c-field descriptions of nonequilibrium polariton fluids

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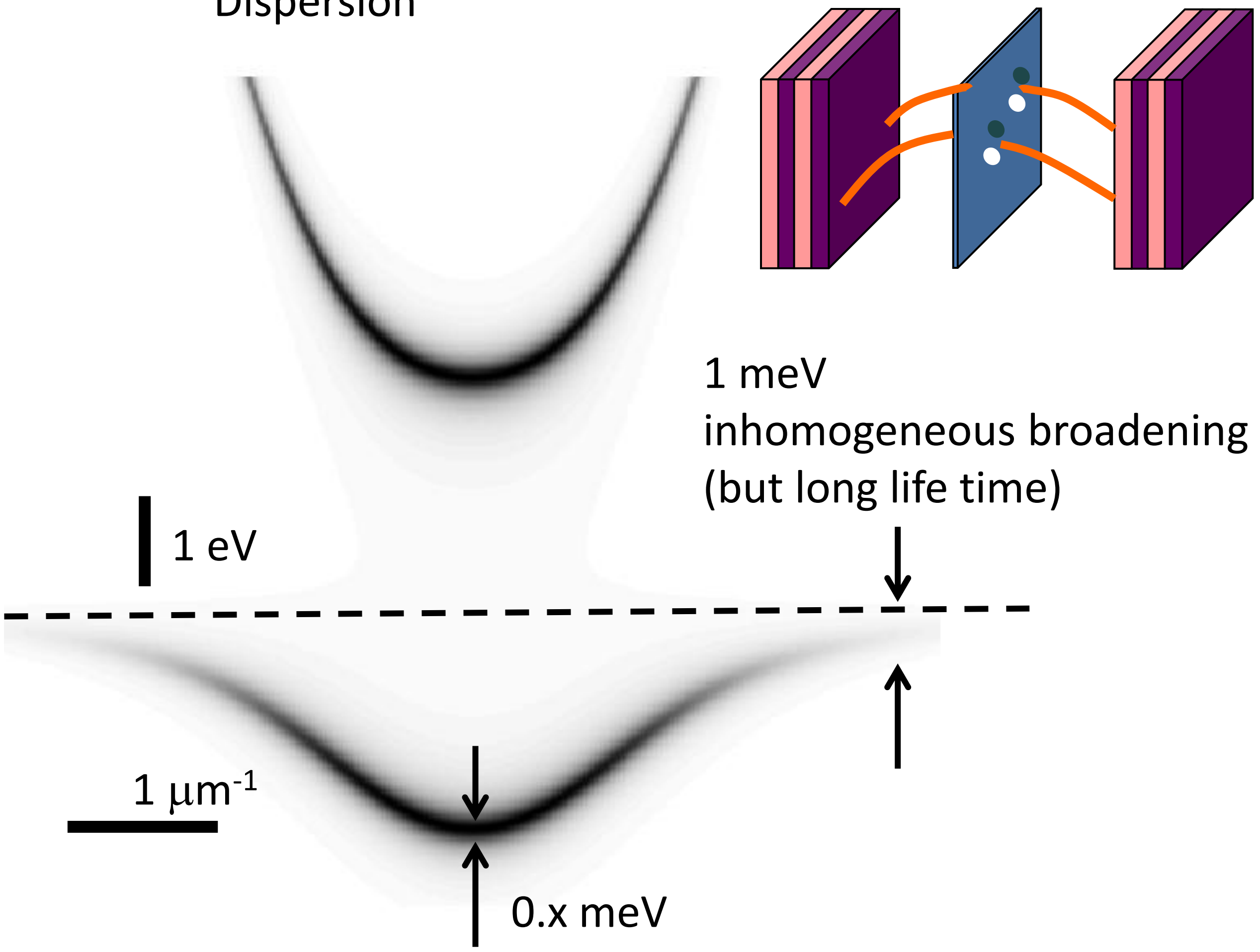
polariton characteristics

Dispersion

2D / 1D / 0D



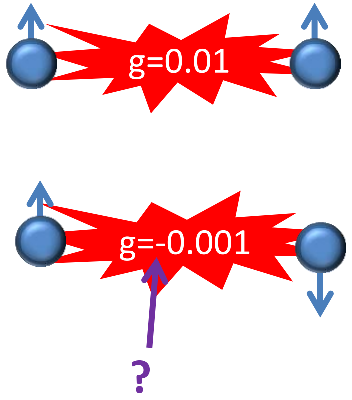
1 meV
inhomogeneous broadening
(but long life time)

