EPR entanglement strategies in two-well BEC

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Criteria suitable for measuring entanglement between two different potential wells in a Bose-Einstein condensation (BEC) are evaluated. We show how to generate the required entanglement, utilizing either an adiabatic two-mode or dynamic four-mode interaction strategy, with techniques that take advantage of s-wave scattering interactions to provide the nonlinear coupling. The dynamic entanglement method results in an entanglement signature with spatially separated detectors, as in the Einstein-Podolsky-Rosen (EPR) paradox.

One of the most important questions in modern physics is the problem of macroscopic spatial entanglement, which directly impinges on the nature of reality. Here we analyse how rapid advances in Bose-Einstein condensation (BEC) in ultra-cold atoms can help to resolve this issue. Recently, the observation of spin-squeezing has shown that measurement beyond the standard quantum limit is achievable [1–3]. Spin squeezing is known to demonstrate entanglement between atoms [4], but not which subsystems have been entangled. An important step forward beyond this would be to realise quantum entanglement in the Einstein-Podolsky-Rosen (EPR) sense; that is, having two spatially separated condensates entangled with each other [5]. This is an important milestone towards future experiments involving entanglement of macroscopic mass distributions, thereby demonstrating quantum Schroedinger cat type superpositions of distinct mass distributions.

In this Letter, we analyse some achievable entangled quantum states using a two-well BEC, and the measurable criteria that can be used to signify entanglement. The types of quantum state considered include number anti-correlated states prepared using adiabatic passage, as well as dynamically prepared spin-squeezed states. In particular, we focus on spin-entanglement, as a particularly useful route for achieving measurable EPR entanglement, without requiring atomic local oscillators. We note that spin orientation is easily coupled to magnetic forces to allow superpositions of different mass distributions, once spin entanglement is present. We consider different types of spin entanglement criteria, and analyze which quantum states these are sensitive to.

We show that existing experimental techniques appear capable of generating spatial entanglement, with relatively minor changes. There are several possible routes available. Our most significant conclusion is that the criterion used to measure entanglement must be chosen carefully. Not all measures of entanglement are equivalent, and there is an important question as to what one regards as the fundamental subsystems, ie, particles or modes. The appropriate choice of measure depends on the entangled state, how it is prepared, and what type of detection is technologically feasible. To demonstrate and analyse this need to adapt the criterion to the state, we choose here to analyse two and four mode models of a BEC, indicated schematically in Fig. 1, where a_1, a_2 are operators for two internal states at A and b_1, b_2 are operators for two internal states at B.

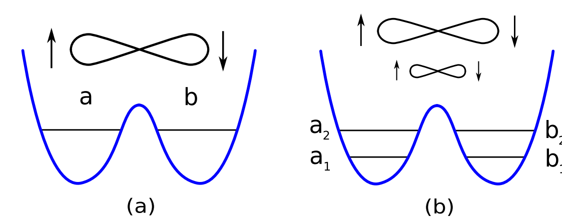

Figure 1. (a) Two internal modes a, b with spatial entanglement; (b) two pairs of modes a_1, a_2 and b_1, b_2 are entangled.

In the limit of tight confinement and small numbers of atoms, this type of system can be treated using a simple coupled mode effective Hamiltonian, of form:

$$\hat{H}/\hbar = \kappa \sum_i \hat{a}_i^\dagger \hat{b}_i + \frac{1}{2}\left[\sum_{ij} g_{ij}\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i\right] + \left\{\hat{a}_i \leftrightarrow \hat{b}_i\right\} \ . \tag{1}$$

Here κ is the inter-well tunneling rate between wells, while g_{ij} is the intra-well interaction matrix between the different spin components.

Adiabatic preparation: We first consider two-mode states having a single spin orientation, with number correlations established using adiabatic passage in the ground state. This makes them practical to prepare following earlier experimental approaches [1, 6], as shown in Fig. 1(a). A recent multi-mode analysis shows that effects of other spatial modes may be relatively small [7]. In a two-mode analysis, we assume that a_1 and b_1 have been prepared in the many-body ground state of Eq (1) with a fixed number of atoms N, while the second pair of spin states a_2 and b_2 remain in the vacuum state, so that we can write $a \equiv a_1$ and $b \equiv b_1$. In these cases there is only one nuclear spin orientation, and there is existing experimental data on phase coherence and number correlations [1, 6], with $10dB$ relative number squeezing being maximally indicated. A number of previous analyses have used entropic measures specific to pure states to study entanglement. These signatures cannot be readily measured, and are not applicable to realistic mixed states that are typically created in the laboratory.

