Quantifying Entanglement in Bose-Einstein-Condensates using Entropic Uncertainty Relations

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Summary

Central question:
How to quantify entanglement in quantum many-body systems?

- Use entropic uncertainty relations to gain bounds on entanglement entropy
- Experimental implementation using spinor Bose-Einstein condensates

Long Term Goal:

- Testing local thermalization and fluid dynamics through quantum information

Entropic Uncertainty Relations

Classical Shannon Entropy:

$$H(X) = -\sum_x P(x)\log P(x)$$

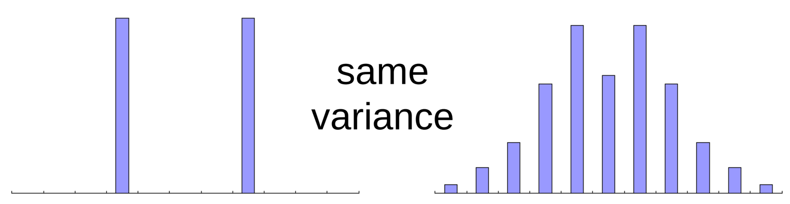
Quantum Von-Neumann Entropy:

$$S(\rho) = -\mathrm{Tr}\{\rho \ln \rho\}$$

Uncertainty relations:

Robertson uncertainty relation

$$\sigma(X)\sigma(Z) \geq \frac{1}{2}|\langle[X,Z]\rangle|$$



Entropic uncertainty relation

$$H(X) + H(Z) \geq \log\frac{1}{c} + S(\rho)$$

$$c = \max_{x,z}|\langle\phi_x|\psi_z\rangle|^2$$

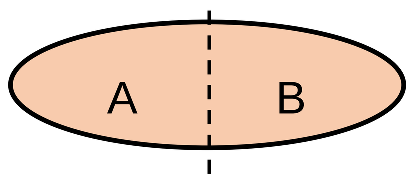
maximal eigenstate overlap

Coles *et al.* RMP **89**, 015002 (2017)

Bipartite Systems:

$$H(X_A|X_B) + H(Z_A|Z_B) \geq \log\frac{1}{c} + S(A|B)$$

Classical conditional entropy (measurable) — Quantum conditional entropy

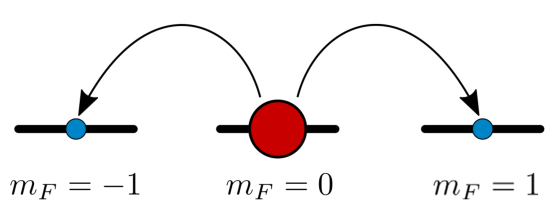


Quantum-Conditional Entropy:

- $S(A|B) = S(\rho_{AB}) - S(\rho_B)$
- For pure states $S(A|B) = -S(\rho_B)$ ⟵ entanglement entropy
- For separable States $S(A|B) \geq 0$, so $S(A|B) < 0$ is entanglement witness
- $-S(A|B)$ lower bound on distillable entanglement (entanglement of formation)

Experimental Setup

Rubidium BEC: Create two-mode squeezed state through spin-changing collisions



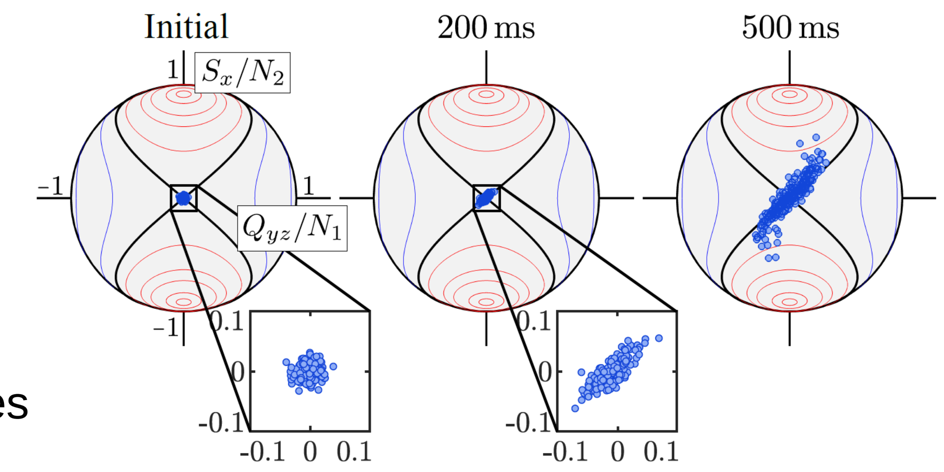
$$|\psi(t)\rangle = \sum_n c_n|n, N_0 - 2n, n\rangle$$