0.x meV

⇒Polariton life time 1-10's ps

⇒ nonequilibrium

Interactions

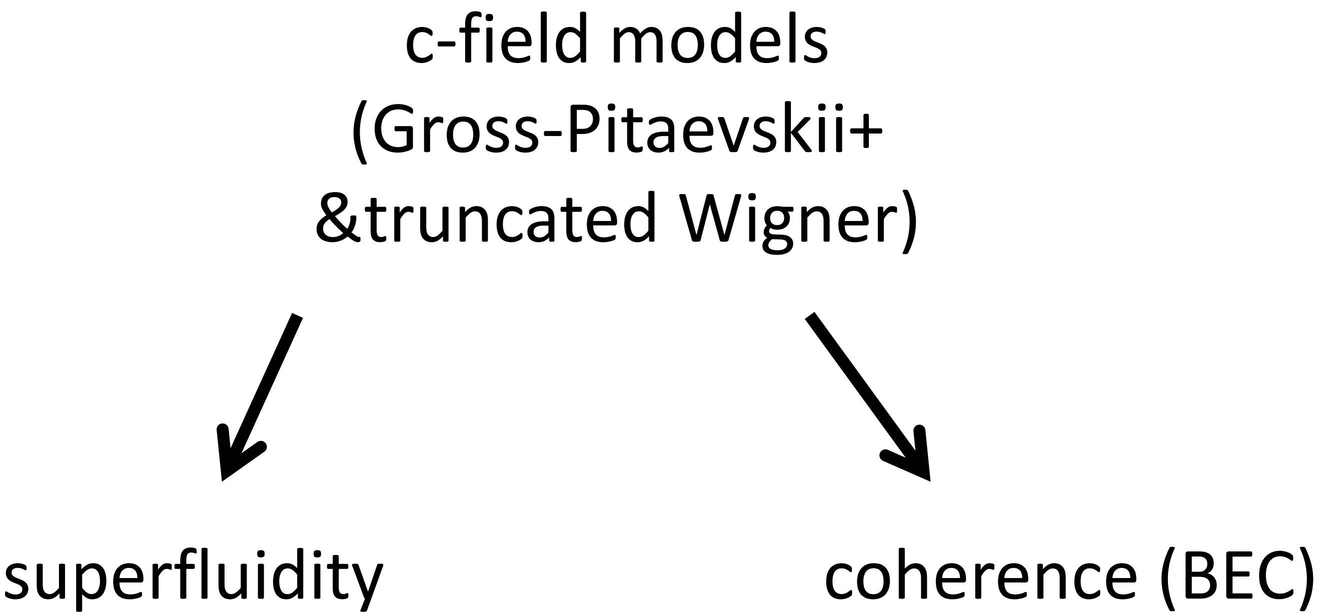


- short range $(R \approx 0.01\mu m)$
- polarization dependent (bi-exciton "feshbach" resonance?)
- weak: $gm/\hbar^2 \ll 1$

⇒ mean field theory OK

[C. Ciuti et al. PRB 1998]
[Kwong et al. PRB 2007]
[M.W. PRB 2007]

overview



c-field models
(Gross-Pitaevskii+
&truncated Wigner)

superfluidity

coherence (BEC)

mean field theory (GPE)

$$\hat{\psi} \rightarrow \langle \hat{\psi} \rangle = \psi$$

$$i\frac{\partial}{\partial t}\psi = \underbrace{\Big(\varepsilon(-i\nabla)+V_{ext}+g|\psi|^2}_{\text{usual GPE}} \underbrace{-i\frac{\gamma}{2}}_{\text{finite life time (dissipation @all scales)}}\Big)\ \psi \quad + \underbrace{F_L(x,t)}_{\text{external pumping laser}}$$

Resonant excitation

$$F_L(x,t) = F_L \ e^{i(k_L x - \omega_L t)}$$

creates a plane wave coherent polariton quantum fluid

inherited from laser

super?

