Revealing Quantum Statistics with a Pair of Distant Atoms

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(Received 13 June 2017; revised manuscript received 27 July 2017; published 16 October 2017)

Quantum statistics have a profound impact on the properties of systems composed of identical particles. At the most elementary level, Bose and Fermi quantum statistics differ in the exchange phase, either 0 or π, which the wave function acquires when two identical particles are exchanged. In this Letter, we demonstrate that the exchange phase can be directly probed with a pair of massive particles by physically exchanging their positions. We present two protocols where the particles always remain spatially well separated, thus ensuring that the exchange contribution to their interaction energy is negligible and that the detected signal can only be attributed to the exchange symmetry of the wave function. We discuss possible implementations with a pair of trapped atoms or ions.

DOI: 10.1103/PhysRevLett.119.160401

The symmetrization postulate of quantum mechanics asserts that the wave function of a system of identical particles is either completely symmetric or antisymmetric under particle exchange [1]. A plethora of physical phenomena observed in experiments investigating atoms, molecules, and solids, as well as the statistical properties of light supports the (anti)symmetrization requirement. While more general quantum statistics [2] are in principle conceivable, they seem not to be realized by elementary particles in nature [3].

The influence of the wave function symmetry has been spectacularly demonstrated in few-particle systems with Hong-Ou-Mandel-like interference experiments [4–8], and in many-body systems with ultracold quantum gases [9]. Spectroscopic experiments have also tested the symmetrization postulate for massive particles [10–14] and for photons [15,16] with high precision. Recently, exchange interactions have been applied in engineered quantum systems for entangling pairs of atoms or electrons [17–20].

At the most elementary level, the wave function symmetry manifests itself when two identical particles are exchanged in position [Fig. 1(a)]: Their state acquires an exchange phase φ_{ex}, which is 0 for bosons but π for fermions. Exchange of identical particles can naturally occur in molecules where identical, distant nuclei may be interchanged as a result of a rotation [21]. Prior experiments [10–14] have exploited this naturally occurring exchange of identical particles to show that only certain rotational states are permitted by the symmetrization postulate. However, a direct interferometric measurement of the exchange phase φ_{ex} has never been attempted. In this Letter, we propose to use the high controllability of trapped atoms or ions for a direct measurement of this phase. To this end, we devise experiments where the two-particle wave function is superposed with the wave function of the same particles having swapped positions. We further request that, if the interferometric sequence is interrupted at any time, the two particles are always found at distant positions. This condition of vanishing overlap between the two particles ensures that the interference signal depends only on the wave function symmetry.

Figure 1(b) schematically illustrates the general interferometric scheme we envision for detecting φ_{ex}: Initially, two identical particles are tightly localized by a confining potential so that their wave functions have vanishing overlap.

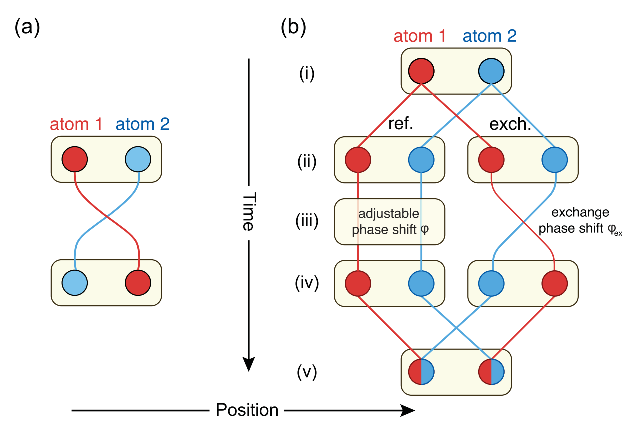

FIG. 1. Detection of the wave function symmetry in a two-particle interference experiment. (a) Exchanging two identical particles multiplies the wave function by a global phase factor $e^{i\varphi_{\mathrm{ex}}} = \pm 1$, which—without a reference state—is not observable. Dynamical and geometrical phases are assumed to vanish. (b) By splitting the wave function into a reference path and another path for which the particles' positions are switched, φ_{ex} can be detected by correlation measurements after recombining the two paths. The interference signal is controlled by an additional phase φ, induced by a potential or by the geometry.