However, one generally demonstrate spatial entanglement between the two wells a and b using the non-Hermitian operator product criterion of Hillery and Zubairy (HZ) [8]. This is also related to a recently developed continuous-variable Bell inequality criterion [9]. A sufficient entanglement criterion between A and B is the operator product measure:

$$|\langle a^\dagger b\rangle|^2 > \langle a^\dagger a b^\dagger b\rangle \,. \tag{2}$$

Interwell spin operators have already been measured in this environment. These are defined as: $\hat{J}^X_{AB} = \left(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger\right)/2$; $\hat{J}^Y_{AB} = \left(\hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger\right)/(2i)$; $\hat{J}^Z_{AB} = \left(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}\right)/2$; $\hat{J}^{\pm}_{AB} = \hat{J}^X_{AB} \pm i\hat{J}^Y_{AB}$; $\hat{N}_{AB} = \hat{N}_A + \hat{N}_B = \hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}$.

In spin language, the HZ criterion shows that spatial entanglement is proved for any state when

$$\begin{aligned} E_{HZ} &= \frac{\langle \Delta\hat{J}^+_{AB}\Delta\hat{J}^-_{AB}\rangle}{\langle\hat{N}_A\rangle} \\ &= \frac{\frac{1}{4}\langle[N_A+N_B]^2\rangle - \langle\left[\hat{J}^Z_{AB}\right]^2\rangle}{|\langle J^X_{AB}\rangle|^2 + |\langle J^Y_{AB}\rangle|^2} < 1 \,. \end{aligned} \tag{3}$$

This has similarities to the spin squeezing criterion [10] which has now been measured experimentally [1, 2]. However, a crucial difference is that the spatial entanglement criterion (3) involves an increased relative number fluctuation, rather than the reduced relative number fluctuations found with the spin-squeezing criterion. Theoretically, we find that two-well entanglement exists in the ground state with the HZ criterion, although suppressed for increasingly strong repulsive interactions. This behaviour is also known from previous studies using an entropic $\varepsilon(\rho)$ entanglement measure [5, 11]. The strongest theoretical entropic entanglement is found when all atom numbers are equally represented in the superposition. We find that the closest state to this 'super-entangled' limit is obtained at a critical value of $Ng/\kappa \simeq -2$. This attractive interaction regime (as found in ^{41}K and ^{7}Li isotopes) gives rise to a maximal spread in the distribution of numbers in each well. Maximum entanglement results for this model have also been found [11] for entropic entanglement measures. In our calculations, we account for effects of finite temperatures by assuming a canonical ensemble of $\hat{\rho} = \exp\left[-\hat{H}/k_BT\right]$, with an interwell coupling of $\hbar\kappa/k_B = 50nK$. Our result for the Hillery-Zubairy operator product signature is graphed below. This shows that two-well spatial entanglement is maximized for an attractive inter-atomic coupling, and the effect is relatively robust against thermal excitations:

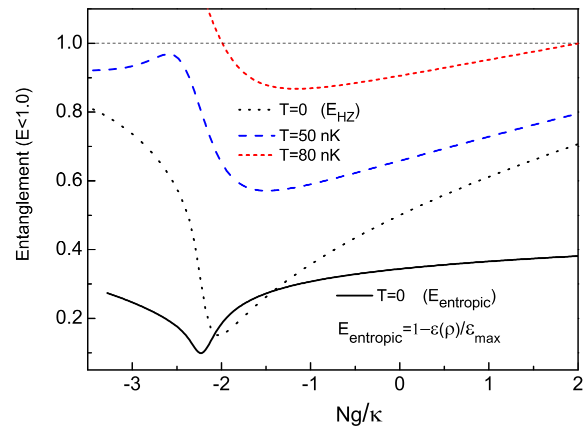

Figure 2. Adiabatic entanglement with interactions in a two-well potential. Dashed and dotted lines: HZ entanglement signature ($E_{HZ} < 1$) at $T = 0K$, $50nK$, $80nK$ - lowest lines at lowest temperature; solid line: entropic entanglement ($E_{entropic} = 1 - \varepsilon(\rho)/\epsilon_{max} < 1$) at $T = 0K$.