$$\psi = \sqrt{n_0}\, e^{i(k_L x - \omega_L t)}$$

independently tunable

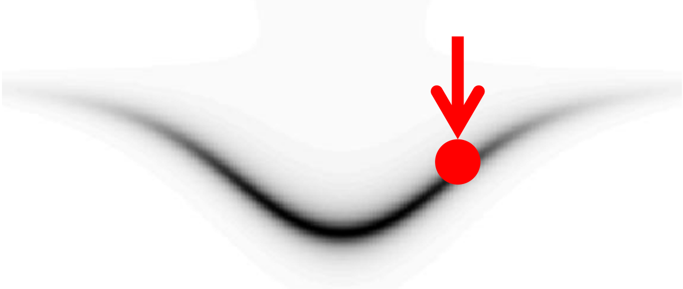
↔ equilibrium condensate

$$\psi = \sqrt{n_0}\, e^{i(k_L x - \mu t)}$$

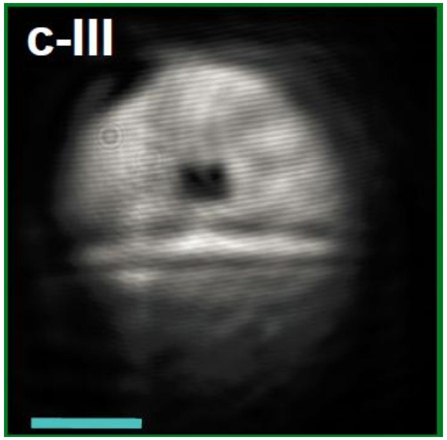
where $\mu = \frac{k_L^2}{2m} + gn_0$

superfluidity

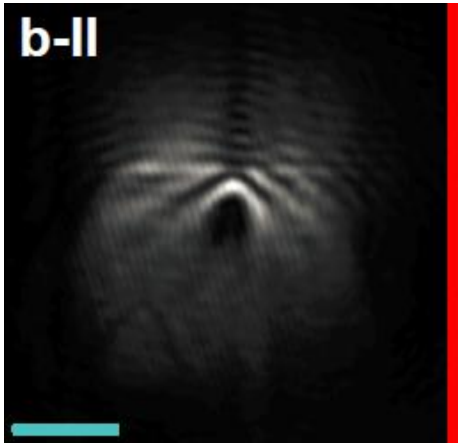
- persistent currents : trivial (driven by laser)



- scattering off weak defects: nontrivial ⍰ Landau criterion



$v<v_c$: flow without scattering



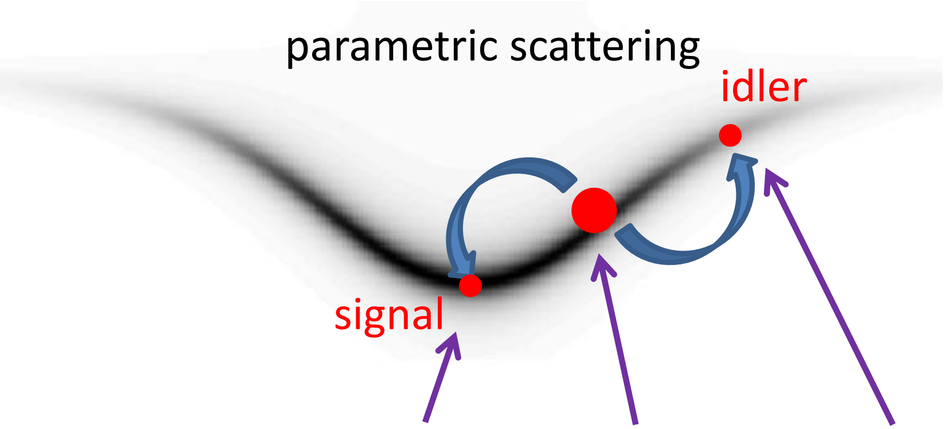
$v>v_c$: Cerenkov radiation

- great similarity with equilibrium GPE

[Carusotto and Ciuti, PRL 2004; Amo et al. Nat. Phys. 2009]

dynamical instability

$$F_L(x,t) = F_L \ e^{i(k_L x - \omega_L t)} \xrightarrow{\text{always ??}} \psi = \sqrt{n_0}\, e^{i(k_L x - \omega_L t)}$$



$$\psi = \sqrt{n_s}\, e^{i(k_s x - \omega_s t)} + \sqrt{n_0}\, e^{i(k_L x - \omega_L t)} + \sqrt{n_i}\, e^{i(k_i x - \omega_i t)}$$