Wave function symmetrization plays no role in the description of the initial state since the particles are initially distinguishable by their positions. Next, by modifying the confining potential, the two-particle wave function is split into two parts, a reference state and a state for which the positions of the particles are subsequently swapped. In the final steps, the two parts of the wave function are recombined and two-particle interference is measured.

The scenario sketched in Fig. 1(b) bears a close resemblance to Hanbury Brown–Twiss [22] and Hong-Ou-Mandel [4] experiments. However, instead of measuring (anti) bunching of particles as in the majority of these experiments, we will focus on schemes where the two particles are measured at distant sites and interference is detected by correlating the internal or motional states of the atoms.

We present two conceptual ways of realizing the exchange of particles and discuss possible experimental implementations: (A) A state-dependent potential transports particles in a way that depends on their internal states; (B) a state-independent potential confining the atoms is adiabatically transformed. Simultaneously, long-range repulsive interactions such as the Coulomb force between a pair of charged particles correlate the atom motion in the potential by keeping them apart.

Protocol A: state-dependent transport.—We consider a pair of bosonic or fermionic atoms with two long-lived internal states labelled $|\uparrow\rangle$ and $|\downarrow\rangle$. Initially, one atom is prepared at site S_1 and the other at site S_2, and both of them in the same internal state $|\uparrow\rangle$. Their state reads $a^\dagger_{S_1,\uparrow}a^\dagger_{S_2,\uparrow}|0\rangle$, where $a^\dagger_{S_i,s}$ are the creation operators for the site S_i and pseudospin state $|s\rangle$. We assume that the spatial wave functions $\psi_{S_i}(\boldsymbol{r}) = \langle \boldsymbol{r}, s|a^\dagger_{S_i,s}|0\rangle$ of the two atoms do not overlap. A $\pi/2$ spin rotation pulse subsequently mixes the internal states and puts the two atoms in a superposition of even- and odd-spin-parity states, $(|\Psi_{\rm even}\rangle - |\Psi_{\rm odd}\rangle)/\sqrt{2}$, defined by

$$|\Psi_{\rm even}\rangle = 2^{-1/2}\left(a^\dagger_{S_1,\uparrow}a^\dagger_{S_2,\uparrow} + a^\dagger_{S_1,\downarrow}a^\dagger_{S_2,\downarrow}\right)|0\rangle, \qquad (1)$$

$$|\Psi_{\rm odd}\rangle = 2^{-1/2}\left(a^\dagger_{S_1,\uparrow}a^\dagger_{S_2,\downarrow} + a^\dagger_{S_1,\downarrow}a^\dagger_{S_2,\uparrow}\right)|0\rangle. \qquad (2)$$

Crucially, a physical transport operation conditionally switches the positions of the atoms if they are in, say, $|\uparrow\rangle$, while it maintains them at the original location if they are in $|\downarrow\rangle$,

$$a^\dagger_{S_1,\uparrow} \to a^\dagger_{S_2,\uparrow}, a^\dagger_{S_2,\uparrow} \to a^\dagger_{S_1,\uparrow}, a^\dagger_{S_i,\downarrow} \to e^{i\varphi/2}a^\dagger_{S_i,\downarrow}, \qquad (3)$$

where we also allow for a precisely adjustable dynamical phase φ acquired during the process. To ensure vanishing exchange interactions, the exchange process must be realized such that $\psi_{S_i}(\boldsymbol{r};t)$ remain disjoint for all times t, i.e., $\psi_{S_1}(\boldsymbol{r};t)\psi_{S_2}(\boldsymbol{r};t) = 0$.

The evolution of $|\Psi_{\rm even}\rangle$ under the transformation in Eq. (3) realizes the situation sketched in Fig. 1(b). The correspondence is apparent once the different terms are reordered according to the commutation rules $a^\dagger_{S_1,s}a^\dagger_{S_2,s} = e^{i\varphi_{\rm ex}}a^\dagger_{S_2,s}a^\dagger_{S_1,s}$, yielding

$$|\Psi_{\rm even}\rangle \to \frac{1}{\sqrt{2}}\left(e^{i\varphi_{\rm ex}}a^\dagger_{S_1,\uparrow}a^\dagger_{S_2,\uparrow} + e^{i\varphi}a^\dagger_{S_1,\downarrow}a^\dagger_{S_2,\downarrow}\right)|0\rangle. \qquad (4)$$

