Noise

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Noise Description

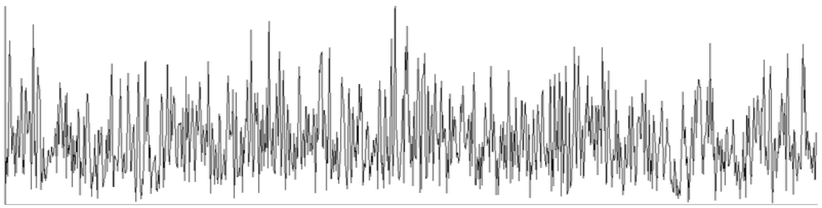
What is Noise ?

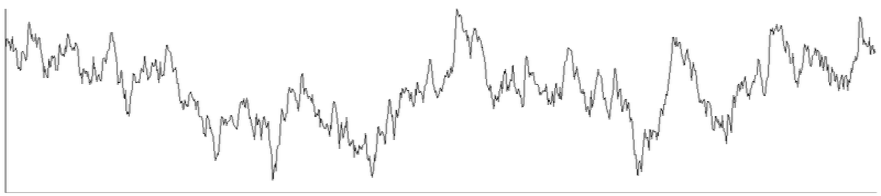
- Noise is a random fluctuation of a voltage / current
- The average noise is zero: $\langle \text{noise} \rangle = 0$
  - A non-zero average is no noise, just a 'bias' or 'offset'
- The noise 'strength' can be defined as the variance:

  $$\text{voltagenoise}^2 = \langle v^2 \rangle \quad \text{or} \quad \text{currentnoise}^2 = \langle i^2 \rangle$$

  where $\langle .. \rangle$ is over time
- The 'RMS Noise' is the square root of the variance

- The same RMS can be obtained by very different noise signals, as seen on an oscilloscope (time domain):





  - Obviously, the left signal contains 'higher frequencies'…

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Spectral Density

- The noisy signal can have different strength for various frequencies.
- We therefore describe noise by its *spectral density*, the (squared) noise voltage (density) as a function of frequency.
  - It has the unit V^2/Hz
  - Sometimes, we use the square root with the unusual unit $V/\sqrt{Hz}$

ANALOG DEVICES

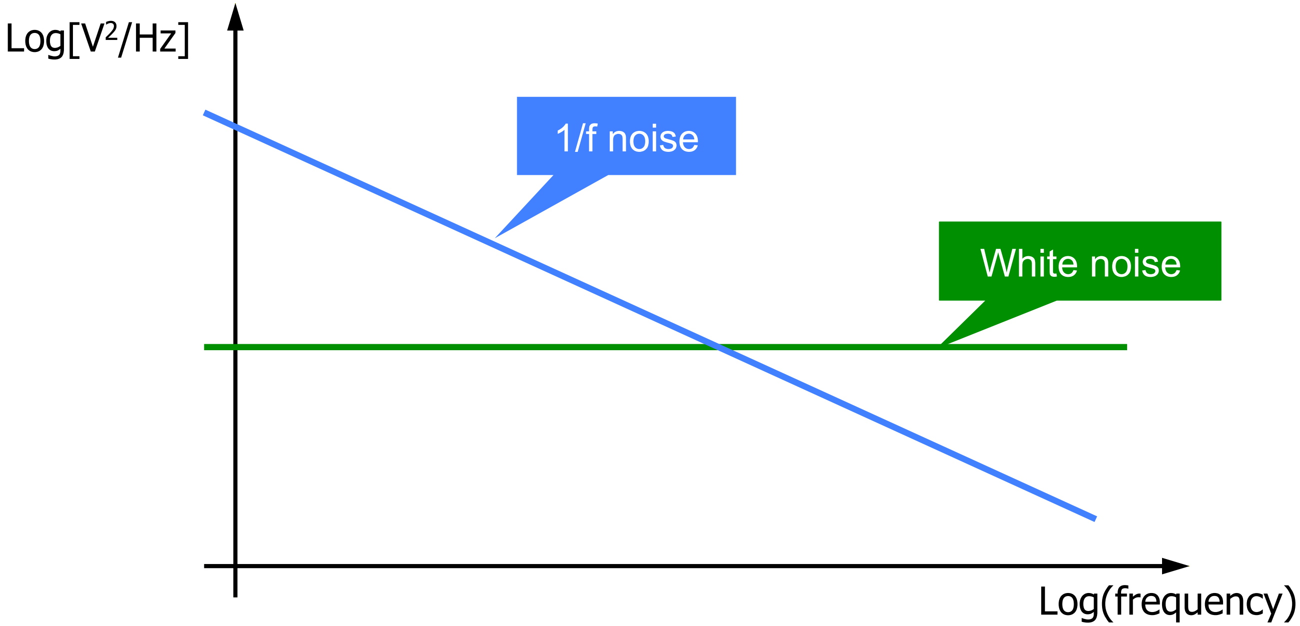
30 V, 8 MHz, Low Bias Current, Single-Supply, RRO, Precision Op Amps

Data Sheet **ADA4622-1/ADA4622-2**

NOISE PERFORMANCE				
Voltage Noise	e_N p-p	0.1 Hz to 10 Hz	0.75	µV p-p
Voltage Noise Density	e_N	f = 10 Hz	30	nV/√Hz
		f = 100 Hz	15	nV/√Hz
		f = 1 kHz	12.5	nV/√Hz
		f = 10 kHz	12	nV/√Hz
Current Noise Density	i_N	f = 1 kHz	0.8	fA/√Hz

Noise Types / Spectra

- Most common types are
  - *White noise* has *constant* spectral density
  - *1/f* noise (*pink* noise) spectrum is ~ 1/f (or $S(f) \propto 1/f^{\alpha}$)
- Be careful: one can use frequency ν, to angular freq. ω!



- The rms noise is the integral of the noise spectral density over all frequencies ($\nu = 0 \ldots \infty$)

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A Closer Look on Thermal Noise

- Problem: a constant spectral density up to infinite frequencies would be infinite noise power.
- Quantum mechanics gives the exact value for the spectral noise density as a function of frequency ν and temperature T:

$$S_{noise}(\nu, T) = \frac{h\,\nu}{e^{h\nu/kT} - 1} \quad \xrightarrow[h\,\nu \ll kT]{} \quad kT$$

  - h = Planck's constant = 6.626×10^{-34} Js,
  - k = Boltzmann's constant = 1.381×10^{-23} J/K
- For 'low' frequencies ($h\nu \ll kT$), this is gives just kT
- The noise starts to drop at $\nu = kT/h \approx 21$ GHz × T/K
  - At room temperature, this is ~ 5THz. The approximation of $S_{noise} = kT$ is therefore valid for (our) practical circuit frequencies.

- (At very high frequencies, there is an additional 'quantum' noise which rises as $h\nu$)

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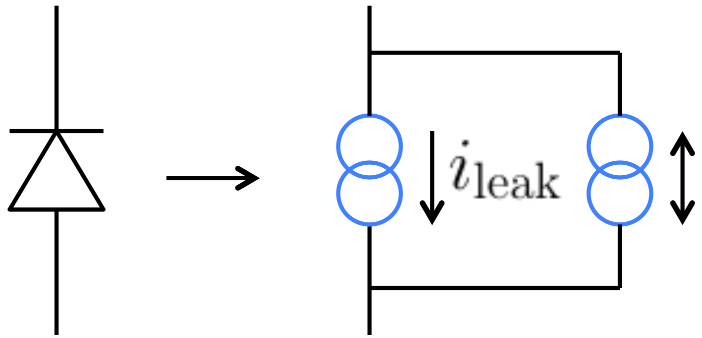
Noise In Components

Noise in Diodes

- The reverse current ('leakage') of a diode is generated by charge carriers which statistically overcome a barrier.
- The statistical fluctuations lead to noise.
  The fluctuations depend on the value of the leakage current.
- This is called shot noise.



$$\frac{\mathrm{d}\langle i_{\mathrm{leak}}^2 \rangle}{\mathrm{d}f} = 2qI_{\mathrm{leak}}$$

- Spectrum is flat (white noise)

  - Check Units: $[2\ q\ I_{Leak}] = \mathrm{As\ A} = \mathrm{A^2/Hz}$

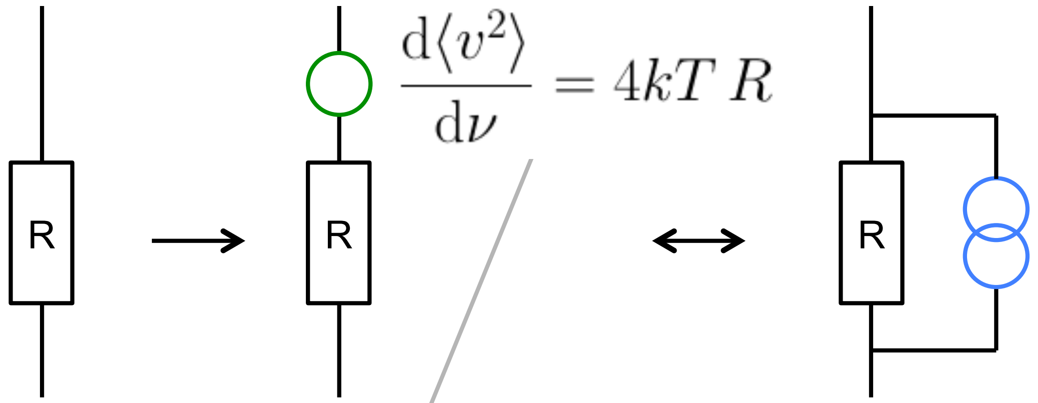
Noise in Resistors

Observe that (only!) these models give correct noise for serial / parallel connections of Rs

- Resistors exhibit noise from thermal motion of charge carriers. This noise is *independent* on current flow!
- This *thermal noise* is *white* noise
- If can be modelled by a serial voltage source

  OR

  by the Thévenin equivalent parallel current source



$$\frac{\mathrm{d}\langle v^2\rangle}{\mathrm{d}\nu} = 4kT\,R$$

$$\frac{\mathrm{d}\langle i^2\rangle}{\mathrm{d}\nu} = \frac{4kT}{R}$$

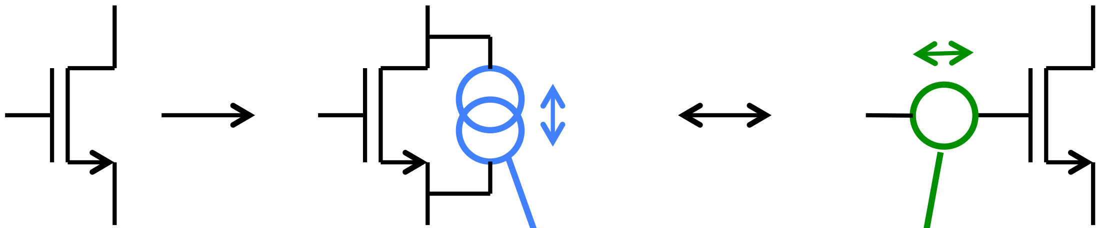
- Check Units: [4kT R] = VA/Hz × V/A = V^2/Hz

  [4kT/R] = VA/Hz / A/V = A^2/Hz

$$4kT \approx 1.6 \times 10^{-20}\, V\,As = 16 \times zV\,A/Hz$$

White Noise in Transistors

- The MOS channel can be seen as a series of (position dependent) resistor (at least in linear operation). Their noise contributions can be integrated up.
- The white current noise in the channel is $\frac{\mathrm{d}\langle i^2\rangle}{\mathrm{d}\nu} = 4kT\,\gamma\,g_m$
    - the factor γ from integration varies depending on operation regime: γ = 2/3 in strong inversion, less in w.i.
- This current noise at the drain can also be written as a voltage noise at the gate (by dividing by g_m^2)



$$\boxed{\frac{\mathrm{d}\langle i^2\rangle}{\mathrm{d}\nu} = 4kT\,\gamma\,g_m}$$

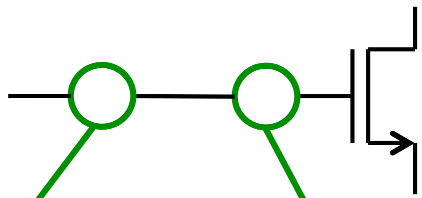
$$\boxed{\frac{\mathrm{d}\langle v^2\rangle}{\mathrm{d}\nu} = \frac{4kT\,\gamma}{g_m} \approx \frac{8}{3}\frac{kT}{g_m}}$$

- Note: noise for very short devices can be increased

1/f Noise in Transistors

- Charge carriers in the channel can be captured ('trapped') at impurities and released later.
  - This happens mostly at the oxide interface
- This leads to an additional noise with *1/f spectrum*
- The importance of this contribution depends on
  - Fabrication process → Technology noise parameter $\mathbf{K_f}$
  - MOS type (JFETs are *very much* (>10 x) better (no Interface)
  - MOS polarity (PMOS are *significantly* (10 x) better than NMOS)
- The effect averages out for larger devices (→WL in formula)

Independent of temperature (as long as traps do not freeze out)



$$\boxed{\frac{\mathrm{d}\langle v_{1/\mathrm{f}}^2\rangle}{\mathrm{d}f} = \frac{K_\mathrm{f}}{C_\mathrm{ox} W L}\frac{1}{f}}$$

1/f noise

$$\boxed{\frac{\mathrm{d}\langle v^2\rangle}{\mathrm{d}\nu} = \frac{8}{3}\frac{kT}{g_m}}$$

Thermal noise (white)

Noise Calculations

Recipes

- Noise contribution are independent → Each source is treated separately and noise contributions are added up (in quadrature) at the end
- For each source
  - calculate the transfer function H(s) to the 'output'
  - multiply the noise spectrum of the source with $|H|^2(s)$ (because we treat squared voltages)
  - integrate the 'output' spectrum over all frequencies.
- Add the resulting variances
- Square root of the sum leads the final noise (at the output)

- The noise rms (in V or A) value must be compared to the signal (which also depends on H(s)) to get a Signal-to-Noise ratio (SNR)