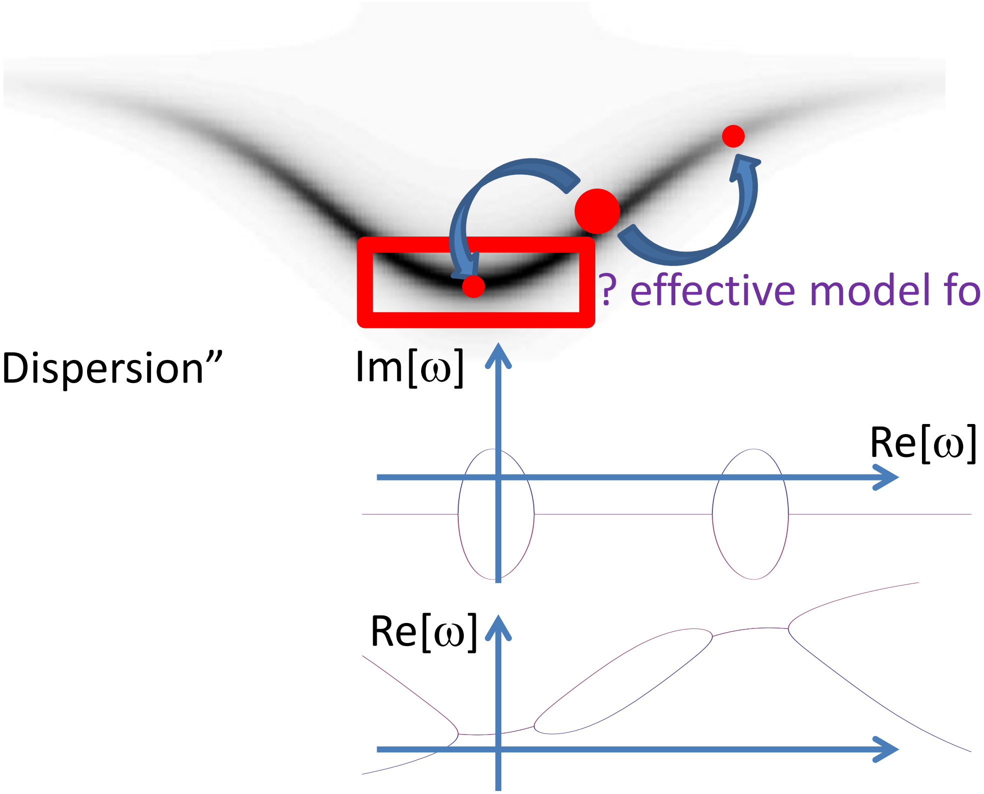
OPO= optical parametric oscillator: occupation of signal/idler in mean field theory

spontaneous U(1) symmetry breaking: $\phi_s + \phi_i = 2\phi_p$ but $\phi_s - \phi_i =$ free

nonequilibrium analog of BEC phase transition

[Houdre et al. PRL 2000] [Stevenson et al. PRL 2000]

dynamics close to bifurcation



? effective model for signal dynamics

“Dispersion”

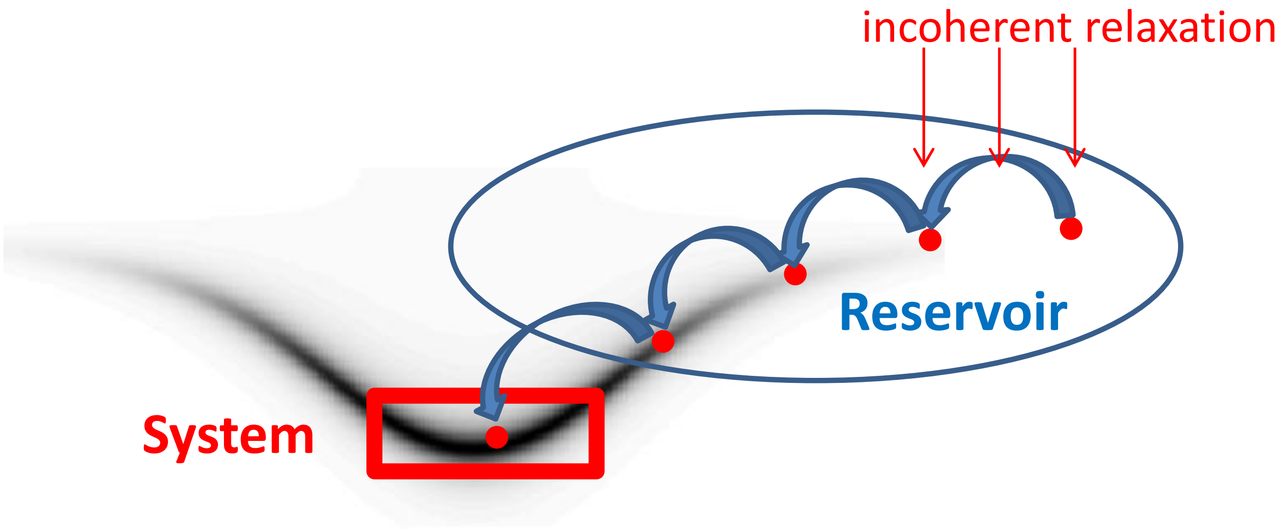
complex Ginzburg Landau-equation (=complex Gross-Pitaevskii Equation)