Thus, the exchange phase $\varphi_{\rm ex}$ now appears in the description of the internal state as a relative phase, which can be detected by correlating local measurements of the particles' internal state [23]: after applying a second $\pi/2$ spin rotation pulse, the expectation value of the spin parity operator Π [24] yields $\langle\Pi\rangle = \cos(\varphi - \varphi_{\rm ex})$. Recording Π for different values of φ allows one to measure $\varphi_{\rm ex}$. The evolution of $|\Psi_{\rm odd}\rangle$ is different, though, and leads to a state with two atoms in the same location, where the exchange phase (as well as the dynamical phase) has no influence on spin correlations between the two particles. If not discarded through postselection, these events would halve the visibility of the parity signal. If state $|\Psi_{\rm even}\rangle$ is directly prepared using an entangling scheme for distant particles, see Refs. [25,26], full visibility of the spin-parity fringe can be ideally obtained without postselection.

Implementation with a pair of neutral atoms in an optical lattice.—Protocol A can be realized using a pair of distant neutral atoms that are transported in spin-dependent optical lattices [27–30]; other forms of state-dependent transport with microwave-dressed potentials in atom chips [31] or with spin-dependent optical tweezers are also conceivable.

We propose a two-particle Ramsey interferometer as is shown in Fig. 2, which, instead of probing first-order coherence, detects second-order coherence revealing $\varphi_{\rm ex}$: A pair of atoms is initially prepared in well-separated lattice sites, denoted L_1 and R_1, with their pseudospin states in $|\uparrow\rangle|\uparrow\rangle$. The lattice depth is chosen sufficiently high to suppress tunneling to neighboring sites [29]. Importantly, both atoms must be cooled to the lowest vibrational state of their respective lattice potential well [30,32,33], in order to make them indistinguishable in the motional degree of freedom, see Supplemental Material [34]. The first $\pi/2$ Ramsey pulse puts both atoms in a superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ states. Subsequently, each atom is split in space and transported conditioned upon its pseudospin state [30] to both end sites L_2 and R_2. Each shift operation can be performed fast on the time scale of 10 μs per lattice site [29]. In particular, polarization-synthesized optical lattices [30] allow one to state-dependently transport atoms in a single operation over few tens of lattice sites, while at the end leaving the atoms in the lowest vibrational state [38]. Finally, the second $\pi/2$ Ramsey pulse erases the information about which way the atoms traveled to reach the end sites. Focusing our attention on atoms detected at distant sites [39,40], local spin measurements yield an equal probability to find $|\uparrow\rangle$ or $|\downarrow\rangle$, meaning that each atom probed individually is found in a statistical mixture of both spin states. However, a parity

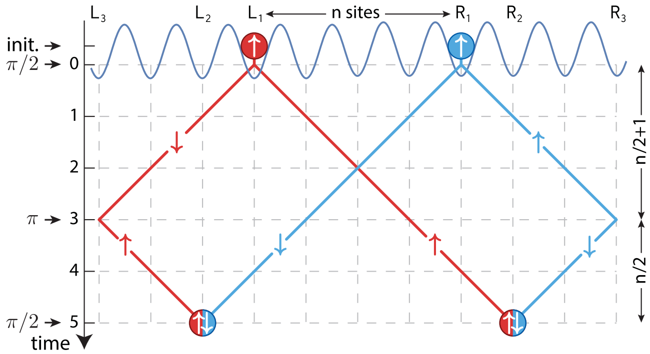

FIG. 2. Two-atom Ramsey interferometer sequence probing quantum statistics with two distant neutral atoms. A spin parity measurement produces a two-atom Ramsey-like fringe, whose phase depends on φ_{ex}. To recombine the atoms, a position-dependent π pulse [41–43] is applied to the outermost sites, L_3, R_3, see Ref. [34]. The arrows indicate the spin state for the different paths, n denotes the initial separation, and time is expressed in units of shift operations. A two-dimensional variant is presented in the Supplemental Material [34], which ensures that the two atoms always stay far apart.

measurement of the spin state [24] yields nontrivial correlations, showing, for example, perfect spin alignment for bosonic and antialignment for fermionic atoms. An interference fringe can be recorded by precisely adjusting the phase difference φ between the outermost and innermost paths, for example, by controlling the relative phase of the position-dependent [34] pulse acting at sites L_3 and R_3. With 90% of atoms prepared in the lowest vibrational state, we expect a visibility of the spin parity signal of ≈80% [34]. Note that while the Ramsey scheme in Fig. 2 preserves the connectedness of the abstract protocol sketched in Fig. 1(b), it is designed to be robust against dephasing mechanisms. Stochastic dynamical phases caused by fluctuating magnetic fields, magnetic field gradients, and state-dependent transport operations cancel out owing to time and space refocusing [34].