Theoretical description:

- 3-mode Fock space $|N_{-1}, N_0, N_1\rangle$
- Observables are spin 1 operators:

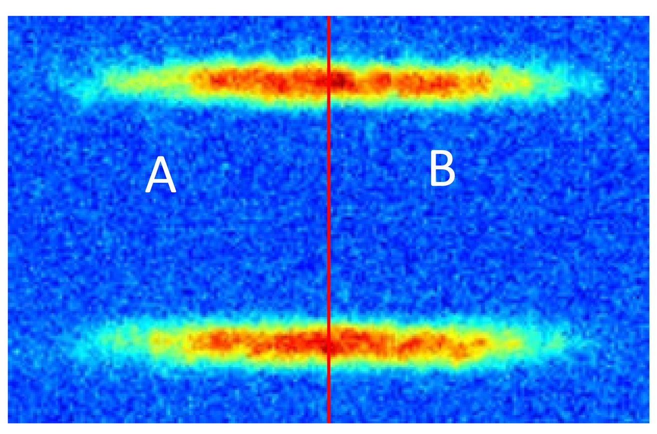
$$S_x, S_y, S_z, Q_{yz}, \ldots$$

- So U(3) set of possible observables



Bipartite system

- Prepare in single spatial mode
- Split into subsystems post measurement



Key theoretical challenge:

- Find ideal observables for entanglement detection

Improved State Dependent Bounds

Previous relations only tight for MUBs: $c_{xz} = |\langle\phi_x|\psi_z\rangle|^2 = const.$
Cannot be realized for our system.