Dynamic preparation: To proceed further, EPR entanglement as we define it requires using measurements O_A and O_B that are individually defined either at well A or well B. Thus, entanglement is shown by performing a set of simultaneous measurements on the spatially separated systems: typically by measuring correlations $< O_A O_B >$ or $P(O_A, O_B)$. This is necessary to justify Einstein's "no action at a distance" assumption, that making one measurement at A cannot affect the outcome of another measurement at B. One could achieve EPR entanglement with this criterion by making quadrature amplitude measurements. That is, by expanding $a = X_a + iP_a$ and $b = X_b + iP_b$, where X_a etc are "quadrature amplitudes", so that the moment $< ab^\dagger >$ is measured as four separate real correlations. Proposed methods for measuring entanglement in BEC experiments include time-reversed dynamics [12], and interference with side-modes of a BEC moving through an optical lattice [13]. This shows that, in principle, such a quadrature-based entanglement measurement is not impossible. However, while feasible optically, this type of measurement is nontrivial with ultra-cold atoms owing to interaction induced phase fluctuations, and we propose a different strategy.

To get good EPR measurements we consider instead the intra-well "spins" J^X, J^Y, and J^Z at site A and B. This means having at least four modes in total. To prove EPR entanglement using these measurements, one can define the spin measurements at A to be in terms of a_1 and a_2: $\hat{J}_A^X = \left(\hat{a_1}^\dagger \hat{a_2} + \hat{a_2}^\dagger \hat{a_1}\right)/2$, $\hat{J}_A^Y = \left(\hat{a_1}^\dagger \hat{a_2} - \hat{a_2}^\dagger \hat{a_1}\right)/(2i)$, $\hat{J}_A^Z = \left(\hat{a_1}^\dagger \hat{a_1} - \hat{a_2}^\dagger \hat{a_2}\right)/2$, $\hat{N}_A = \hat{a_1}^\dagger \hat{a_1} + \hat{a_2}^\dagger \hat{a_2}$; also define raising and lowering operators as: $\hat{J}_A^{\pm} = \hat{J}_A^X \pm i\hat{J}_A^Y$, and similar definition for site B.

These are measurable locally using Rabi rotations and number measurements, without local oscillators being required. The spin orientation measured at each site can be selected independently to optimise the criterion for the state used. One can then show EPR entanglement via spin measurements using the spin version of the Heisenberg-product entanglement criterion [14]

$$E_{product} = \frac{2\sqrt{\Delta^2 \hat{J}_{AB}^{\theta\pm} \cdot \Delta^2 \hat{J}_{AB}^{(\theta+\frac{\pi}{2})\pm}}}{|\langle J_A^Y \rangle| + |\langle J_B^Y \rangle|} < 1 \,, \tag{4}$$

or the sum criterion by Duan et al and Simon [15, 16]

$$E_{sum} = \frac{\Delta^2 \hat{J}_{AB}^{\theta\pm} + \Delta^2 \hat{J}_{AB}^{(\theta+\frac{\pi}{2})\pm}}{|\langle J_A^Y \rangle| + |\langle J_B^Y \rangle|} < 1 \,, \tag{5}$$

with general sum and difference spins $\hat{J}_{AB}^{\theta\pm} = \hat{J}_A^\theta \pm \hat{J}_B^\theta$, and $J^\theta = cos(\theta)J^Z + sin(\theta)J^X$. Here the conjugate Schwinger spin operators J^θ and $J^{\theta+\pi/2}$ obey the uncertainty relation $\Delta^2 J^\theta \Delta^2 J^{\theta+\pi/2} \geq \frac{1}{4}|\langle J^Y \rangle|$.

In order to obtain ultra-cold atomic systems with four-mode entanglement, we consider a dynamical approach to EPR entanglement which utilizes phase as well as number correlations. This requires the BEC's to evolve in time, in a similar way to successful EPR experiments in optical fibres [15, 17, 18]. This is very different to the previous scheme, as the atom-atom interaction appears explicitly as part of the time-evolution. The best entanglement is obtained when the interaction between atoms of different spin is different to the interaction between the atoms of the same spin. In Rubidium, this either requires using a Feshbach resonance to break the symmetry, or else separating the two spin components spatially as in the successful fibre experiments [18] or in spin-squeezing atom-chip experiments [3]. At a Feshbach resonance, for alkali metals like Rubidium-87, the interactions between the different spin orientations can be reduced compared to the self-interactions. This allows an avenue for this type of entanglement with both the spin orientations remaining *in situ* in the same trap potential.

To start with, we consider the conditions required to obtain the best squeezing of Schwinger spin operators by optimizing the phase choice θ: $tg(2\theta) = 2\langle J^Z,\ J^X \rangle/(\Delta^2 J^Z - \Delta^2 J^X)$. Entanglement can be generated by the interference of two squeezed states on a 50 : 50 beam-splitter with a relative optical phase of $\varphi = \pi/2$. This has also been achieved in optical experiments [15], although not yet in BEC.