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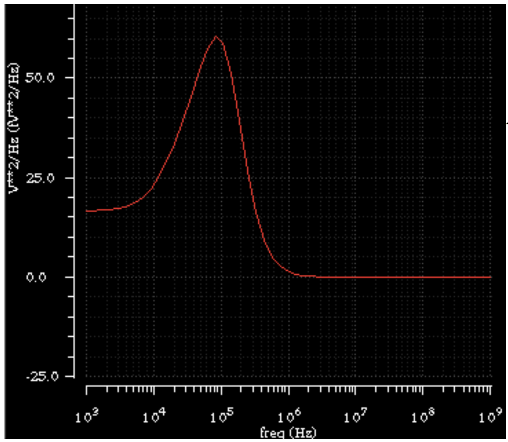
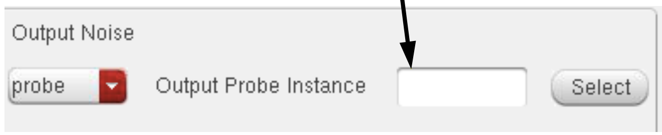
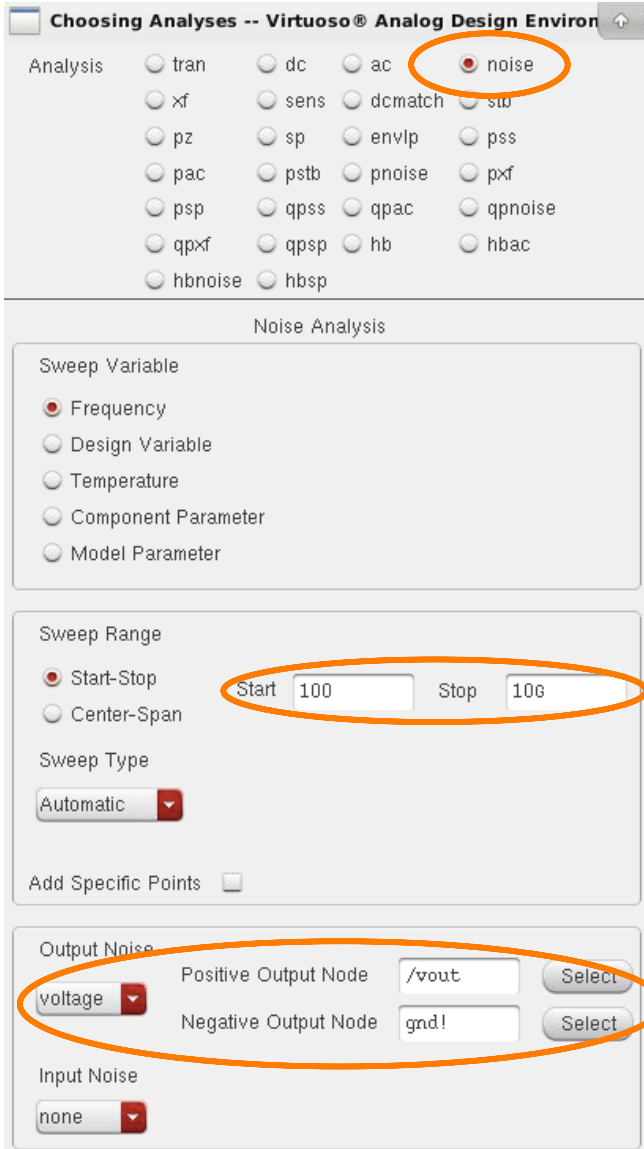
NOISE SIMULATION

Noise Simulation

- Simulation type 'noise' creates AC noise for all components
- Give a (generous) frequency range
- Noise is 'collected' for ONE node
  This can be
  - a voltage (give terminals)
  - a current (give a voltage source)
- You need to select any node for plotting.
  You will see the V^2/Hz spectrum





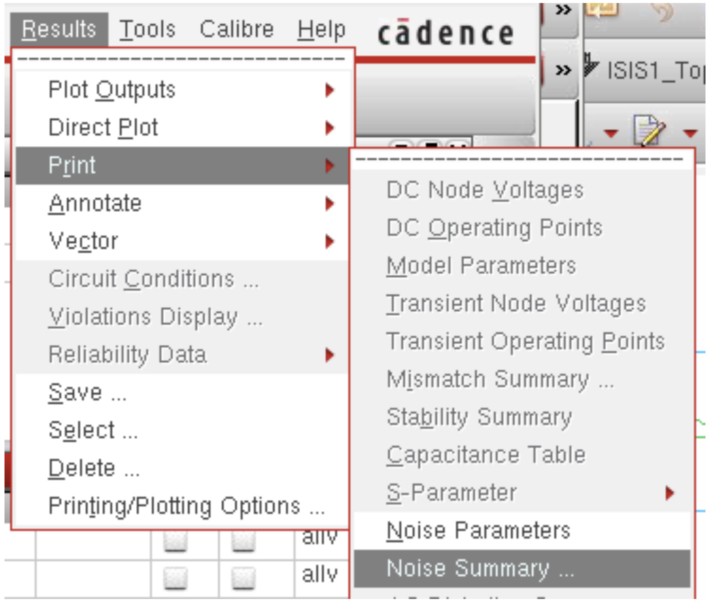


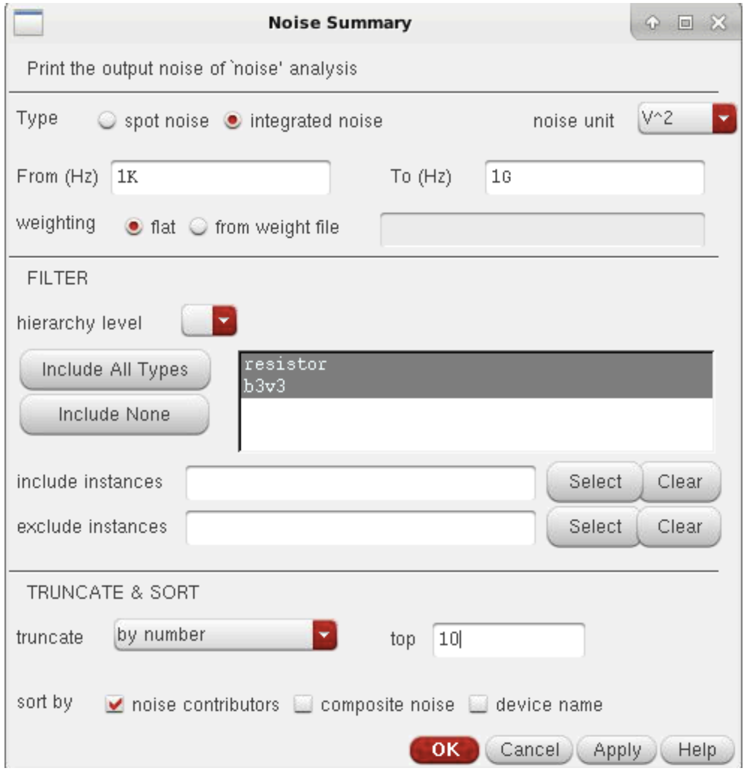
Total Noise

- To integrate a noise spectrum over frequency use the **`totalNoise( "noise" nil nil nil)`** command in ADE
  - Arguments are **`start_freq stop_freq exlusions`**. They can be set to **`nil`** for full range / all components
  - No need to specify a net / node (the one given on the simulation window is taken)
- This will give the *squared* noise voltage (in V^2 or A^2)
- To get RMS noise (in V or A), calculate the square root!

Noise Summary

- In more complex circuits, you can see which components contribute to the output noise:
  - Results → Print → Noise Summary
  - Usually select 'integrated noise'





Noise Summary Result

- Lists most important contributions
- Also gives the type of noise:
  - Resistor noise ('rn')
  - Channel noise ('id')
  - 1/f noise ('fn)
  - Source resistive noise ('rs') (depends on actual layout!)
  - Drain resistive noise ('rd') (depends on actual layout!)
  - Gate resistive noise (?) (depends on actual layout!)

Device	Param	Noise Contribution	% Of Total
/M1	fn	1.0337e-08	58.40
/R1	rn	4.17986e-09	23.62
/R0	rn	2.098e-09	11.85
/C2	rn	9.66773e-10	5.46
/M1	id	7.17454e-11	0.41
/MN	id	3.52872e-11	0.20
/MN	fn	1.07926e-11	0.06
/M1	rs	4.07753e-14	0.00
/MN	rs	3.16337e-15	0.00
/M1	rd	2.69697e-17	0.00

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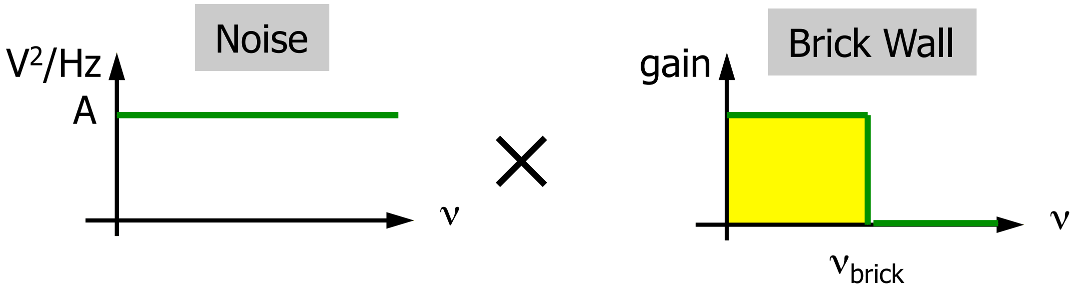
Noise Bandwidth Limitation

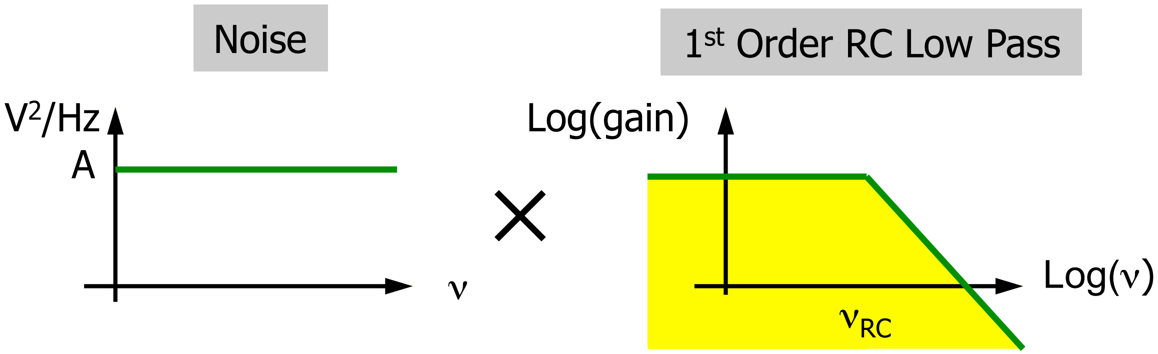
Bandwidth Limitation

- Brick-Wall Low Pass:
  - Assume a Low Pass which passes all frequencies up to ν_{brick} and then stops perfectly



  - When we filter white noise, the overall noise is just **$rms^2 = A\ \nu_{brick}$**

- RC Low Pass:
  - How is the integral now ?



The Integral

- Transfer Function of RC Low Pass: $H(\omega) = \frac{1}{1 + i\,\omega\tau} \quad (\tau = RC)$

- Gain: $$v^2(\omega) = |H(\omega)|^2 = H(\omega)H^*(\omega)$$
$$= \frac{1}{(1+i\,\omega\tau)(1-i\,\omega\tau)} = \frac{1}{1+(\omega\tau)^2}$$

- Integral: $$rms^2 = \int_0^\infty \frac{A}{1+(2\pi\nu\tau)^2}\mathrm{d}\nu$$
$$= \frac{A}{2\pi\tau}\int_0^\infty \frac{\mathrm{d}x}{1+x^2} = \frac{A}{2\pi\tau}\frac{\pi}{2} = \frac{A}{4\tau} = \frac{A\omega}{4} = \frac{A\pi\nu}{2}$$

(an integral of H(ω) ~1/ω would not converge, but $H^2(\omega)$ does)

- To obtain the same noise with a 'brick wall' filter, we need
  - $A\,\nu_{brick} = A\,\pi\,\nu_{RC} / 2 \quad \rightarrow \quad \nu_{brick} = \nu_{RC} \times \pi / 2$

Noise of RC Low Pass

- In the previous calculation, we have assumed R in the Low-Pass as noiseless. In reality, it is noisy
- If we have no *signal* at the input, the remaining input noise is just the *voltage noise* of the resistor, i.e. A = 4kT R
- The resulting output noise is

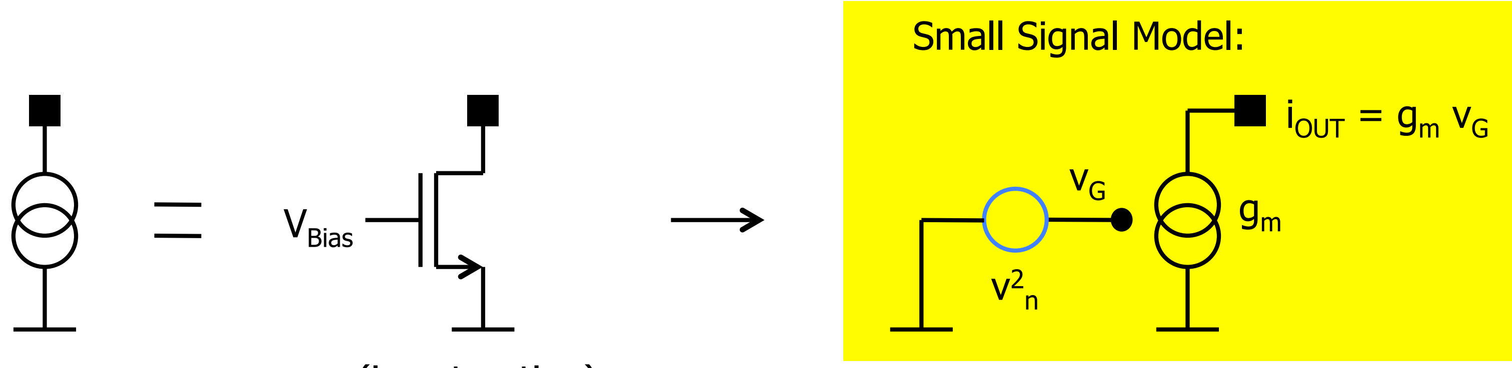
$$rms^2 = \frac{1}{4} A\,\omega = \frac{1}{4}\,4kTR\,\frac{1}{RC} = \boxed{\frac{kT}{C}}$$

- This 'kT/C-Noise' does *not* depend on R, but on C
  - This is because the change in bandwidth (with R) just compensates the change in noise
  - This kTC noise is present whenever signals are sampled to Cs!

Small capacitors 'have' large noise

Current Sources

MOS as Current Source



(in saturation)

- Due to input voltage noise source, we get at the output

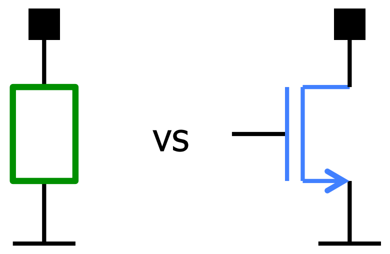
$$i_{\mathrm{OUT}}^2 = g_m^2 v_{\mathrm{G}}^2 = g_m^2 v_{\mathrm{n}}^2 = g_m^2 \frac{4kT\,\gamma}{g_m} = 4kT\,\gamma g_m$$

as before.