$$i\frac{\partial}{\partial t}\psi = \Big[\underbrace{a+ib+(b+ic)\nabla^2}_{\text{linear: dispersion and gain}} + \underbrace{(-d+ie)|\psi|^2}_{\text{interaction: gain saturation and energy shift}}\Big]\ \psi$$

U(1) symmetric

[Cross & Hohenberg, RMP 1993] [Aranson&Kramer RMP 2002] [Keeling & Berloff PRL 2008]

“nonresonant” excitation



excitons thermalize with lattice

complicated relaxation process when polaritons are injected at high energy, but no change in nature of dynamical instability
→ also model with cGLE or with similar phenomenological models

analogy with lasers (Jonathan Keeling)

reservoir model

polariton field dynamics

$$i\frac{\partial}{\partial t}\psi=\left(\varepsilon(-i\nabla)+V_{ext}-i\frac{\gamma-R(n_R)}{2}+g|\psi|^2+g_R n_R\right)\ \psi$$

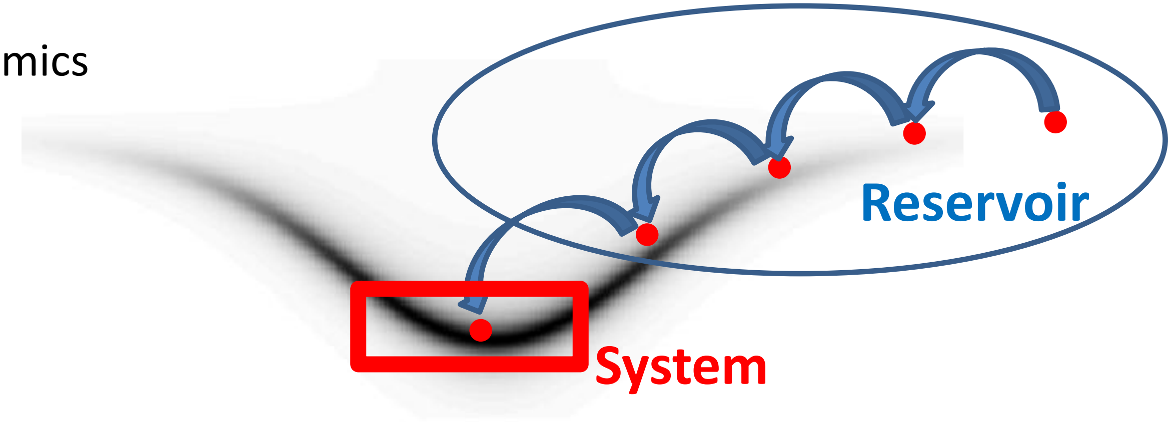
gain

exciton-polariton interaction

coupled to exciton density dynamics

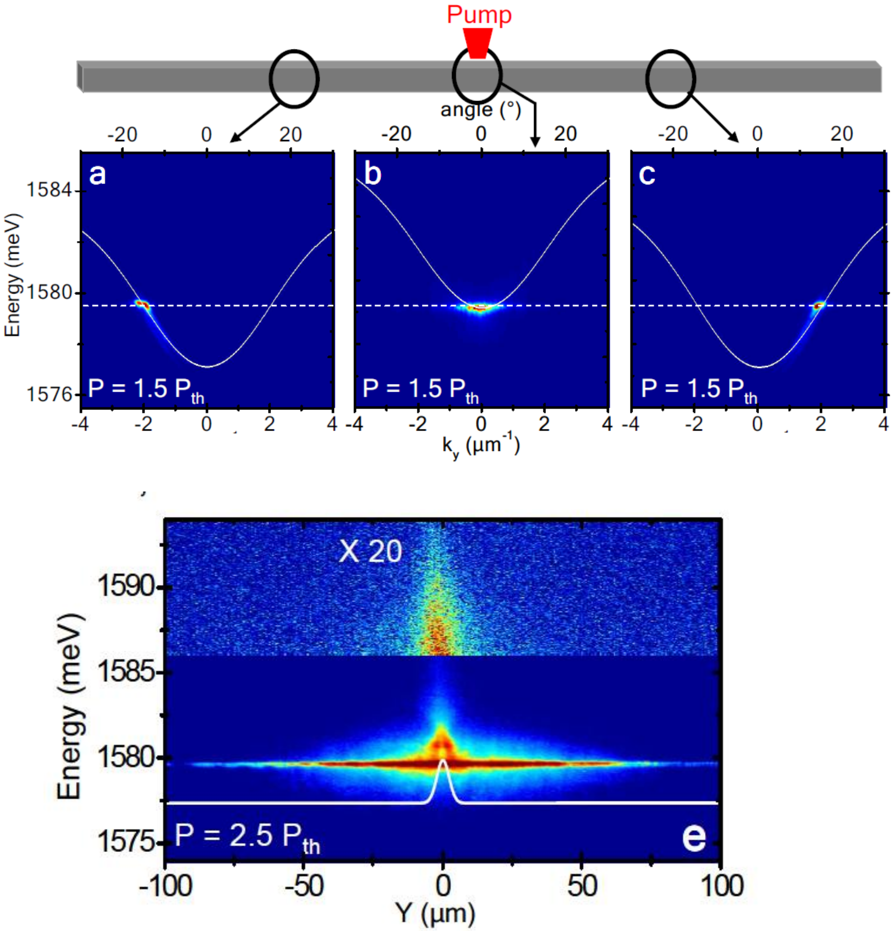
$$\frac{\partial}{\partial t}n_R=P-\gamma_R n_R-R(n_R)|\psi|^2$$

gain saturation

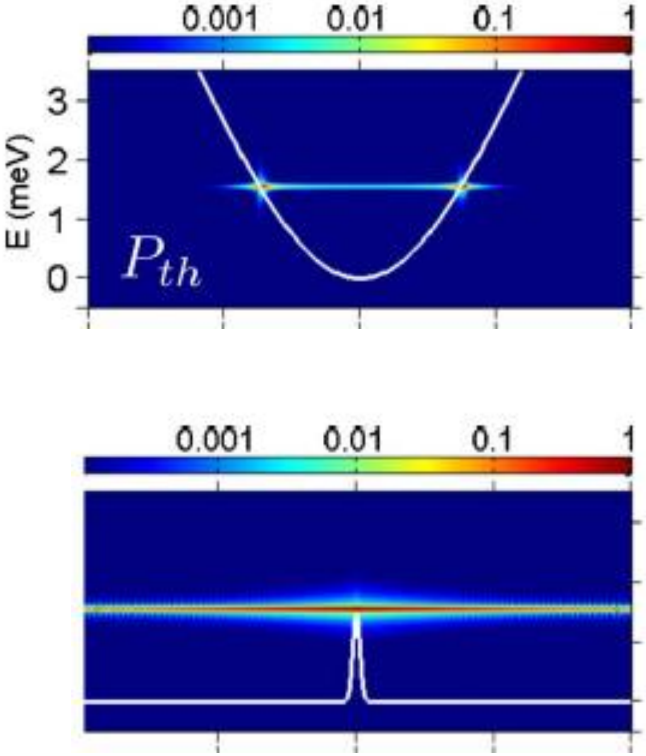


application: accelerated condensate

experiment



theory



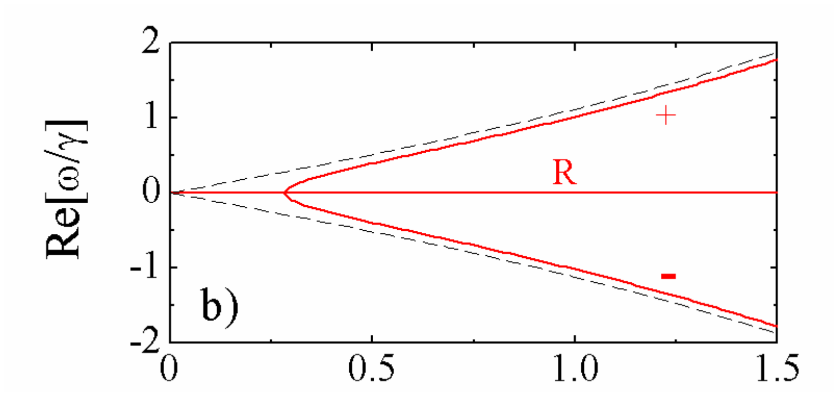
[E. Wertz et al. Nat. Phys. 2010]