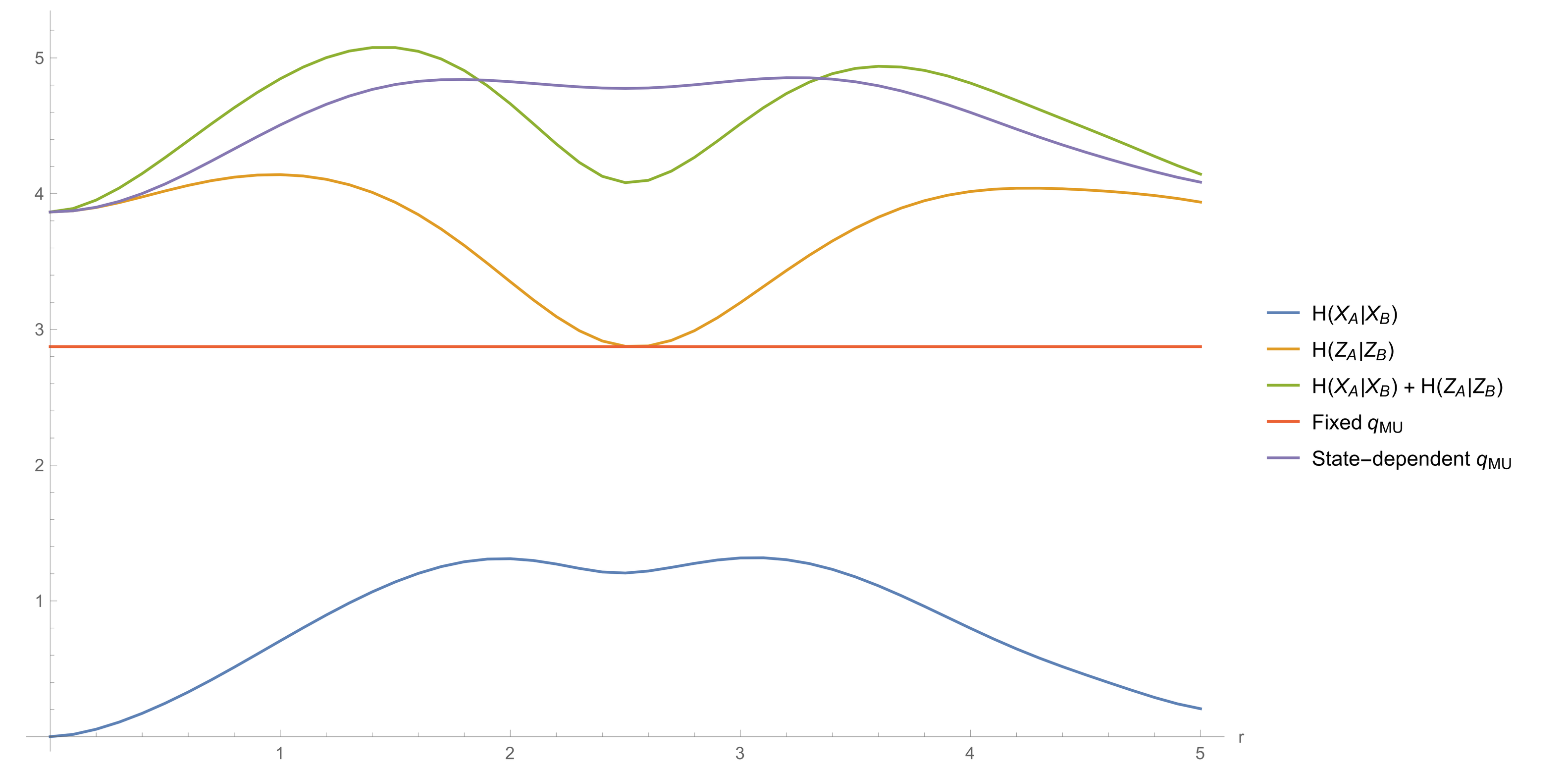
We proved bounds with state-dependent (but measurable) right-hand side:

$$H(X_A|X_B) + H(Z_A|Z_B) \geq S(A|B) - \sum_{x,x} P(x_A, x'_B)\log\left(\sum_z P(z_A|X_B = x'_B)\, c_{xz}\right)$$