For a *low noise current source* we need *small* g_m

MOS vs. Resistor

- For a current I_0, what gives lower noise: MOS or resistor ?
  - Assume we operate the MOS in s.i. just at edge of saturation:



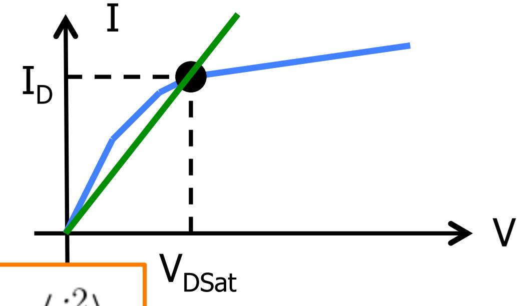
- R: $$\langle i^2 \rangle_R = \frac{4kT}{R} = 4kT\frac{I_\mathrm{D}}{V_\mathrm{DSat}}$$

- MOS: $$I_\mathrm{D} = \frac{\beta}{2}(V_\mathrm{GS} - V_\mathrm{th})^2(1 - \lambda V_\mathrm{DS})$$

$$V_\mathrm{DSat} = V_\mathrm{GS} - V_\mathrm{th} = \sqrt{\frac{2I_\mathrm{D}}{\beta}}$$

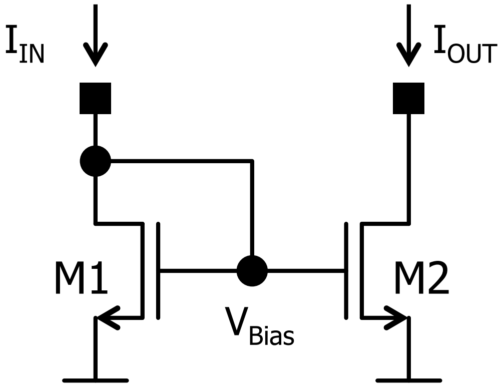
$$g_m = \frac{\partial I_\mathrm{D}}{\partial V_\mathrm{GS}} = \sqrt{2I_\mathrm{D}\beta} = \frac{2I_\mathrm{D}}{V_\mathrm{DSat}}$$

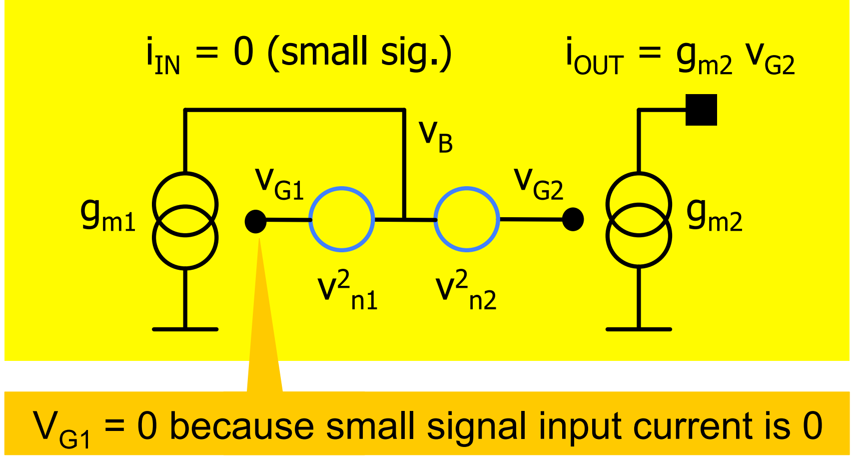
$$\boxed{\langle i^2 \rangle_{MOS}} = 4kT\,\eta\,g_m = 4kT\,\eta\,\frac{2I_\mathrm{D}}{V_\mathrm{DSat}} = \boxed{2\eta \times \langle i^2 \rangle_R}$$



- MOS is slightly worse, but has *much* lower output resistance
  - Also, current cannot be varied with fixed R
  - At higher voltage, R is larger and its noise decreases.
  - R has *no 1/f* noise!
  - Consider this 'old style' approach for very low noise…

Noise in the Simple Current Mirror





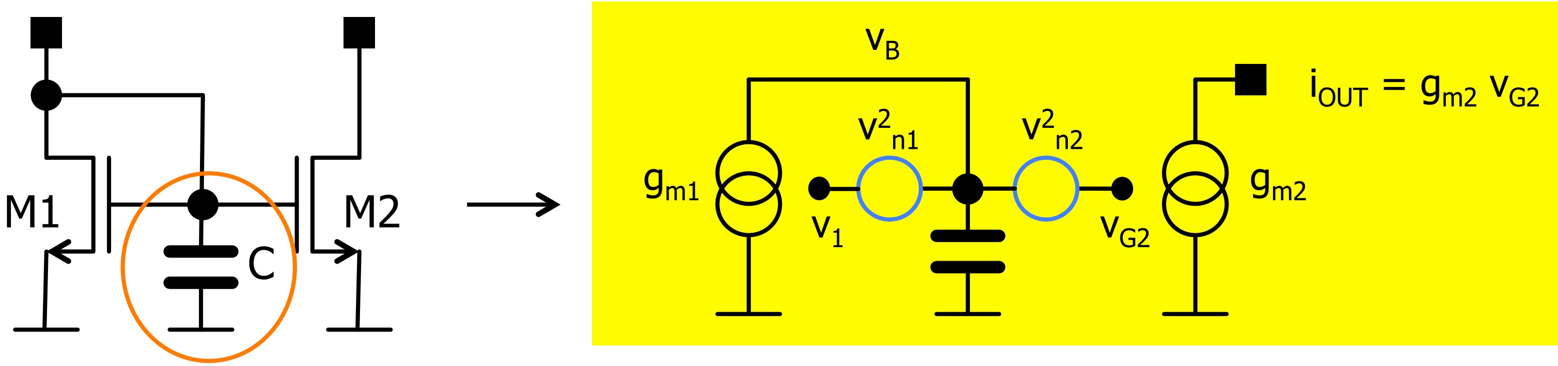
1. Left noise source: $v_{G2}^2 = v_B^2 = v_{n1}^2 \;\rightarrow\; i_{OUT}^2 = g_{m2}^2 \dfrac{4kT\,\gamma}{g_{m1}}$

2. Right noise source: $v_{G2}^2 = v_{n2}^2 \;\rightarrow\; i_{OUT}^2 = g_{m2}^2 \dfrac{4kT\,\gamma}{g_{m2}}$

Sum: $i_{OUT}^2 = g_{m2}\, 4kT\,\gamma \left(1 + \dfrac{g_{m2}}{g_{m1}}\right)$

- For $g_{m1} = g_{m2}$ (1:1 Mirror), noise $\sqrt{i_{OUT}^2}$ increases by $\sqrt{2}$
- For $g_{m1} >> g_{m2}$ (N:1 Mirror), noise is small
- For $g_{m1} << g_{m2}$ (1:N Mirror), noise is large. **DO NOT USE**

Improving the Current Mirror with Decoupling



- The first component is multiplied by

$$\frac{1}{1+\frac{C^2\omega^2}{g_{m1}^2}}$$ so that we get overall

$$g_{m2}\,4kT\,\gamma\left(1+\frac{g_{m2}}{g_{m1}}\,\frac{1}{1+\frac{C^2\omega^2}{g_{m1}^2}}\right)$$

Noise of the input MOS is cut away above $\omega = g_{m1}/C$

In[41]:=
```
EQ1 = gm1 v1 + vb s C == 0;
EQ2 = v1 - vb == vn;
```

In[43]:=
```
Eliminate[{EQ1, EQ2}, v1]
```

Out[43]= $gm1\ vn == (-gm1 - C\ s)\ vb$

In[44]:=
```
Solve[%, vb] // First
```

Out[44]= $\left\{vb \to -\frac{gm1\ vn}{gm1 + C\ s}\right\}$

In[45]:=
```
VB[s_] = vb /. %
```

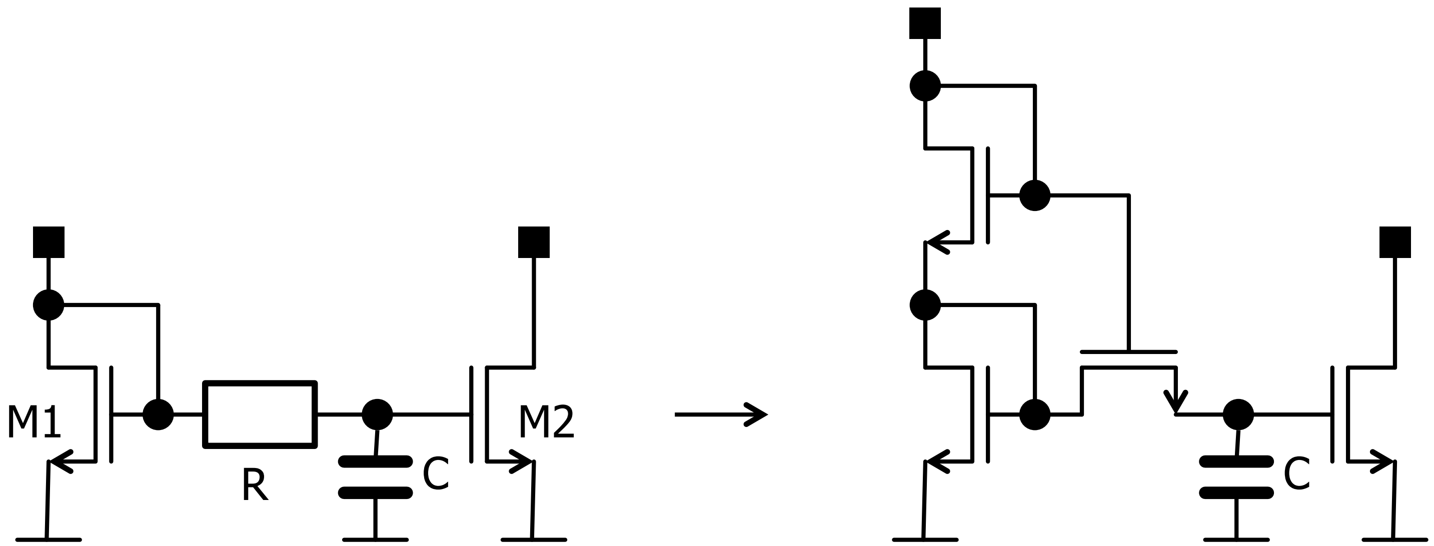
Out[45]= $-\frac{gm1\ vn}{gm1 + C\ s}$

In[46]:=
```
VB[i ω] Conjugate[VB[i ω]]
```

Out[46]= $\frac{gm1^2\ vn^2}{gm1^2 + C^2\ \omega^2}$

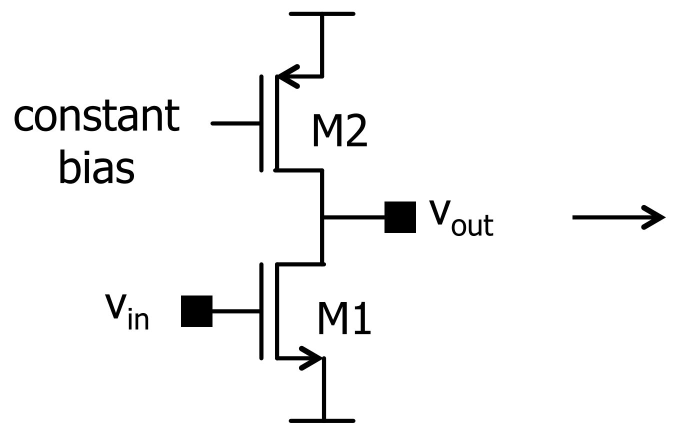
(One Possible further Improvement)

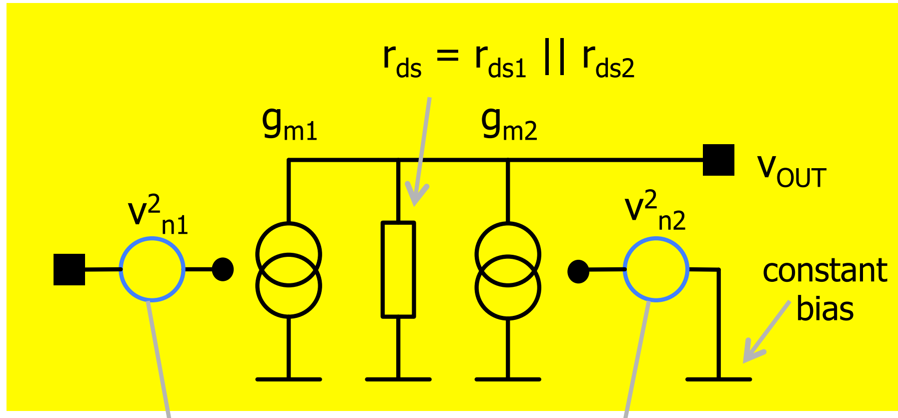
- An additional resistor R can be used to lower the bandwidth of the filter from g_{m1}/C to $1/RC$
- Note that R also adds its own noise...
- R can be implemented as a MOS with proper bias…



AMPLIFIER

Gain Stage






```
EQ1 = gm1 vn1 + vout/rds == 0
Solve[EQ1, vout] // First
{vout → -gm1 rds vn1}
vout2in = vout^2 /. %
gm1^2 rds^2 vn1^2
```

```
EQ2 = vout/rds + gm2 vn2 == 0
Solve[EQ2, vout] // First
{vout → -gm2 rds vn2}
vout2load = vout^2 /. %
gm2^2 rds^2 vn2^2
```

- Output noise:

```
vout2 = vout2in + vout2load /.
  {vn1^2 → 4 k T γ / gm1, vn2^2 → 4 k T γ / gm2}
```

$$\boxed{4\,(gm1 + gm2)\; k\, rds^2\, T\, \gamma}$$

- Referred to input (divide by gain):

```
gain = gm1 rds; vin2 = vout2 / gain^2
```

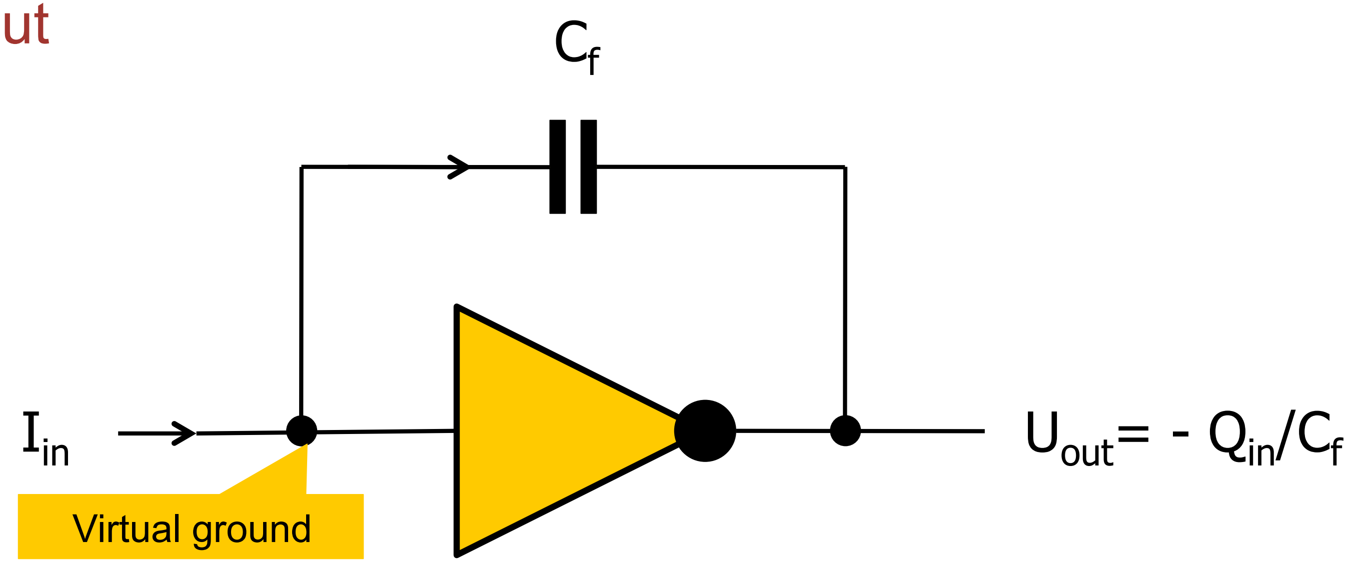
$$\boxed{\frac{4kT\gamma}{g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right)}$$