Remarkably, nontrivial correlations are predicted in the proposed scheme even though the two particles have never met nor interacted with each other. These correlations are purely quantum and, as such, incompatible with a macrorealistic worldview [44] where atoms travel either the outermost or the innermost paths. Correlations from accidental interactions between the two atoms at the intersection point in the center can be made vanishingly small by increasing the transport velocity and by softening the transverse confinement; in a two-dimensional scheme using two-dimensional spin-dependent optical lattices [45], interactions are completely avoided, see Ref. [34].

Conceptually, the closest analog to this scheme is the Franson interferometer [46] suggested to test local hidden-variable theories with two photons independently emitted at consecutive times. However, here the massive particles are "emitted" (namely, transported) simultaneously. It also shares a resemblance with Fano's interpretation [22] of the Hanbury Brown–Twiss experiment, although here we detect spin correlations instead of (anti)bunching of particles. As a potential application, the proposed interference scheme would allow one to test nonlocal correlations [47,48] between macroscopically distant atoms [19]. We expect that entangling atoms separated by macroscopic distances on the scale of few thousands of lattice sites should be doable with currently available technology [30].

Suitable atomic species for such experiments are discussed in detail in Ref. [34]: Rb [28] and Cs [29] for bosons, and alkali-earth-like atoms [49] for fermions. Moreover, aluminum is an attractive atomic species for a direct comparison of the exchange phase of fermionic (^{26}Al) and bosonic (^{27}Al) isotopes with the same experimental setup.

Protocol B: long-range interactions.—In the presence of a confining potential, long-range repulsive interactions turn two particles into a moleculelike quantum rotor. We assume a potential that is strongly confining in one dimension, effectively freezing out the rotor's motion in this direction, and that has a single minimum in the orthogonal plane. As in the case of homonuclear diatomic molecules, the symmetry of the spin state here also controls the symmetry of the spatial wave function $\Psi(\theta)$ of the rotor [50,51] with orientation angle θ in the weakly confining plane. For clarity, we focus on the case of fermionic particles. If the particles are prepared in, for example, $|\downarrow\rangle|\downarrow\rangle$, the rotor's wave function must be antisymmetric, $\Psi(\theta) = -\Psi(\theta + \pi)$, as sketched in Fig. 3(a). Apart from that, the spin state plays no role in this protocol in contrast to the previous one. Although the wave function can be completely specified by limiting the angle to a range of $0 \le \theta < \pi$, it is convenient to represent it over the full range $0 \le \theta < 2\pi$.

We assume that the rotor is initially prepared in the ground state of the potential $V(\theta, t = 0) = V_0 \cos^2\theta$. If the initial potential well located, e.g., at $\theta = \pi/2$ is adiabatically split into a double well, the Gaussian wave packet of the ground state will be transformed into an even superposition of wave packets. By slowly separating the two minima of the double well (and of the double well at $\theta = 3\pi/2$ as well), the wave packets originating from opposite sides of the ring will eventually meet and merge into a wave packet with uneven parity. The final potential again consists of a single well, but now located at $\theta = \pi$ (or $\theta = 2\pi$). Importantly, for spatial wave functions that are antisymmetric under particle exchange, the adiabatic transport maps the even states of the initial potential onto odd states of the final potential and vice versa, whereas for spatially symmetric wave functions the state's parity is preserved. Because of this property, bosonic and fermionic atoms can be distinguished by measuring whether the parity of the motional state has changed at the end of the adiabatic transport. The analysis of the rotor's angular motional state is equivalent to a correlation measurement of local modes of motion of the two atoms.