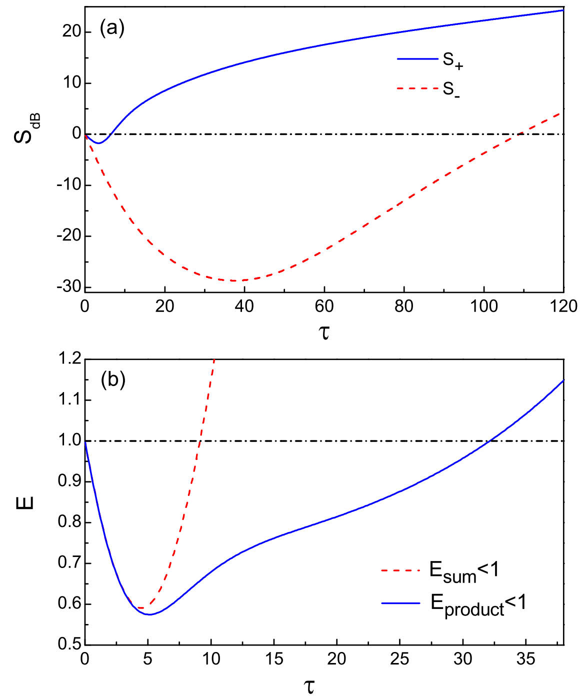

Figure 3. (a) Squeezing of Schwinger spin operators S_{dB}: $S_+ = 10log_{10}\left[\Delta^2(J_A^\theta - J_B^\theta)/n_0\right]$ (solid), $S_- = 10log_{10}\left[\Delta^2(J_A^{\theta+\pi/2} + J_B^{\theta+\pi/2})\right]/n_0$ (dashed), and $n_0 = \frac{1}{2}(|\langle J_A^X \rangle| + |\langle J_B^Y \rangle|)$ is shot noise (dotted). (b) Entanglement ($E_{product}$) based on the criterion (4) by the solid curve and E_{sum} in sum criterion (5) by the dashed curve. Here the parameters correspond to Rb atoms at magnetic field $B = 9.131G$, with scattering lengths $a_{11} = 100.4a_0$, $a_{22} = 95.5a_0$, and $a_{12} = 80.8a_0$. $a_0 = 53pm$. The coupling constant $g_{ij} \propto 2\omega_\perp a_{ij}$. The number of Rb atoms is $N_A = 200$. $\tau = g_{11}N_At$.

Here we again take a four modes approach, explicitly assuminga_1, b_1 and a_2, b_2 that are initially in coherent states for simplicity, i.e., assuming we have coherence between the wells. For simplicity, we suppose that the initial state is then prepared in an overall four-mode coherent state using a Rabi rotation. It is also possible to choose a constrained total particle number, but we have used the simplest model of coherence between the wells:

$$|\psi>=|\alpha>_{a_1}|\alpha>_{b_1}|\alpha>_{a_2}|\alpha>_{b_2} \tag{6}$$

Next, we assume that the inter-well potential is increased so that each well evolves independently. Finally, we decrease the inter-well potential for a short time, so that it acts as a controllable, non-adiabatic beam-splitter [19], to allow interference between the wells, followed by independent spin measurements in each well.

For dynamics, we assume a simple two-spin evolution per well, which is exactly soluble. We can treat this using either Schroedinger or Heisenberg equations of motion. In the Heisenberg case, since the number of particles is conserved in each mode, this has the solution:

$$\hat{a}_i(t) = \exp\left[-i\sum g_{ij}\hat{N}_j t\right]\hat{a}_i(0) \ , \qquad (7)$$

where the couplings g_{ij} are obtained from the known Rb scattering lengths at a Feshbach resonance.

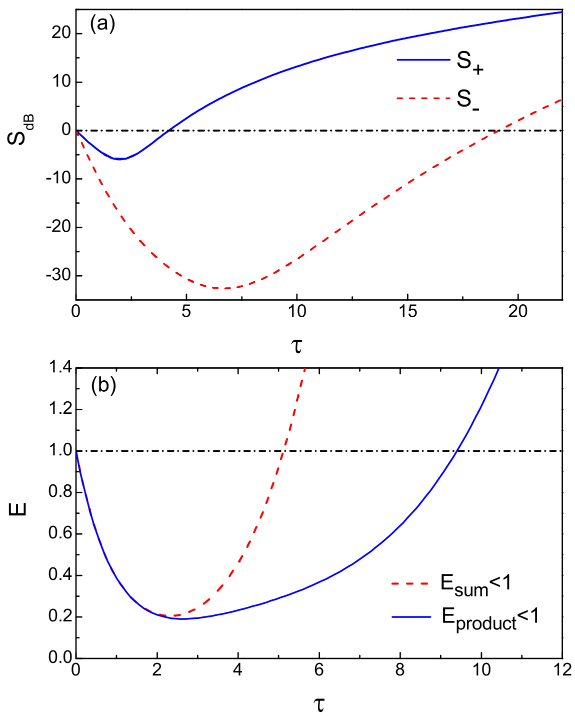

Figure 4. Same as Fig. 3 but assuming NO cross-couplings, i.e., $g_{12} = 0$.