Input MOS must have *high* g_{m1}
Bias Source must have *low* g_{m2}

Charge Amplifier

The Charge Amplifier

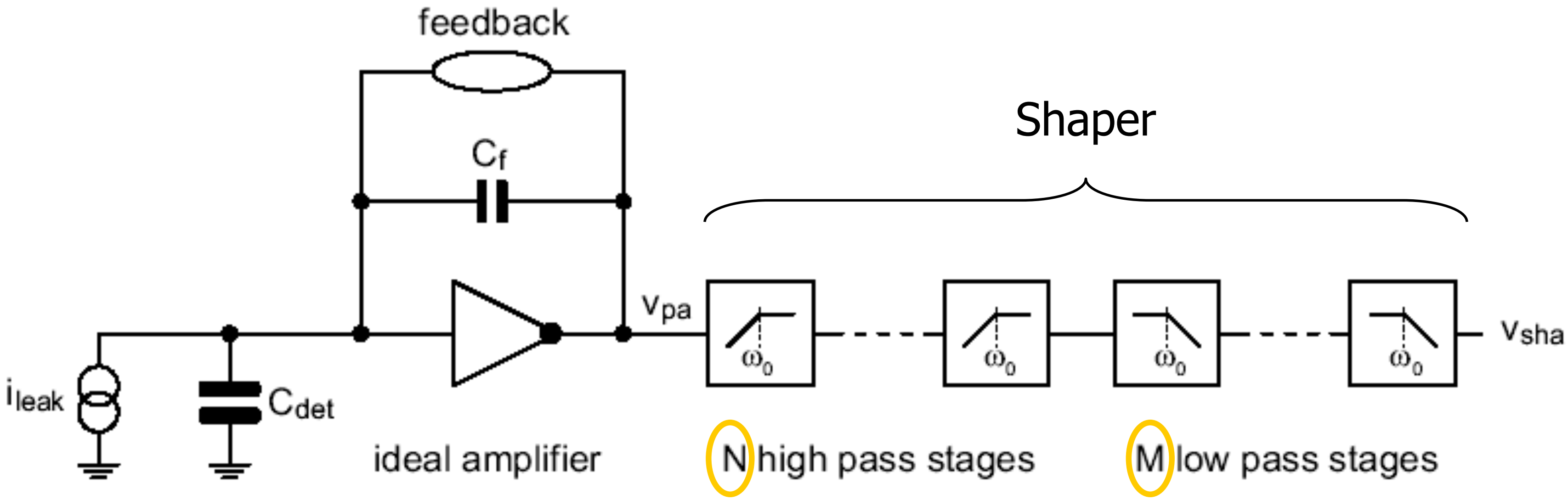
- The amplifier with feedback generates a *virtual ground* at its input



- Current (flowing charge) cannot stay on the input node (because the voltage is fixed) and must flow onto C_f
- The total input charge is the integral over I_{in}
- Therefore $Q_{in} = \int I_{in}\, dt = Q_f = U_f\, C_f \quad \rightarrow U_{out} = -U_f = -Q_{in}/C_f$

Classical System have a Filter = Shaper

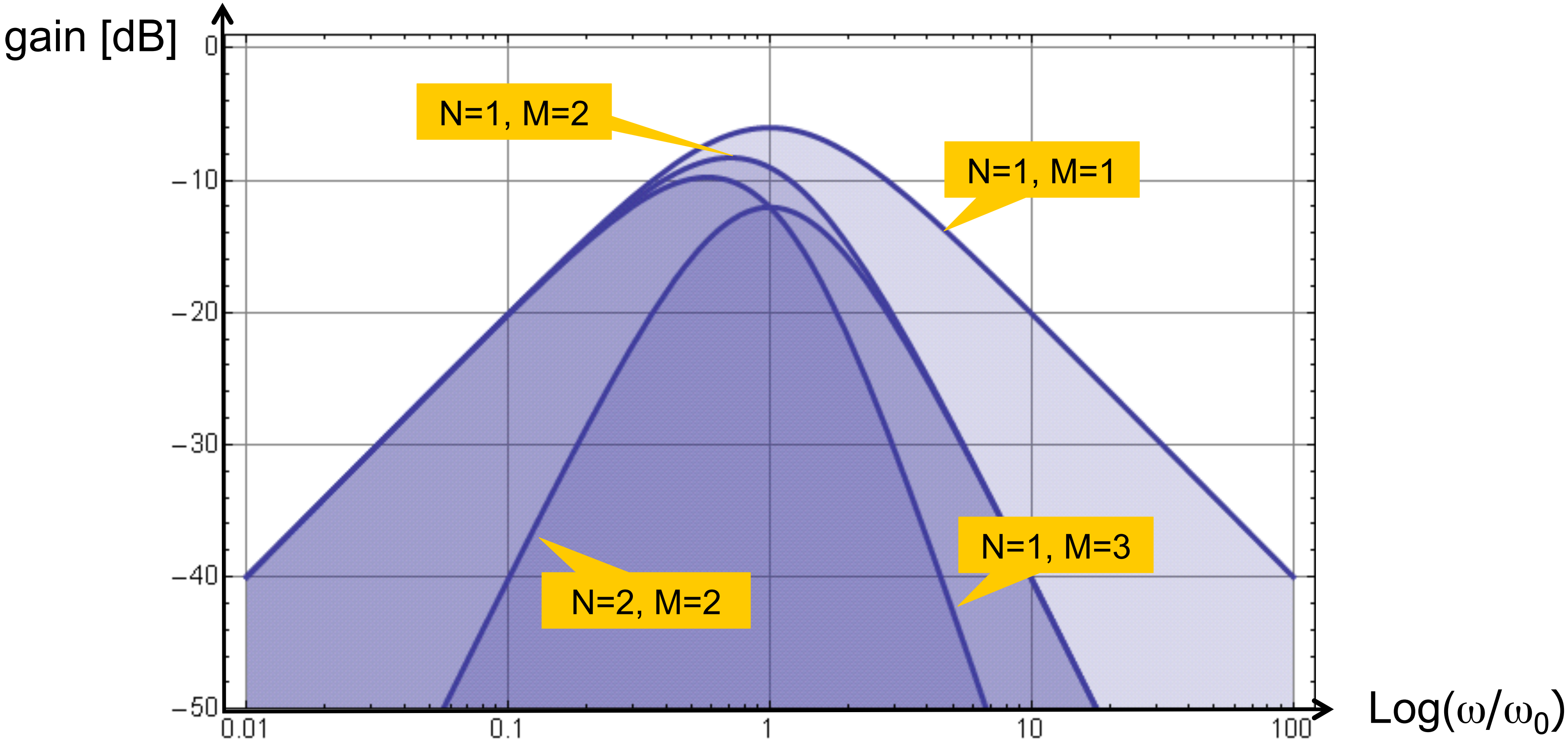
- Filter are added for pulse shaping & noise reduction:
  - *High pass stages* eliminate DC components & low freq. noise
  - *Low pass stages* limit bandwidth & therefore high freq. noise



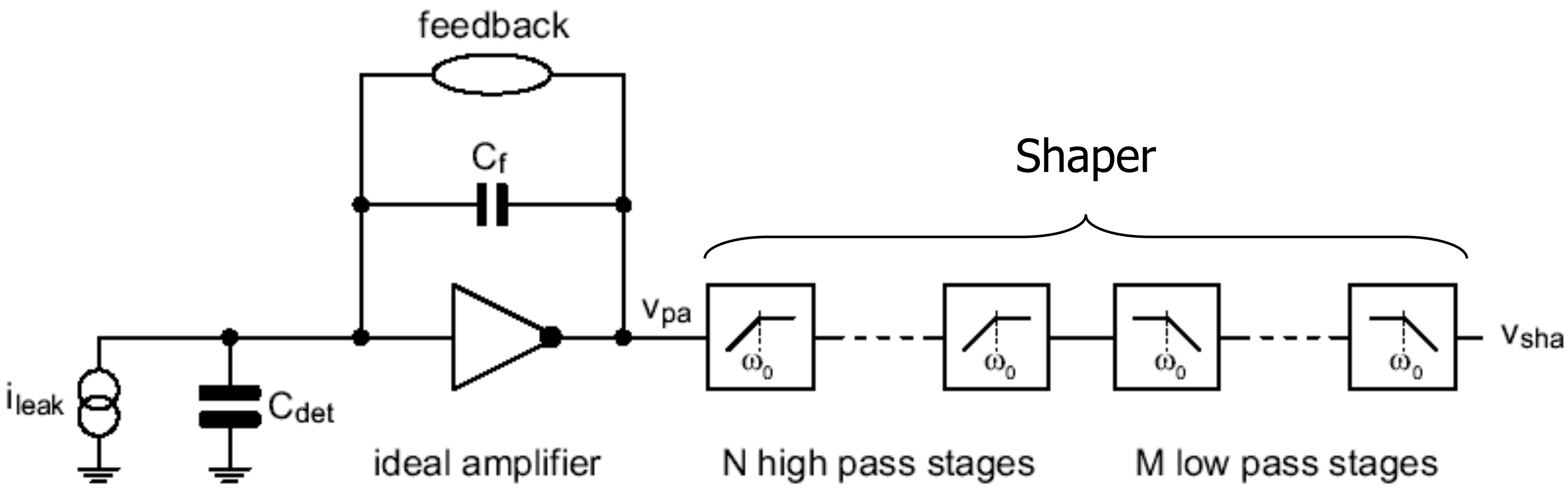
- Due to its output shape (see later), this topology is often called a ‘Semi Gaussian Shaper’
- Nearly always N = 1. Often M = 1, sometimes M up to 8

Frequency Behaviour of Shaper

- Low and High frequencies are attenuated
- Corner frequency (here: 1) is transmitted best
- Bode Plot (log/log) of transfer characteristic:



What is the output signal?



- For a delta current pulse, the output voltage v_{pa} is a step function
- This has a Laplace-Transform ~1/s
- The transfer functions of the high / low pass stages multiply to:

$$\mathcal{L}^{(N,M)}(s) = \frac{1}{s}\left(\frac{s\tau}{1+s\tau}\right)^N\left(\frac{1}{1+s\tau}\right)^M = \frac{\tau^N s^{N-1}}{(1+s\tau)^{N+M}}$$

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Pulse shape after shaper

- The time domain response is the inverse Laplace transform.
- The Laplace integral can be solved with residues:
  There is an (N+M)-fold pole at -1/τ

$$
\begin{aligned}
f^{(N,M)}(t) &= \operatorname{Res} \left. \frac{\tau^N s^{N-1} \mathrm{e}^{st}}{(1+s\tau)^{N+M}} \right|_{s=-1/\tau} \\
&= \frac{\tau^N}{(N+M-1)!} \lim_{s \to -\frac{1}{\tau}} \frac{\mathrm{d}^{N+M-1}}{\mathrm{d}s^{N+M-1}} \left[\frac{s^{N-1} \mathrm{e}^{st}}{(1+s\tau)^{N+M}} \left(s + \frac{1}{\tau} \right)^{N+M} \right] \\
&= \frac{1}{(N+M-1)!} \left(\frac{t}{\tau} \right)^M \sum_{i=0}^{\infty} \frac{(-\frac{t}{\tau})^i}{i!} \frac{(M+i+N-1)!}{(M+i)!} \qquad (1.42)
\end{aligned}
$$

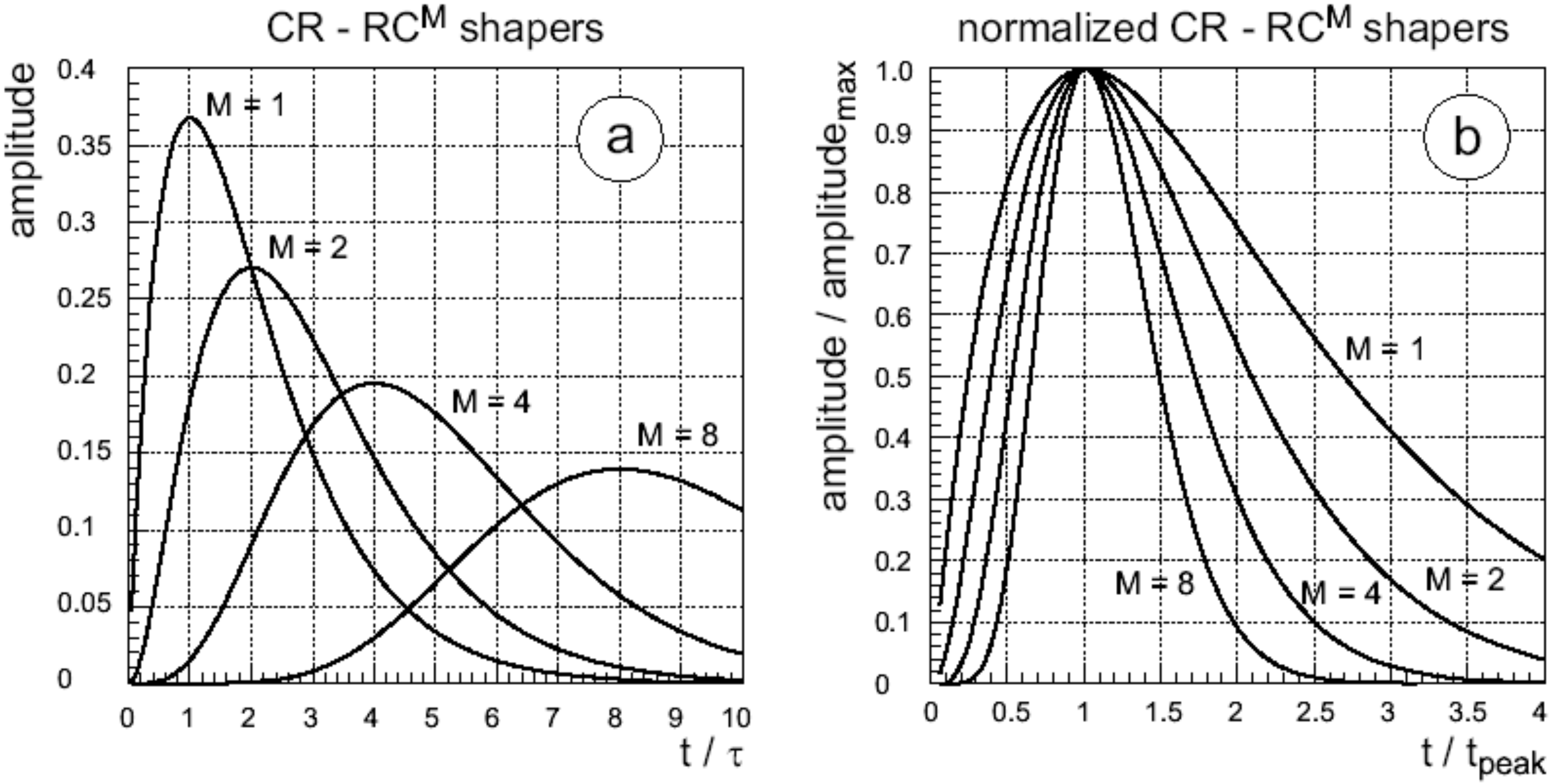
- For only ONE high pass section (N=1), this simplifies to:

$$
f^{(1,M)}(t) = \frac{1}{M!} \left(\frac{t}{\tau} \right)^M \mathrm{e}^{-t/\tau} \qquad t_{\text{peak}}^{(1,M)} = M\tau = \frac{M}{\omega_0} \qquad f_{\max}^{(1,M)} = \frac{1}{M!} \left(\frac{M}{\mathrm{e}} \right)^M
$$

