Hemmerling, R. Gati, and T. Ottenstein. We acknowledge support from NSF, DFG, and AvH. J. E. acknowledges support from the EC MC-EIF program.

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