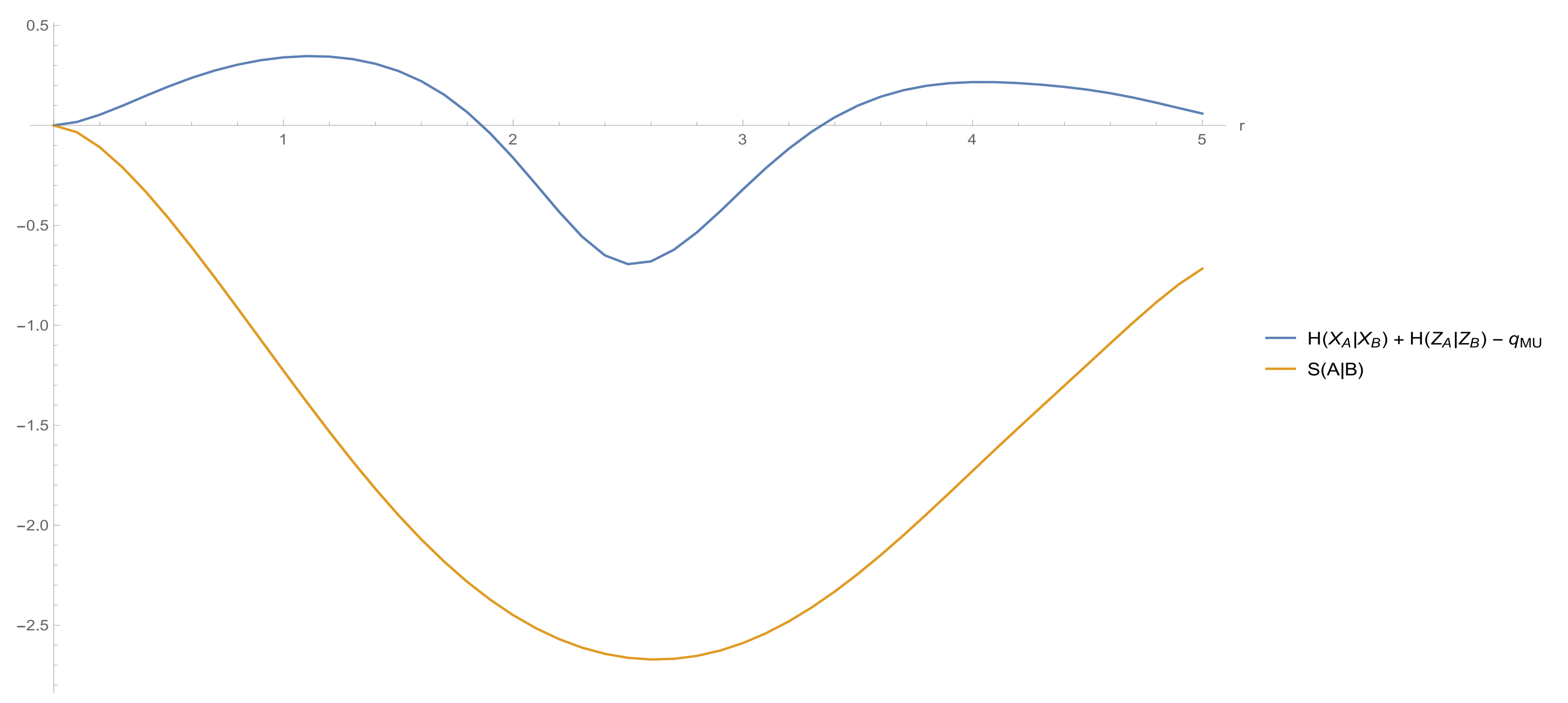
Strict generalization of state-independent relations

Numerical Simulations

Numerical simulations show the possibility to get some entanglement quantification for certain states in our system (depends significantly on chosen observables)

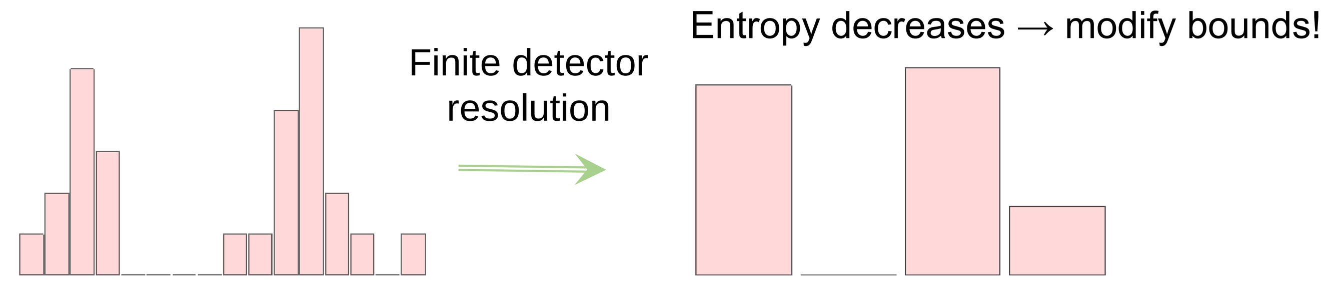


Bounds are not tight, no entanglement detected at small squeezing



Remaining Questions

- Finding optimal observables: so far can only guess and then calculate
- Numerically optimizing over U(3) is very expensive
- Investigate measurement statistics: Need full bipartite probability distribution
- Include total particle number fluctuation and effects of experimental noise
- Investigate effects of coarse graining



Long Term Goals

- Get access to $S(A|B)$ such that entanglement growth in scenarios as thermalization or fluid dynamics can be quantified
- Expand the single mode setup to multimode and do a continuum limit to experimentally probe quantum information quantities in a QFT setting