$f_{max}^{(1,1)} = 1/e$

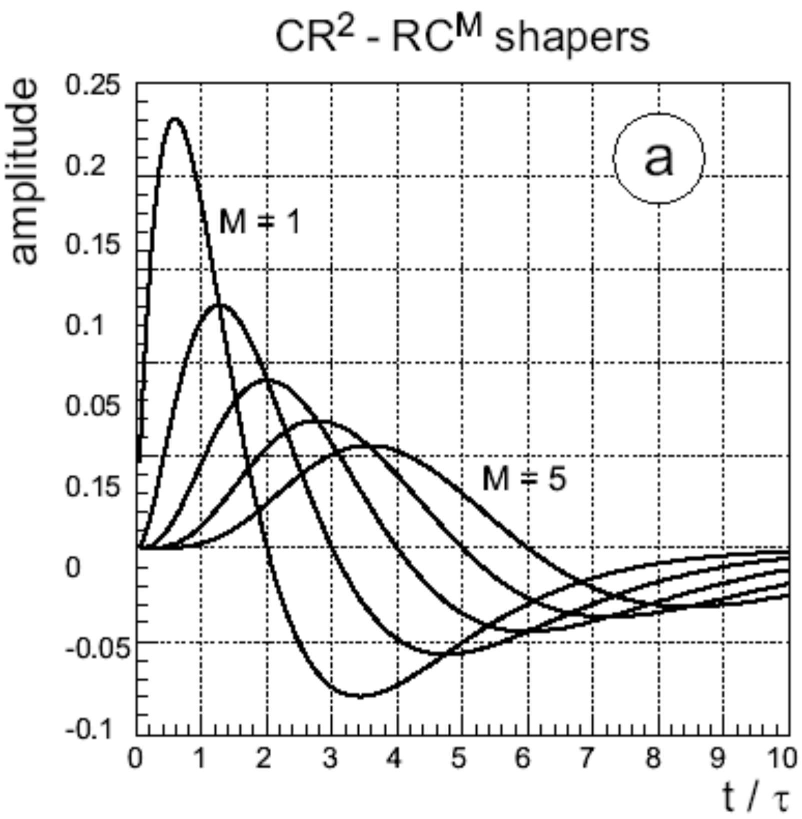
Pulse Shapes for N=1 (Only ONE High – Pass)

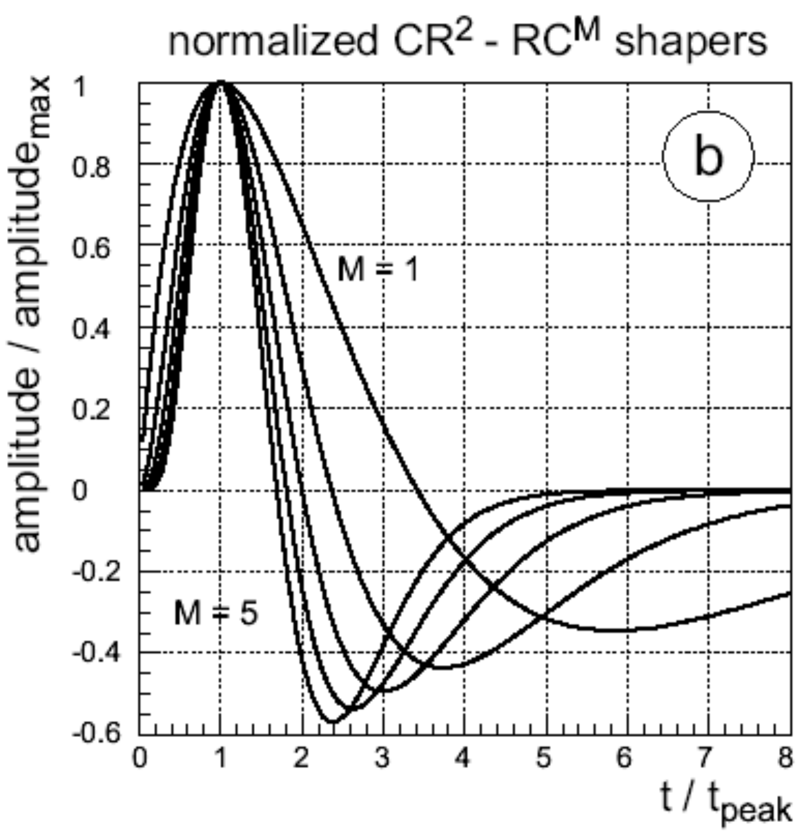
- Pulses from higher order are slower. To keep peeking time, τ of each stage must be decreased
- Right plots shows normalized pulses (same peak amp. & time)
- For high orders, pulses become narrow (width / peaking time), this is good for high pulse rates!



(Pulse Shapes for N=2)

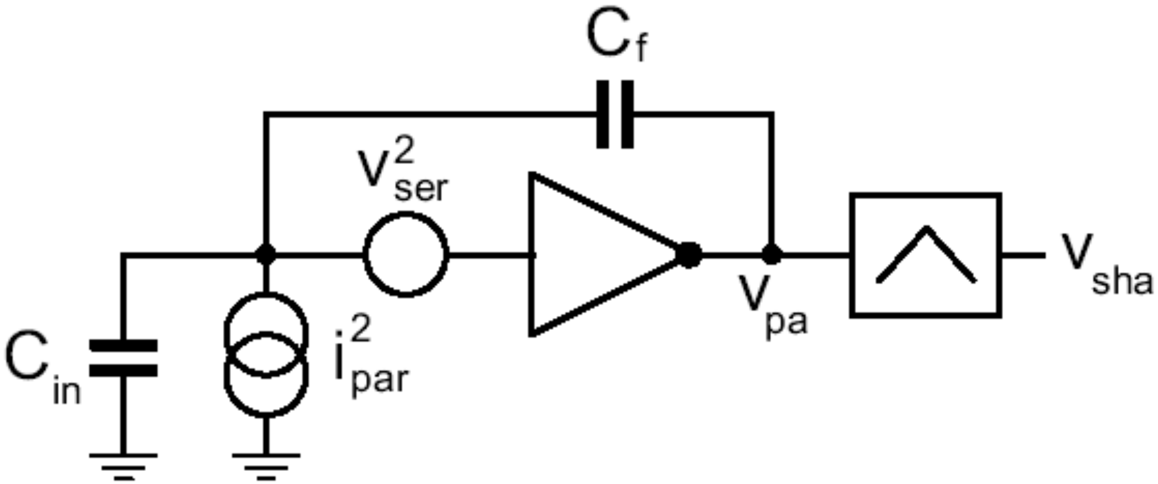
- This gives an undershoot which is often undesirable → N=1.
  - But: The zero crossing time is *independent* of amplitude.
    It can be used to measure the pulse arrival time with no time walk





Noise calculation: Noise sources

- Equivalent circuit with (ideal) amplifier, input capacitance, feedback capacitance and (dominant) noise sources:



- Spectral densities of noise sources:

$$\text{serial noise voltage}: \quad \frac{\mathrm{d}\langle v^2(f)\rangle}{\mathrm{d}f} = V_0 + V_{-1} f^{-1}$$

$$\text{parallel noise current}: \quad \frac{\mathrm{d}\langle i^2(f)\rangle}{\mathrm{d}f} = I_0$$

white (channel)

1/f noise (MOS)

white (leakage)

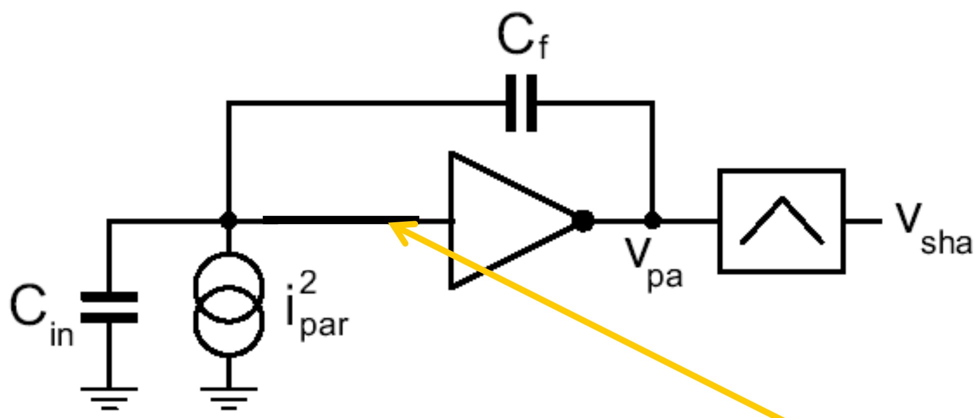
What is the total noise at the output ?

- Recipe (again):
  1. Calculate what effect a voltage / current noise *of a frequency f* at the input has at the output
  2. For each noise source: Integrate over all frequencies (with the respective densities)
  3. Sum contributions of all noise sources

- This yields the total rms voltage noise at the output

- Then compare this to a ‘typical’ signal.
  It is custom to use *one* electron at the input as reference.

Parallel Noise Current



- We assume a perfect virtual ground at the amplifier input

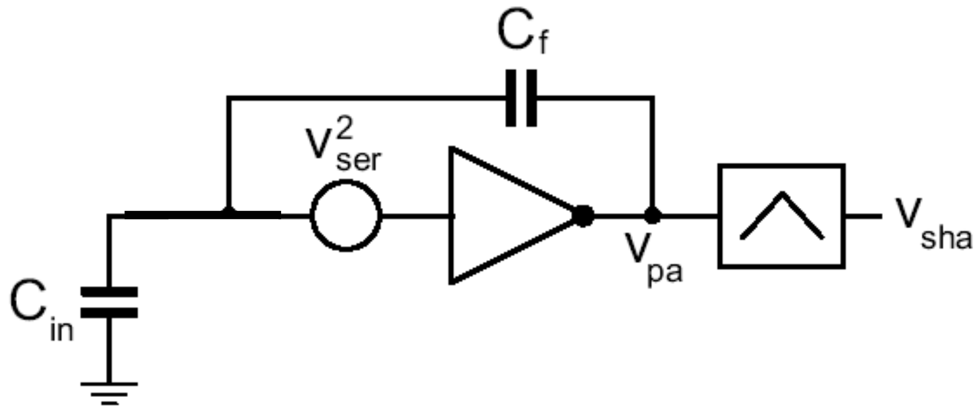
$\rightarrow$ No charge can the go to C_{in} (voltages are fixed)

$\rightarrow$ Noise current must flow through C_f: $v_{out} = i_{in} \times Z_{Cf}$

$$\text{parallel noise :} \quad \frac{\mathrm{d}\langle v_{\mathrm{pa}}^2(\omega)\rangle}{\mathrm{d}\omega} = \frac{\mathrm{d}\langle i_{\mathrm{par}}^2(\omega)\rangle}{\mathrm{d}\omega} \frac{1}{(\omega C_{\mathrm{f}})^2} = \frac{I_0}{2\pi} \frac{1}{(\omega C_{\mathrm{f}})^2}$$

(note the change of the frequency variable from ν to ω)

Serial Noise Voltage



- Output noise is determined by the capacitive divider made from C_f and C_{in}: $v_{ser} = v_{pa} \times Z_{Cin} / (Z_{Cin}+Z_{Cf})$

or:

$$v_{\mathrm{pa}}^2 = v_{\mathrm{ser}}^2 \left(\frac{C_{\mathrm{in}} + C_{\mathrm{f}}}{C_{\mathrm{f}}} \right)^2$$

Therefore:

$$\text{serial noise}: \quad \frac{\mathrm{d}\langle v_{\mathrm{pa}}^2(\omega)\rangle}{\mathrm{d}\omega} \approx \left(V_{-1}\omega^{-1} + \frac{V_0}{2\pi} \right) \left(\frac{C_{\mathrm{in}}}{C_{\mathrm{f}}} \right)^2$$

($C_{in}=C_{det} + C_{preamp} + C_{parasitic}$)

Total *Output* Noise (after the amplifier)

- In total, the output noise can be written as a sum of contributions with different frequency dependence:

$$\frac{\mathrm{d}\langle v_{\mathrm{pa}}^2(\omega)\rangle}{\mathrm{d}\omega} = \sum_{k=-2}^{0} c_k \omega^k$$

frequency dependence is here

with

$$c_{-2} = \frac{I_0}{2\pi C_{\mathrm{f}}^2}, \quad c_{-1} = V_{-1}\frac{C_{\mathrm{in}}^2}{C_{\mathrm{f}}^2} \quad \text{and} \quad c_0 = V_0 \frac{C_{\mathrm{in}}^2}{2\pi C_{\mathrm{f}}^2}.$$

leakage (white)

MOS gate (1/f)

MOS channel (white)

from leakage current I_{leak}: $\quad I_0 = 2qI_{\mathrm{leak}}$

from transistor channel noise: $\quad V_0 = \frac{8}{3}\frac{kT}{g_m}$

from 1/f noise: $\quad V_{-1} = \frac{K_{\mathrm{f}}}{C_{\mathrm{ox}}WL}$

Noise Transfer Function

- (N,M) - Shaper transfer function: $H^2_{N,M}(\omega) = A^2 \frac{\left(\frac{\omega}{\omega_0}\right)^{2N}}{\left(1+\left(\frac{\omega}{\omega_0}\right)^2\right)^{N+M}}$.

