Mathematischer Vorkurs WS18/19

Martin Gote

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Übungsblatt 4

Aufgabe 1

$$sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

(a)

$$\sum_{n=0}^{5}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{39916800}x^{11}$$

(b)

$$\frac{d}{dx}\sum_{n=0}^{5}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{45360}x^8 - \frac{1}{3628800}x^{10}$$

(c)

$$cos(x) = \frac{d}{dx}sin(x) = \frac{d}{dx}\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n(2n+1)\frac{x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$$

Aufgabe 2

$$tan(x) = \frac{sin(x)}{cos(x)}$$

$$\frac{d}{dx}[tan(x)] = \frac{cos(x)\cdot cos(x) - sin(x)\cdot(-sin(x))}{cos^2(x)} = \frac{sin^2(x)+cos^2(x)}{cos^2(x)} = \frac{1}{cos^2(x)} = sec^2(x)$$

Aufgabe 3

(a)

$$x = \sum_{i=1}^{5}\frac{3i}{i+5} = \frac{3}{6} + \frac{6}{7} + \frac{9}{8} + \frac{12}{9} + \frac{15}{10} = \frac{893}{168} \approx 5,315$$

(b)

$$x = \sum_{i=1}^{3}i! = 1! + 2! + 3! = 1 + 2 + 6 = 9$$

(c)

$$x = \sum_{i=1}^{3} = 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$$