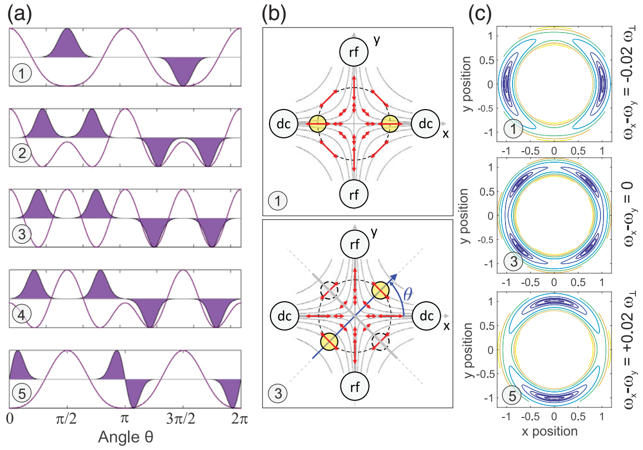

FIG. 3. Trapped-ion protocol. (a) The two-ion wave function is treated as a one-dimensional quantum rotor (shown for fermions in the same internal state). An adiabatic transformation splits a single-well (1) into a double well potential (3) that is subsequently merged again into a single well, but at a different position (5). In the case of fermions (bosons), the final state (5) has opposite (same) parity as compared to that of the initial state (1). (b) In the radial rf-quadrupolar potential of a linear trap, a two-ion rotor (yellow spheres) can be aligned with the x axis by dc voltages reducing the confinement along the x axis (top). For zero dc voltage, the radial symmetry of the confining potential is broken by the orientation of the micromotion (red arrows) with respect to the rotor axis [34]. The rotor will align under an angle $\theta = \pm\pi/4$ with respect to the x axis (bottom). (c) Contour lines of the time-averaged Coulomb and trapping potential as a function of the relative position vector $\boldsymbol{r}$ for three different potentials.

Such a rigid ion rotor behaves similarly to homonuclear diatomic molecules, where techniques such as pendular state spectroscopy or rotational coherence spectroscopy reveal the effect of the exchange symmetry on allowed rotational states. In contrast, in the experiment proposed here, complete control over the rotor enables exchanging the particles without rotating their electronic wave function [21] and, most importantly, a direct measurement of the exchange phase.

Implementation with a pair of trapped ions.—For the realization of protocol B, we consider a linear radiofrequency (rf) trap confining the ions in a harmonic potential with oscillation frequencies ω_x, ω_y in the radial directions and ω_z in the axial direction. The difference between the radial oscillation frequencies can be controlled by a static voltage U_{dc}. A pair of laser-cooled ions forms a crystal in the radial plane if $\omega_z > \omega_x, \omega_y$. At the ions' equilibrium positions, the trapping force is balanced by the ions' mutual Coulomb repulsion.

Because of the harmonic confinement, the ion dynamics separates into the center-of-mass motion and the relative motion $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$. The latter is governed by the Hamiltonian

$$H_r = -\frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{\mu}{2}(\omega_x^2 r_x^2 + \omega_y^2 r_y^2) + V_{\mathrm{coul}}(\boldsymbol{r}),$$

where μ is the ions' reduced mass and r_x, r_y are the transverse components of $\boldsymbol{r}$. Because of the micromotion of the ions in the radial plane, one has to time average the Coulomb energy over one period of the rf-driving field, leading to a modified Coulomb potential

$$V_{\mathrm{coul}}(\vec{r}) = \frac{e^2}{4\pi\epsilon_0|\vec{\boldsymbol{r}}|}\left(1 + \frac{3}{16}q^2\cos^2(2\theta)\right)$$

where q is the trap's q parameter [52] and θ denotes the orientation of the crystal in the radial plane (see Fig. 3(b) and Ref. [34]). For the case where $\omega_x = \omega_y$, the asymmetry of the micromotion lifts the rotational symmetry of the potential and leads to two equivalent sets of equilibrium orientations of the crystal under $\theta = \pm\pi/4$ [53,54].