- 'Filtered' noise at the output of the shaper:

$$\langle v^2_{\mathrm{sha}} \rangle = \int_0^\infty H^2_{N,M}(\omega)\,\mathrm{d}\langle v^2_{\mathrm{pa}}(\omega)\rangle = \sum_{k=-2}^{0} \int_0^\infty c_k\,\omega^k H^2_{N,M}(\omega)\mathrm{d}\omega$$

$$= \frac{A^2}{2}\frac{1}{\Gamma(N+M)}\sum_{k=-2}^{0} c_k\,\omega_0^{k+1}\,\Gamma\left(N+\frac{k+1}{2}\right)\Gamma\left(M-\frac{k+1}{2}\right)$$

$$\Gamma(z+1) = z\,\Gamma(z)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi},$$

- For simplest shaper (N=M=1),
  Squared rms noise voltage at the shaper output:

$$\langle v^2_{\mathrm{sha}} \rangle = A^2\frac{\pi}{4}\left(\frac{c_{-2}}{\omega_0} + \frac{2}{\pi}c_{-1} + \omega_0\,c_0\right)$$

Calculation of ENC

- The **equivalent noise charge, ENC** is the (rms) noise at the output of the shaper expressed in Electrons input charge, i.e. divided by the 'charge gain'
- The 'charge gain' is (see before): Vmax = q/C_f × A × 1/e

charge of 1 electron (1.6e-19C)

Shaper dc gain

Peak amplitude for N=M=1

leakage gives noise for slow shaping

1/f – noise cannot be reduced by charging shaping time

$$\mathrm{ENC}^2_{\mathrm{CR-RC}} = \frac{\langle v^2_{\mathrm{sha}} \rangle}{V^2_{\mathrm{max}}} = \frac{\mathrm{e}^2}{4q^2}\left(\frac{\tau}{2} I_0 + \frac{1}{2\tau} V_0 C^2_{\mathrm{in}} + 2\, V_{-1} C^2_{\mathrm{in}}\right)$$

C_{in} is bad for **fast** shaping. Reducing V_0 requires large g_m

Noise contributions

- Real noise contributions for the coefficients I_0, V_0, V_{-1}:

$$\text{from leakage current } I_\text{leak}: \qquad I_0 = 2qI_\text{leak}$$

$$\text{from transistor channel noise:} \qquad V_0 = \frac{8}{3}\frac{kT}{g_m}$$

$$\text{from 1/f noise:} \qquad V_{-1} = \frac{K_\text{f}}{C_\text{ox}WL}$$

- For a 0.25µm technology (C_{ox}=6.4 fF/µm^2, K_f=33×10^{-25} J, L=0.5µm, W=20µm) and C_{in}=200fF, I_{leak}=1nA and τ=50ns, g_m=500µS (typical LHC pixel detector):

$$\left(\frac{\text{ENC}}{\text{e}^-}\right)^2 = 115 \cdot \frac{\tau}{10\,\text{ns}} \cdot \frac{I_\text{leak}}{1\,\text{nA}} \qquad \rightarrow 575$$

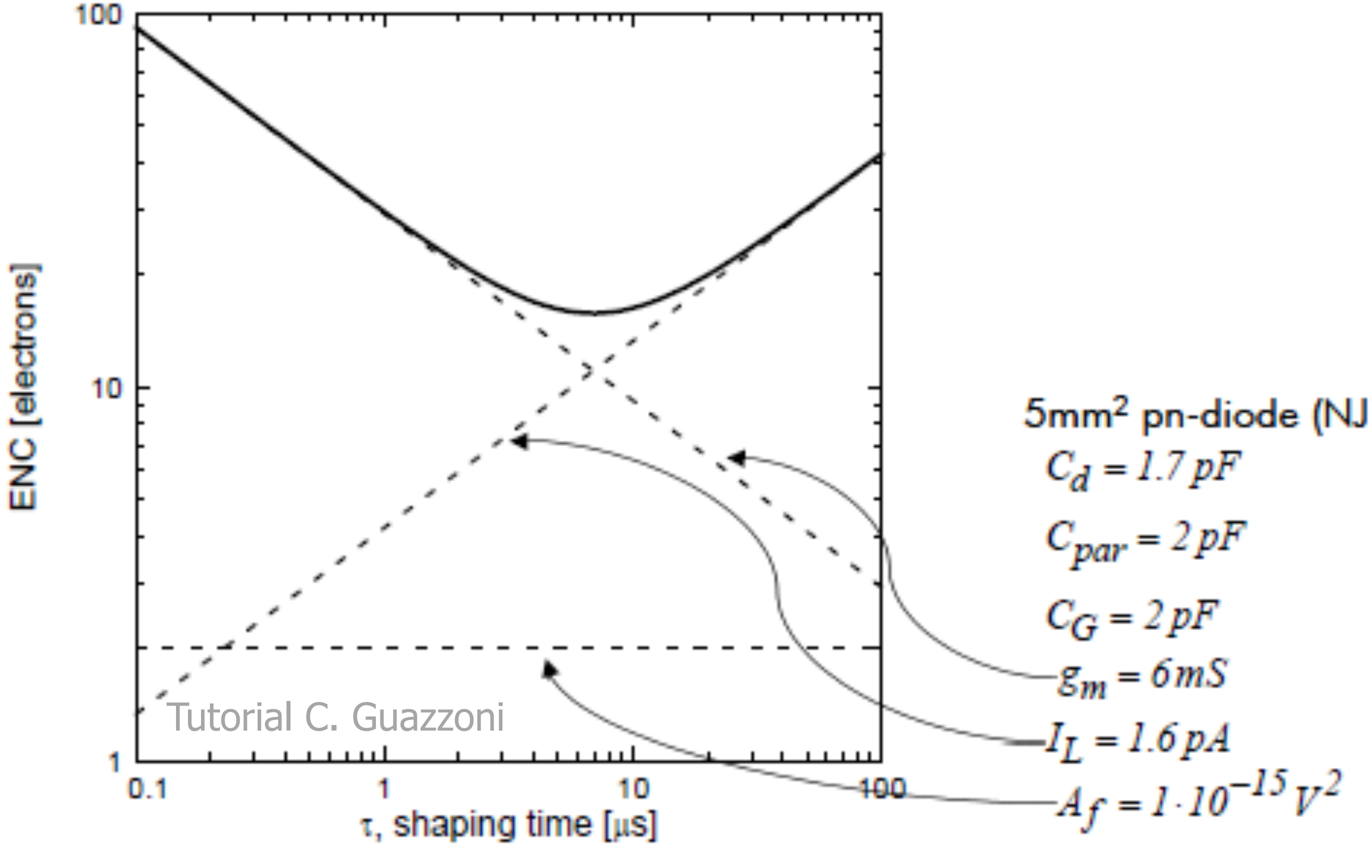
$$+ 388 \cdot \frac{10\,\text{ns}}{\tau} \cdot \frac{\text{mS}}{g_m} \cdot \left(\frac{C_\text{in}}{100\,\text{fF}}\right)^2 \qquad \rightarrow 621$$

$$+ 74 \cdot \left(\frac{C_\text{in}}{100\,\text{fF}}\right)^2. \qquad \rightarrow 296$$

ENC=40 e$^-$

Noise vs. Shaping Time

- Long shaping: leakage noise contributes more
- Short shaping: Amplifier white noise, worsened by C_{Det}
- Always: Amplifier 1/f noise, worsened by C_{Det}



MISC

- For large MOS, the gate can have a significant resistance which can add thermal noise. Must use layout with multiple gate contacts.
- The noise coefficient γ can increase (quite) a lot for very short channel MOS. In general do NOT use shortest MOS if you need low noise.
- Noise models are often not very reliable. In particular 1/f noise can be run dependent. You may want to include noise test structures (i.e. large MOS arrays of the geometry you use)

- Noise can also be treated in transient simulation (enable 'transient noise' button). Good for nonlinear systems (comparator) and 'quick look'. Large simulation effort due to small time steps (high freq. noise components). Provides less understanding where noise comes from…

Summary

- Resistor, MOS and Diodes (and BJTs) have noise
- Noise is described by its spectral density
- Noise contributions are propagated through the circuit with the respective frequency transfer function

- Total RMS noise is the integral over the spectra

- Current sources require MOS with low g_m.
  - This leads to larger saturation voltages…
- Amplifiers require MOS with large g_m

- Low bandwidth → Low Noise
- Filters limit bandwidth and thus reduce noise, but also decrease the signal…

Exercise 1: Noise in Resistors

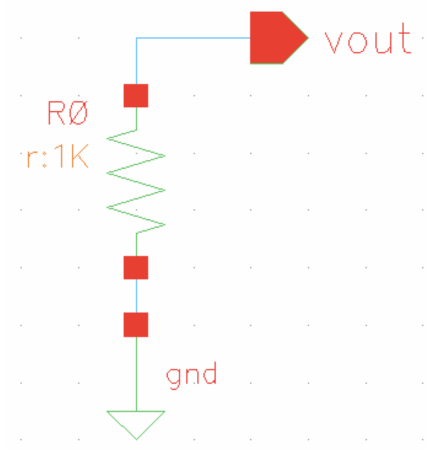
- Connect a 1kΩ resistor (from AnalogLib) on one side to gnd!

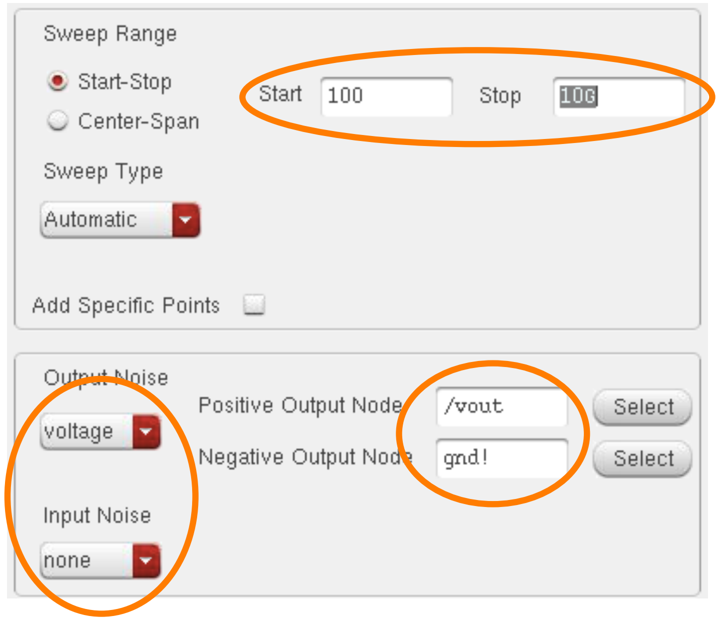
- Simulate the voltage noise spectrum on the other side
  - Is it flat?
  - Is the absolute value what you expect?
    Note how the prefixes (e.g. 'a' for Atto) are used…

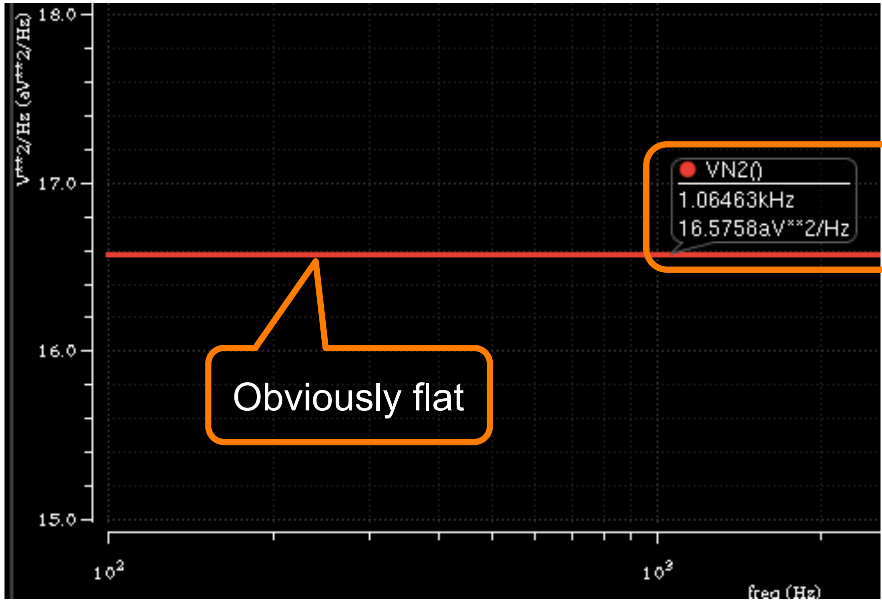
- Add an ideal low pass filter with corner frequency 10 MHz
  - Use a simple RC and set the 'Generate Noise?' flag of R to 'No'
  - Choose R much larger than 1kΩ to not 'load' the 'source'
  - How does the spectrum look like?
  - How much has noise decreased at the corner?
  - Integrate over a large frequency range to get the rms noise
  - Is it what you expect?

- Determine the overall RMS noise (totalNoise("noise" nil nil))
  - Is it what you calculate?

