Drell-Yan Cross Section

- QED process: $e^+e^- \to \mu^+\mu^-$ $\rightsquigarrow$ $\sigma = \frac{4\pi\alpha^2}{3s}$
- DY: $q\bar{q} \to \mu^+\mu^-$ $\rightsquigarrow$ $\hat{\sigma} = \frac{4\pi\alpha^2}{3\hat{s}} \frac{1}{N_c} \cdot Q_q^2$
  - N_c: # colors; Q_q: quark charge
- $\hat{s}$ not fixed:

$$\frac{d\hat{\sigma}}{dM^2} = \frac{4\pi\alpha^2}{3} \frac{1}{N_c} \cdot Q_q^2 \cdot \delta(\hat{s} - M^2)$$

- Incoming partons: $p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1)$, $p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)$

$$\to p_1 + p_2 = \frac{\sqrt{s}}{2}(x_1 + x_2, 0, 0, x_1 - x_2)$$

$$\Rightarrow M^2 = (p_1 + p_2)^2 = \frac{s}{4}(x_1^2 + x_2^2 + 2x_1x_2 - x_1^2 - x_2^2 + 2x_1x_2) = s \cdot x_1 \cdot x_2$$

$\Rightarrow$ Rapidity (of lepton pair): $y = \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right) = \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right)$

note: $M^2 = s x_1 x_2 \Leftrightarrow x_2 = \frac{M^2}{s x_1} \rightsquigarrow y = \frac{1}{2} \ln\left(\frac{s \cdot x_1^2}{M^2}\right)$

$$\Rightarrow x_1 = \frac{M}{\sqrt{s}} e^y \quad , \quad x_2 = \frac{M}{\sqrt{s}} \cdot e^{-y}$$

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c} \int_0^1 dx_1 dx_2 \; \delta(s x_1 x_2 - M^2) \left[\sum_j Q_j^2 \left(q_j(x_1, M^2) \cdot \bar{q}_j(x_2, M^2) + (1 \leftrightarrow 2) \right) \right]$$

- Double differential: $\frac{dx_{1/2}}{dy} = x_{1/2}$ $\mapsto$ do both integrals

$$\to \frac{d^2\sigma}{dM^2 dy} = \frac{4\pi\alpha^2}{3N_c} \frac{1}{s} \left[\sum_j Q_j^2 \left(q_j(x_1, M^2)\, \bar{q}_j(x_2, M^2) + (1 \leftrightarrow 2) \right) \right]$$

- Analog: $\hat{\sigma}(q\bar{q}' \to W) = \frac{\pi}{3}\sqrt{2}\, G_F\, m_W^2\, |V_{qq'}|^2\, \delta(\hat{s} - m_W^2)$
  - $|V_{qq'}|^2$: CKM

$$\hat{\sigma}(q\bar{q} \to Z) = \frac{\pi}{3}\sqrt{2}\, G_F\, m_Z^2 \left(g_{V,q}^2 + g_{A,q}^2\right) \delta(\hat{s} - m_Z^2)$$

($g_{V,q}, g_{A,q}$: Z couplings (vector/axial))