[M.W. et al PRB 2010]

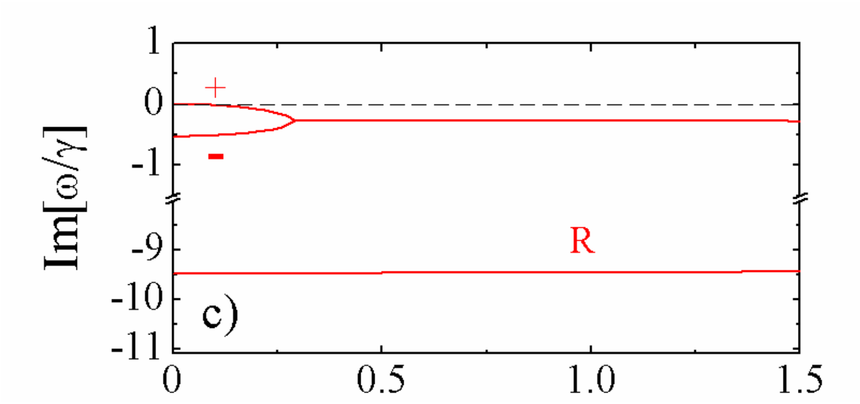
excitation spectrum

excitation spectrum

real part



imaginary part

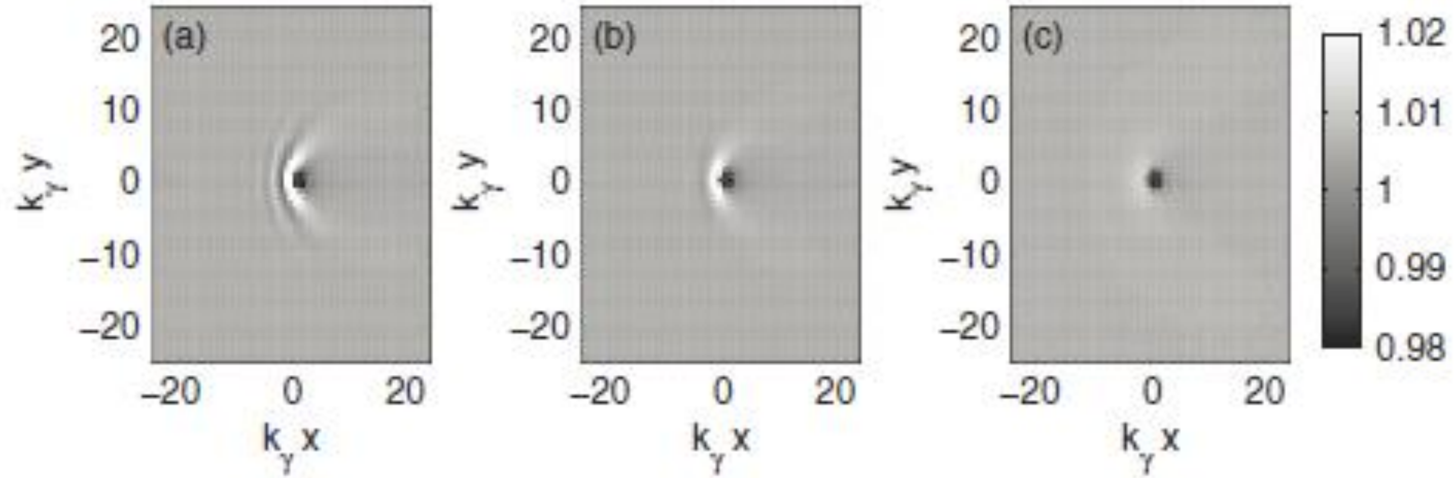


→ zero critical velocity ?

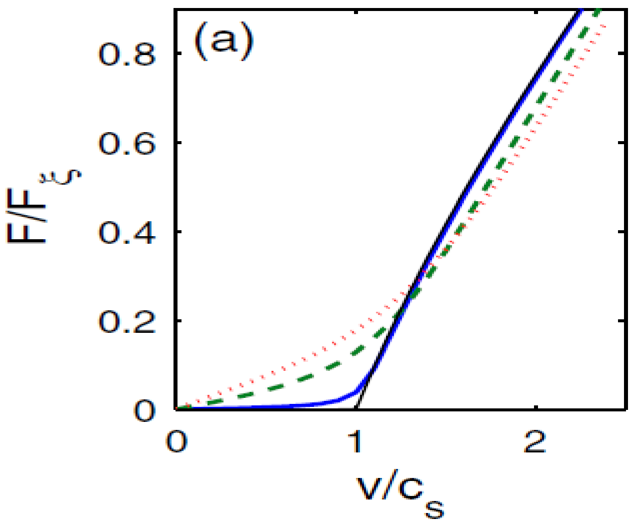
[M.Szymanska et al. PRL 2006)] [M. Wouters and I. Carusotto, PRL 2007]