This effect opens up the prospect of implementing protocol B with a pair of ions by ramping U_{dc} from positive to negative voltages. The relative motion is described by two normal modes, which we assume to be cooled to the ground state. If initially $\Delta\omega = \omega_y - \omega_x > 0$, the ion crystal is aligned with the x axis [Fig. 3(b), top]. Lowering $\Delta\omega$ by reducing U_{dc} softens the normal mode perpendicular to the crystal's axis (the rocking mode) while hardly affecting the other mode. At the critical value $\Delta\omega_{\mathrm{crit}} = \frac{3}{4}q^2\omega_\perp$, with $\omega_\perp = \sqrt{(\omega_x^2 + \omega_y^2)/2}$, the rocking mode potential becomes quartic and subsequently splits into a double well. The wells separate and move to $\theta = \pm\pi/4$ when U_{dc} becomes zero [Fig. 3(b), bottom, and Fig. 3(c)]. At this point, the ion rotor is in a coherent superposition of two perpendicular orientations. Ramping U_{dc} to negative values combines a different pair of wells which will finally merge, resulting in an ion rotor oriented along the y axis. In this way, the two possible paths of rotating the ion rotor clockwise or counterclockwise interfere, and a measurement by sideband spectroscopy of the motional state [55] of the rocking mode reveals the bosonic or fermionic character of the ions. For further information, see Ref. [34].

The quantum coherence of the process can be checked by initially preparing the internal state of the ions in a Bell state $(|\uparrow\rangle|\downarrow\rangle + e^{i\phi}|\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$. The phase ϕ controls the symmetry of the spatial wave function [56], which for the special case $\phi = 0$ (π) is antisymmetric (symmetric). As a consequence, this phase determines whether the protocol maps the rocking mode's state onto the ground or first excited state.

The experiment could be carried out with ion species like the fermionic $^{40}Ca^+$ or the bosonic $^{43}Ca^+$, for which ground state cooling and Bell state generation are routinely done [57,58]. A numerical simulation of the time-dependent Schrödinger equation suggests that an adiabatic transfer is achievable in less than 2 ms [34], much shorter than the time scale on which heating of the relative ion motion occurs.

In the absence of imperfections, this protocol constitutes an interferometer with completely symmetric arms. Therefore, it should be immune against dynamical phases. A nonzero magnetic flux through the circle on which the ions move, however, would lift the symmetry and give rise to a small, but measurable geometric Aharonov-Bohm phase [59].

An experimental challenge is to suppress stray electric field gradients, which, by breaking the symmetry of the confining potential, would cause dynamical phases or even compromise the process of splitting the minimum of the potential into two. After compensation of such fields, it should be possible to independently measure the remaining dynamical phases (see Ref. [34]) in order to extract φ_{ex} from the measured signal.

The proposed protocol shows that quantum statistics can become important for trapped ions [60] in experimentally accessible parameter regimes. A quantum gate entangling the pair of ions based solely on particle exchange could be realized by first carrying out the protocol and then running it backwards again after a suitable waiting time. Since triplet and singlet states have different symmetry, and therefore are transiently mapped to different motional states, they pick up different phase factors. In this way, a $\sqrt{\mathrm{SWAP}}$ gate could be realized as used for solid-state quantum computing based on exchange interactions [20,61,62] (and for linear-optical quantum information processing [63]). The protocol could even be applied to a pair of molecular ions. In addition, it could lead to ion-based quantum sensors complementing single-particle interferometry schemes based on structural phase transitions [64,65].

Conclusions.—The proposed experiments show that the exchange phase can be precisely measured with massive particles. By ensuring that the particles' wave functions have vanishing overlap, a situation not encountered so far in Hong-Ou-Mandel-like experiments [4–8,66,67], these experiments would demonstrate the effect of exchanging two identical particles at the most elementary level. Moreover, the two protocols open novel perspectives for entanglement generation and sensing applications based on a pair of identical particles.

C. R. acknowledges helpful discussions with P. Zoller on analogies between the trapped-ion proposal and molecular physics. The authors wish to thank W. Alt, M. Baranov and the anonymous referees for insightful discussions and helpful comments. A. A. and D. M. acknowledge financial support from the Deutsche Forschungsgemeinschaft (SFB/TR 185 OSCAR) and the ERC advanced grant DQSIM (Project ID 291401). P. H. acknowledges support from the Austrian Science Fund (FWF) through the SFB FoQuS (Project No. F4016-N23), and from the European Commision through the ERC synergy grant UQUAM and the ERC advanced grant EntangleGen (Project ID 694561).

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