Parton Luminosity

- Starting point: factorization ansatz

$$\sigma = \sum_{jk} \int dx_j\, dx_k\, f_j(x_j, \mu_F^2)\, f_k(x_k, \mu_F^2)\, \hat{\sigma}_{jk}(\underbrace{x_j x_k s}_{\hat{s}}, \ldots)$$

- Physics: $\hat{\sigma} \sim \frac{1}{\hat{s}}$ → take out this dependence

$$\frac{d\sigma}{d\hat{s}} = \sum_{jk} \int dx_j\, dx_k\, f_j\, f_k\, \hat{\sigma}_{jk}\, \delta(\hat{s} - x_j x_k s)$$

- Introduce scaling variable: $\tau \equiv \frac{\hat{s}}{s}$

$$\rightarrow \delta(\hat{s} - x_j x_k s) \underset{\uparrow}{=} \frac{1}{\hat{s}} \delta\left(1 - x_j k_k \frac{s}{\hat{s}}\right) = \frac{1}{\hat{s}} \delta\left(1 - \frac{x_j k_k}{\tau}\right)$$

$$\delta(ax) = \frac{\delta(x)}{|a|}$$

$$\Rightarrow \frac{d\sigma}{d\tau} = \sum_{jk} \int dx_j\, dx_k\, f_j\, f_k\, \frac{\hat{\sigma}_{jk}}{\tau}\, \delta\left(1 - \frac{x_1 x_2}{\tau}\right)$$

- Carry out one of the $\int dx_i$ integrals using δ distribution: x_k

$$\frac{d\sigma}{d\tau} = \sum_{jk} \frac{\sigma_{jk}}{\tau} \int_\tau^1 dx_j\, \frac{\tau}{x_j}\, f_j(x_j, \mu_F^2) \cdot f_k\left(\frac{\tau}{x_j}, \mu_F^2\right)$$

$$= \sum_{jk} \sigma_{jk} \cdot \frac{dL_{jk}}{d\tau} \quad \text{with} \quad \frac{dL_{jk}}{d\tau} = \int_\tau^1 \frac{dx}{x}\, f_j(x, \mu_F^2)\, f_k\left(\frac{\tau}{x}, \mu_F^2\right)$$

- Sometimes $\frac{dL_{jk}}{d\tau}$ is symmetrized:

$$\frac{dL_{jk}}{d\tau} = \frac{1}{1+\delta_{jk}} \int_\tau^1 \frac{dx}{x} \left[f_j(x, \mu_F^2)\, f_k\left(\frac{\tau}{x}, \mu_F^2\right) + f_j\left(\frac{\tau}{x}, \mu_F^2\right) f_k(x, \mu_F^2) \right]$$