Exercise 1: Solution





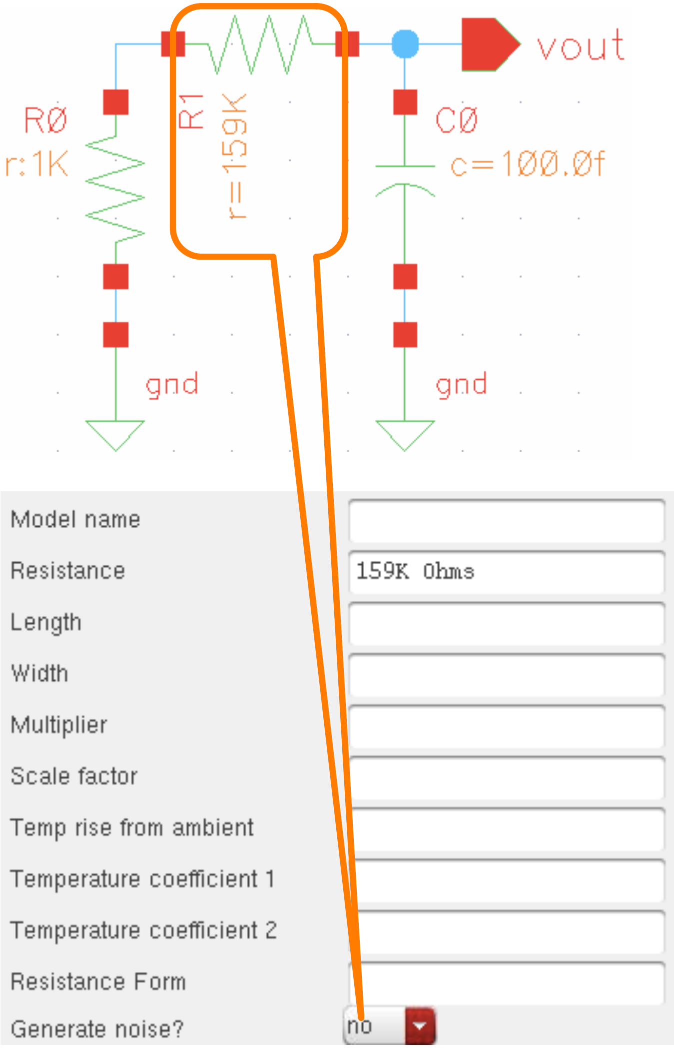


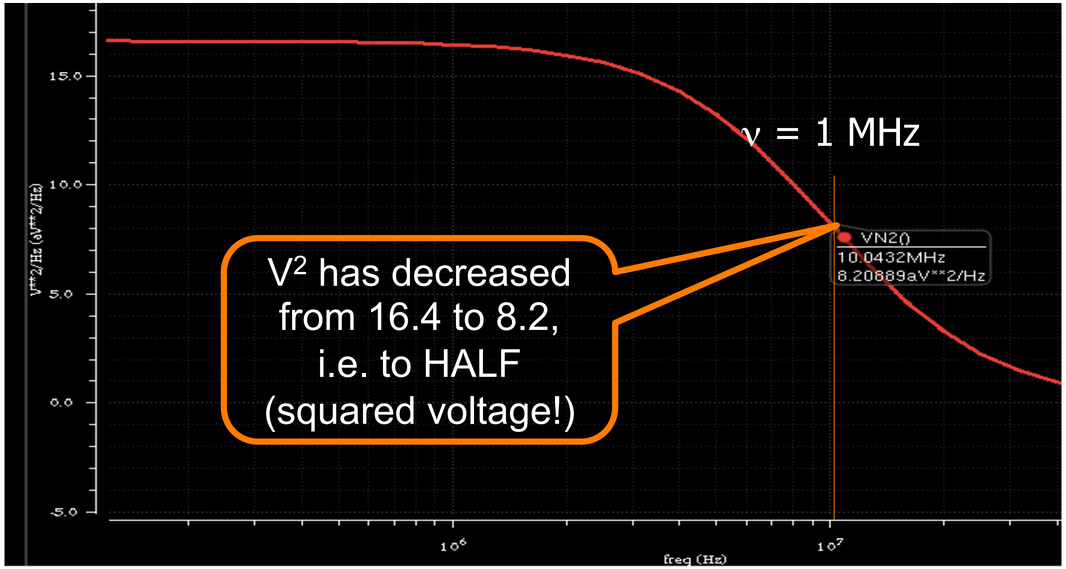
$4kT \times 10k\Omega = 1.64 \times 10^{-17}\ V^2/Hz$
$= 16.4\ aV^2/Hz$ - OK

$aV^2 = a \times V^2$
NOT: $(aV)^2$

Exercise 1: Solution

- $\omega = 2\,\pi\,\nu = 1/(RC) \rightarrow R = 1/(2\,\pi\,\nu\,C) = 159\text{k}\Omega$ (for C =0.1pF)





- $v^2_{RMS} = 16.4\ \text{aV}^2 \times \nu_{Brick}$
  $= 16.4\ \text{aV}^2 \times \pi/2 \times 10\ \text{MHz}$
  $= 257\ \text{pV}^2$

Outputs

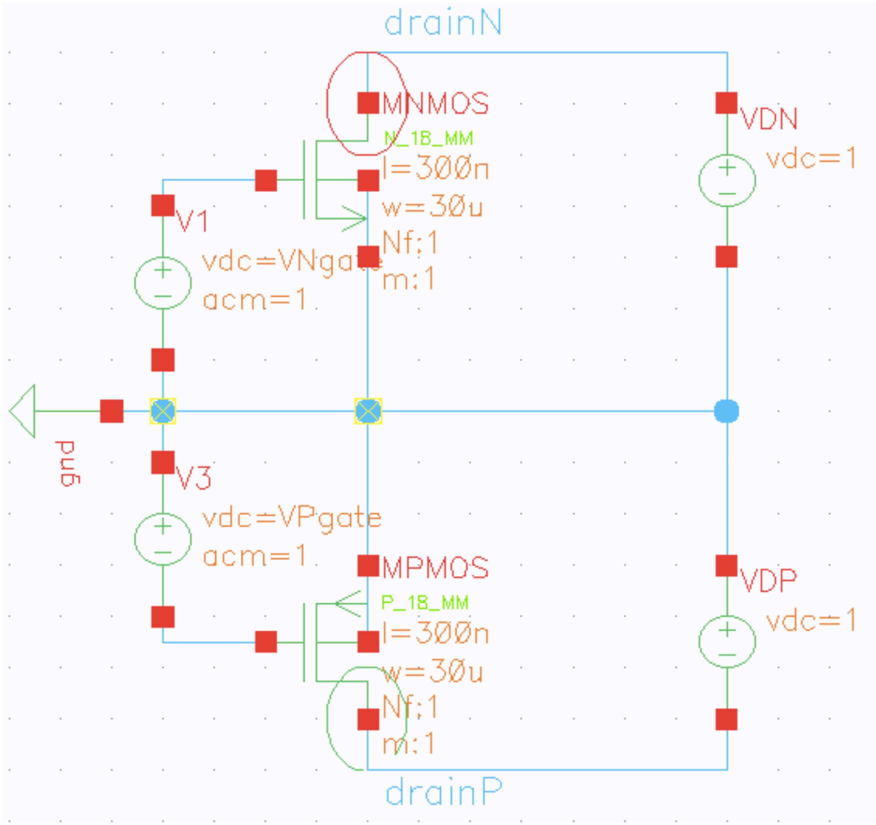
	Name/Signal/Expr	Value	Plot
1	vout		✔
2	totalNoise("noise" nil nil nil)	259.4p	✔

Exercise 2: Noise in MOS Transistors

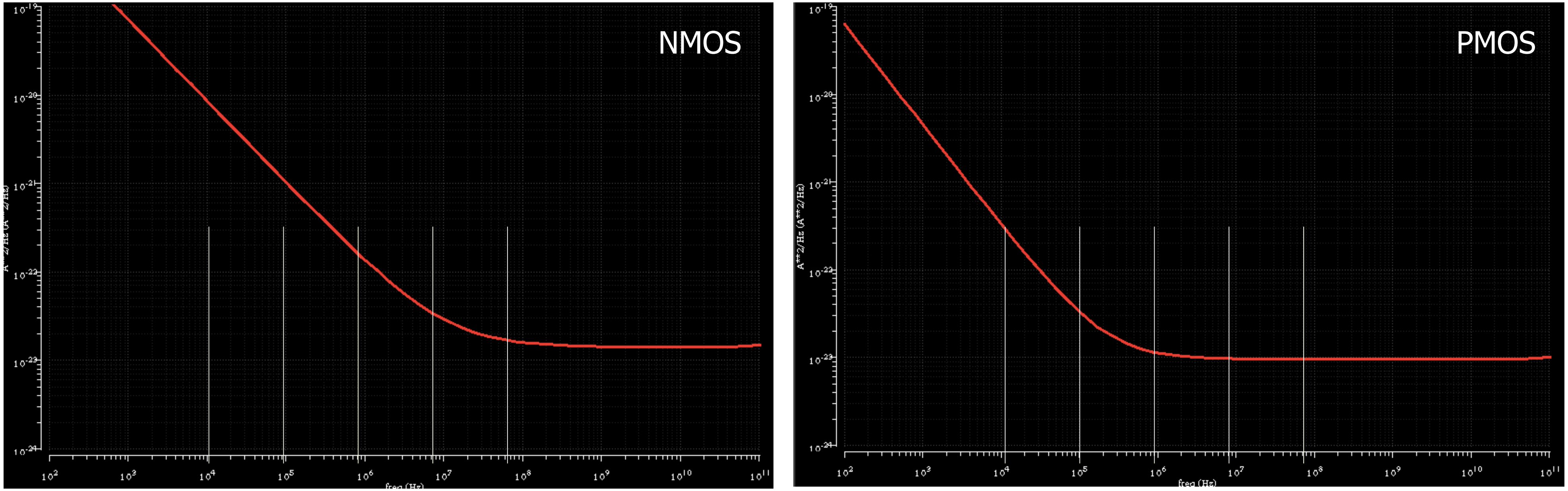
- Instantiate an NMOS and a PMOS of W/L = 30µm/0.3µm
- Apply a fixed drain voltage of, say, 1V
- Find the gate voltages required for drain currents of 100µA
  - Use a DC sweep
- Determine the transconductance for this operation point
  - Use an AC analysis
  - What g_m values do you get? Compare NMOS and PMOS.
- Observe the noise current spectra at the drains
  - Set the drain voltage source as *probe instance* to see *currents*
  - You have to run NMOS and PMOS separately
- What are the white noise magnitudes?
  - Do the values roughly match with what you expect from g_m?
- Where are the 1/f corners? (Use Log Plot!)
- For one device, increase the current and observe the spectrum

Exercise 2: Solution

- For my technology (UMC180nm):
  - Vgate: NMOS = 511 mV, PMOS = 649 mV (PMOS needs more because geometry and current are the same, but not so much, because we are close to w.i. given the 'wide' devices)
  - gm: NMOS: 1.7mS, PMOS: 1.1mS



Exercise 2: Solution

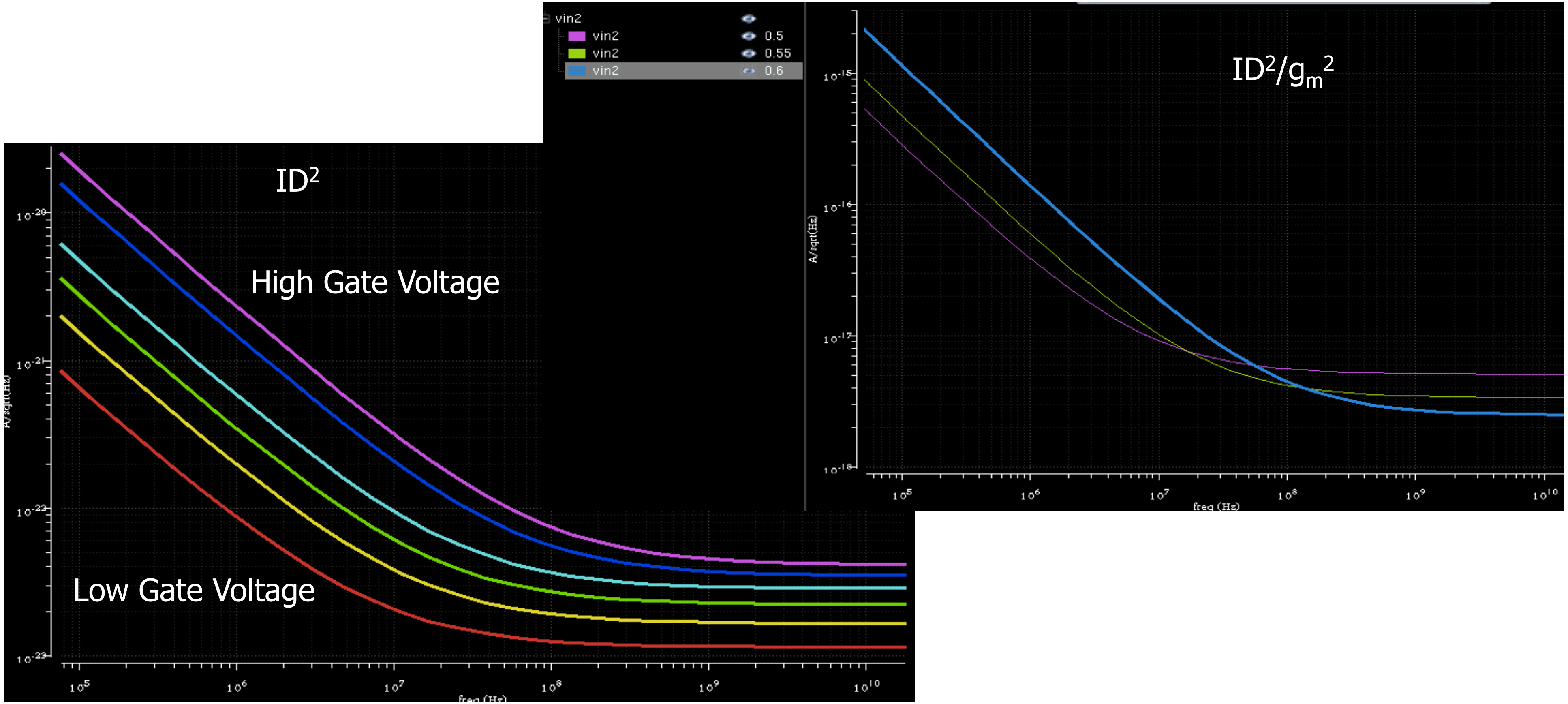


- Expect noise currents (NMOS/PMOS):
  - 2/3 × 16.4 aVA/Hz × 1.7/1.1mS = 19/11.9 yA²/Hz
- Simulated:
  - NMOS: 14.3/9.7 yA²/Hz - OK

- Corners: ~ 50 / 1 MHz.
  PMOS is much better @ comparable thermal noise

Exercise 2: Solution

- With higher gate voltage (higher current, higher g_m (1.5->4mS)) output current noise increases (left), but input voltage noise decreases (right)
- 1/f noise at input is 'fairly' constant