After dynamical evolution from an initial coherent state, we find spin-squeezing in each well, prior to using the beam-splitter as shown in Fig. 3(a).

After using the beam-splitter, entanglement can be detected in principle as $E < 1$, as shown in Fig. 3(b). Note, Fig. 4 shows that assuming NO cross-couplings, i.e., $g_{12} = 0$ gives much better results than using the cross-couplings obtained in a Rb Feshbach resonance. As discussed in the adiabatic approach, this would require spatially separated wells for the different spin-orientations, or a different type of Feshbach resonance, in order to eliminate cross-couplings.

In summary, we have shown two feasible techniques for measuring EPR-type spatial BEC entanglement, using currently available double-well BEC approaches combined with available atomic detection methods. The simplest method employs an attractive ground-state adiabatic method, with a single spin orientation. This requires an essentially nonlocal detection strategy, in which the two BEC's are expanded and interfere with each other. To obtain a spatially separated EPR entanglement strategy, appropriate for investigating questions of local realism, we propose a four-mode, dynamical strategy that employs two distinct spin orientations in each spatial well.

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[1] J. Esteve, C. Gross, A. Weller, S. Giovanazzi, and M. K. Oberthaler, Nature **455**, 1216 (2008).
[2] C. Gross, T. Zibold, E. Nicklas, J. Esteve, and M. K. Oberthaler, Nature **464**, 1165 (2010).
[3] M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, and Philipp Treutlein, Nature **464**, 1170(2010).
[4] A. Sorensen, L.-M. Duan, J. I. Cirac and P. Zoller, Nature **409**, 63 (2001).
[5] Andrew P. Hines, Ross H. McKenzie, and Gerard J. Milburn, Phys. Rev. A. **67**, 013609 (2003).
[6] G.-B. Jo, Y. Shin, S. Will, T. A. Pasquini, M. Saba, W. Ketterle, D. E. Pritchard, M. Vengalattore and M. Prentiss, Phys. Rev. Lett. **98**, 030407 (2007).
[7] A. J. Ferris and M. J. Davis, New Journal of Physics **12,** 055024 (2010).
[8] M. Hillery and M. S. Zubairy, Phys. Rev. Lett. **96**, 050503 (2006).
[9] E. G. Cavalcanti, C. J. Foster, M. D. Reid, and P. D. Drummond, Phys. Rev. Lett. **99**, 210405 (2007).
[10] A. S. Sorensen and K. Molmer, Phys. Rev. Lett. **86**, 4431 (2001).
[11] Q. Xie and W. Hai, Eur. Phys. J. D **39**, 277 (2006).
[12] M. Zhang, Kristian Helmerson, and L. You, Phys. Rev. A **68**, 043622 (2003).
[13] A. J. Ferris, M. K. Olsen and M. J. Davis, Phys. Rev. A **79**, 043634 (2009).
[14] W. P. Bowen, N. Treps, R. Schnabel, and P. K. Lam, Phys. Rev. Lett. **89**, 253601 (2002).
[15] V. Giovannetti, S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. A **67**, 022320 (2003); R. Dong, J. Heersink, J. I. Yoshikawa, O. Glockl, U. L. Andersen, and G. Leuchs, New Journal of Physics **9,** 410 (2007).
[16] L. M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000); R. Simon, Phys. Rev. Lett. **84**, 2726 (2000).
[17] S. J. Carter, P. D. Drummond, M. D. Reid, and R. M. Shelby, Phys. Rev. Lett. **58**, 1841 (1987); P. D. Drummond and S. J. Carter, JOSA B **4**, 1565 (1987).
[18] P. D. Drummond, R. M. Shelby, S. R. Friberg, and Y. Yamamoto, Nature **365**, 307 (1993); R. Dong, J. Heersink, J. F. Corney, P. D. Drummond, U. L. Andersen, and G. Leuchs, Opt. Lett. **33**, 116 (2008).
[19] T. J. Haigh, A. J. Ferris, M. K. Olsen, Optics Communications **283,** 3540–3547 (2010).