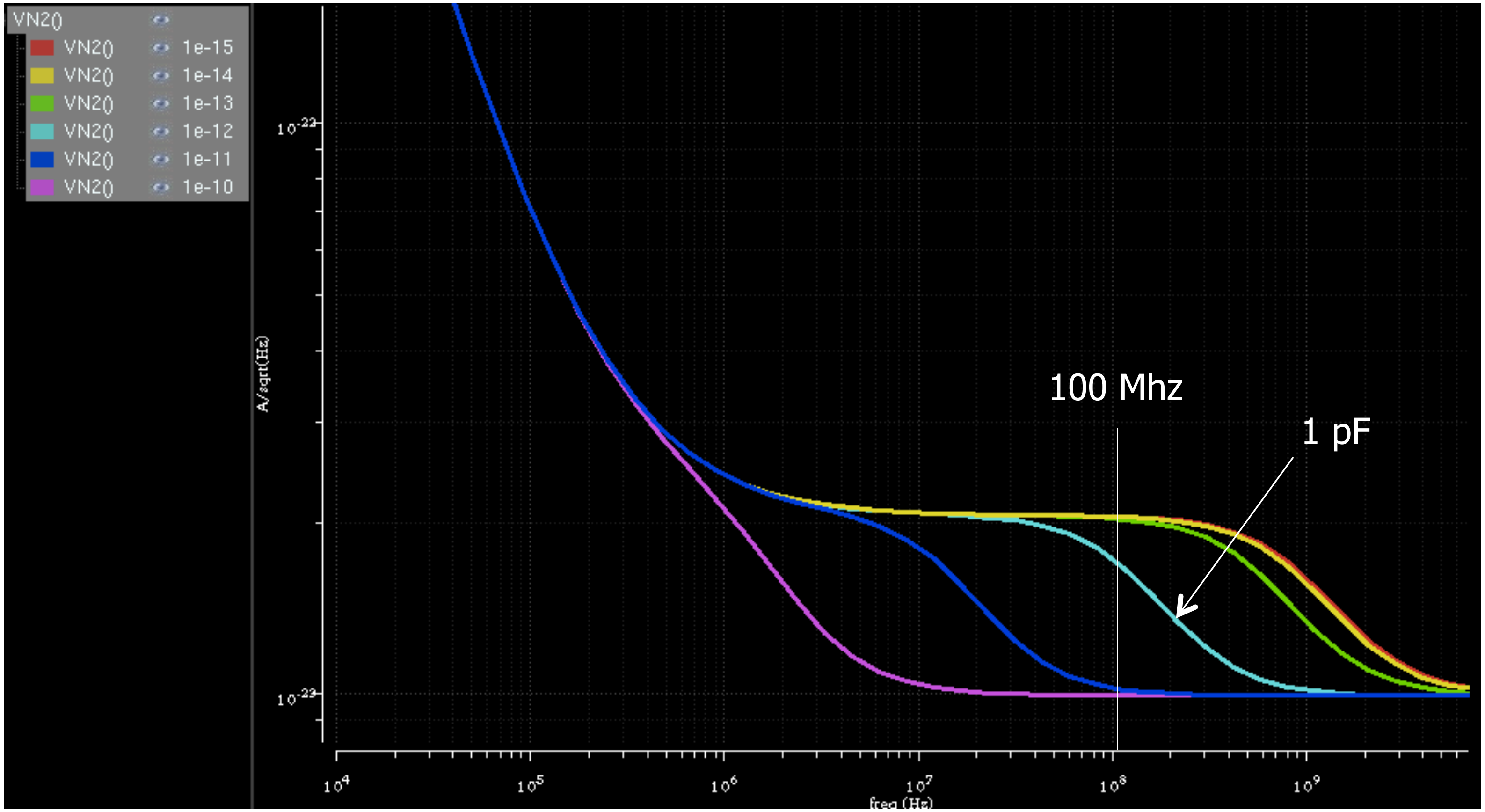


Exercise 3: Noise in Current Mirror

- Make an 1:1 PMOS current mirror, using the same PMOS as in the previous exercise.
  - (PMOS is better for lower 1/f noise here)
- Load the output with 1V. Inject a current at the input and verify that you get the current at the output
- Plot the noise at the output.
- Now add a decoupling capacitor (to gnd!) to the bias node.
- Plot the noise spectrum for C_{dec} = 1/10/100/… fF
  - Observe how larger decoupling cuts down the noise at lower frequencies
  - Why is there no added noise at very high frequency ?
  - Check that the 'corner' frequencies for various caps are where you expect them !

Exercise 3: Solution

- Simulation for C=1fF (red),10fF,…100pF (pink)



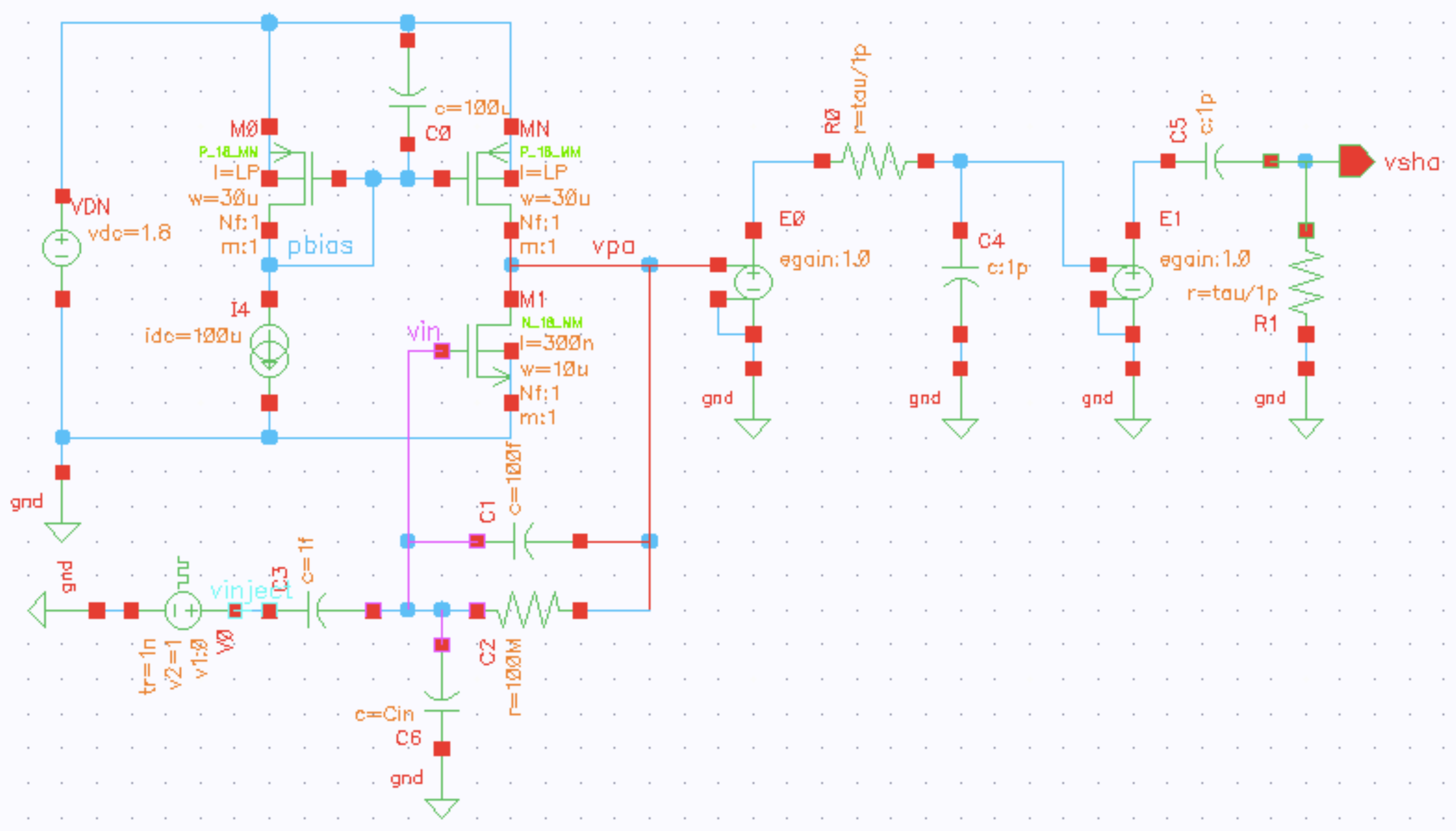
- The corner is at $\omega = g_m/C$ with $g_m \sim 1$mS.
  For 1pF this gives $\omega \sim 1$GHz, i.e. $\nu \sim 130$ MHz

Exercise 4: Charge Amplifier

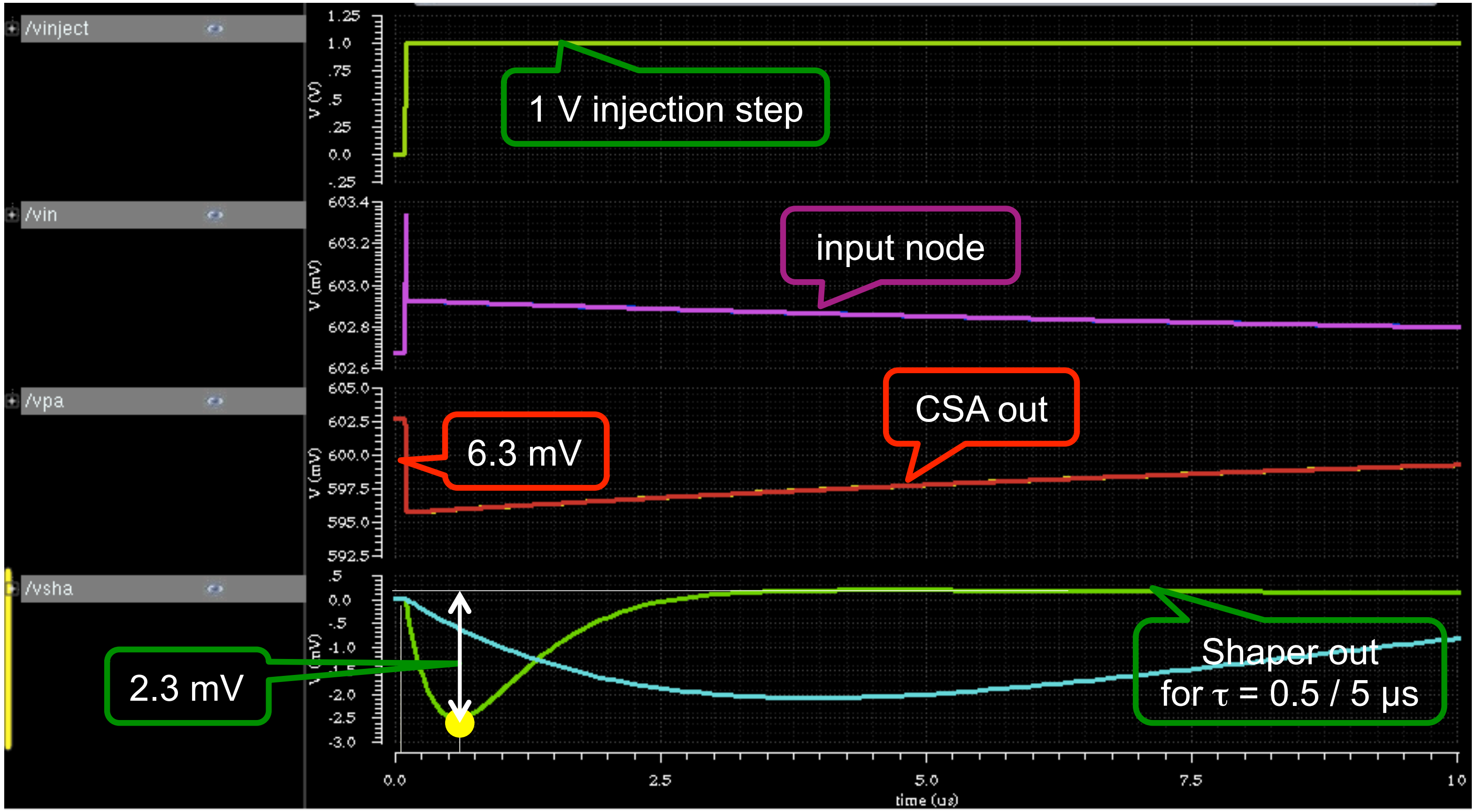
- Design a charge amplifier followed by a (passive) CR-RC filter with corner frequency τ ~ 1µs with input cap of 1 pF.
  - For the amplifier, use a simple NMOS (W/L~10µ/0.3µ) gain stage with a PMOS current mirror load (30µ/0.3µ). Decouple the bias node.
  - Bias at ~100µA.
  - Use a feedback capacitor of 100 fF. Put a 100MΩ resistor in parallel to set the dc operation point.
  - Implement the filter as passive RCs, with parameterized τ. Use vcvs buffering between the stages. You may want to switch off noise in the resistors…
- In a transient simulation, inject a 1fC charge (1V step across a 1fF injection capacitor)
  - Observe the output of preamp / shaper, and the preamp input
  - Is the virtual ground at the input 'ok'?
  - Is the shaper signal as expected (amplitude, peaking)? Vary τ!

Exercise 4: Solution

- Schematic



Exercise 4: Transient Simulation



$\Delta V_{CSA} = Q_{in}/C_f = V_{in} \times C_{in}/C_f$ = 1V × 1fF/100fF = 10 mV (see 6.3 mV)

$\Delta V_{SHA} = \Delta V_{CSA} / e$ (p.43) = 3.7 mV (see 2.3 mV)

-> Gain x C_f is not much larger than C_{in} so that input moves

-> Shapes for long shaping are degraded by preamp discharge

Exercise 4: Charge Amplifier

- Perform a noise simulation for τ = 0.5 µs and 5 µs

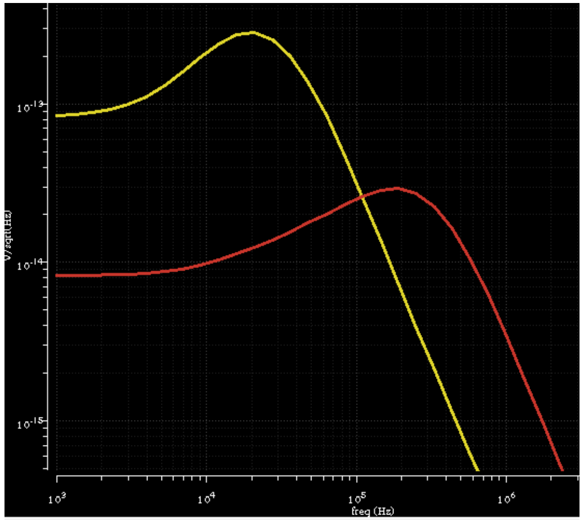
- For τ = 1 µs:
  - Determine the total noise at the shaper output by integration
  - Take the square root to get the RMS (voltage) noise
  - Divide by the signal for one electron to get the ENC
  - What do you get ?

- Double C_{in} to 2 pF
  - What is the ENC now?

- Find out which noise type / contribution dominates your circuit by using Noise Analysis

Exercise 4: Solution

- Noise sim:



- For τ = 1 µs, C_{in} = 1 pF:
  - Noise Integral: 17 × 10^{-9} V^2 (30 × 10^{-9} V^2 for 2 pF)
  - RMS noise (square root): 130 × 10^{-6} V (173)
  - We get 2.3/2.0 mV for 1fC (~ 6250 e) → 3.7/3.2 × 10^{-7} V/e-
  - For 1pF: ENC = 130 × 10^{-6} V / 3.7 × 10^{-7} V/e = *350 e-*
  - For 2pF: ENC = 173 × 10^{-6} V / 3.2 × 10^{-7} V/e = *540 e-*
- Notes:
  - Gain of amplifier is very low
  - More noise components than just input….

Exercise 4: Solution

- Noise Summary:

Device	Param	Noise Contribution	% Of Total
/M1	fn	9.72616e-09	57.09
/R1	rn	4.17986e-09	24.53
/R0	rn	2.098e-09	12.31
/Rf	rn	8.74026e-10	5.13
/M1	id	6.28512e-11	0.37
/MN	fn	4.92349e-11	0.29
/MN	id	4.7543e-11	0.28
/M1	rs	3.66483e-14	0.00
/MN	rs	1.25303e-14	0.00
/M1	rd	2.44555e-17	0.00

Integrated Noise Summary (in V^2) Sorted By Noise Contributors
Total Summarized Noise = 1.70377e-08

1/f noise input MOS

Oups: Noise in the Rs of the filter…

Noise in Feedback Resistor

Channel Noise in input MOS

- We are totally 1/f noise limited…. (slow shaping)
  → larger area MOS, PMOS,…
- I forgot to switch off the resistive noise in the shaper...