superfluidity : interaction with a defect

$v/c_s =$ 1.5 1 0.4



Force on the defect



finite superfluid fraction

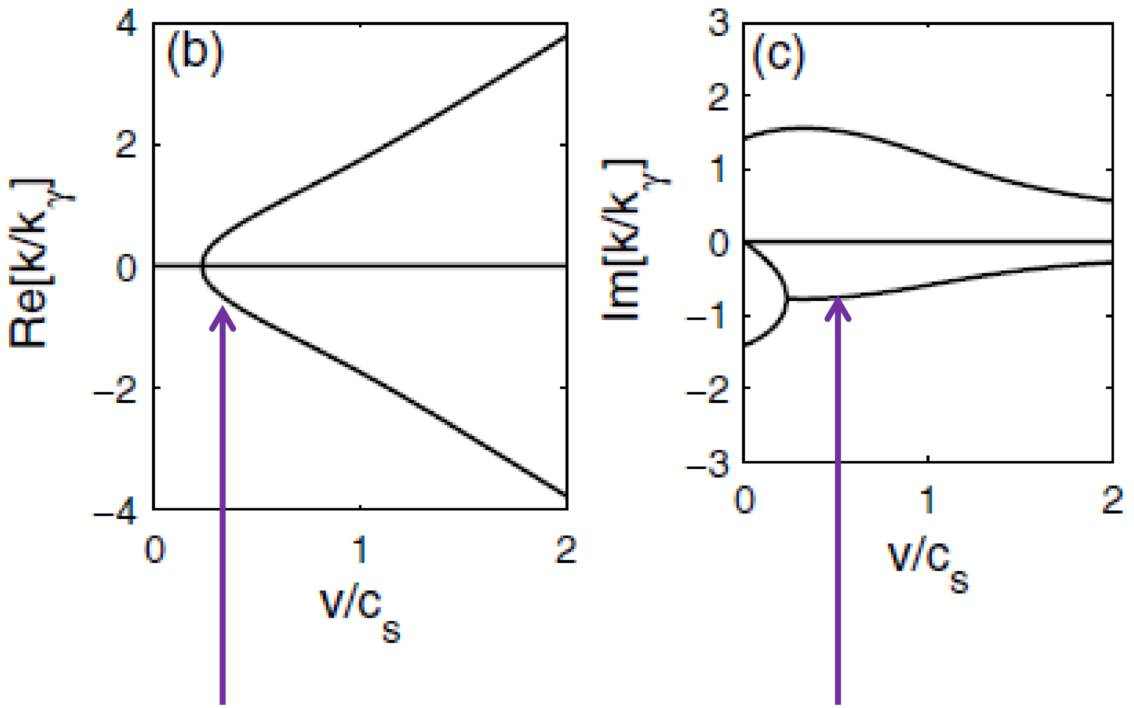
[J. Keeling, PRL 2011]

[M. W. and I. Carusotto, PRL 2010]

excitation spectrum II

Static defect excites modes for which $\omega(k) = 0$

Re(k) Im(k)

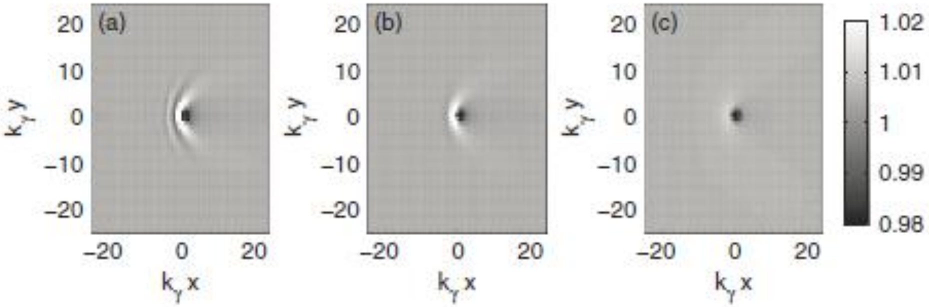


Condensate perturbation oscillation

Spatial decay rate

metastable flows

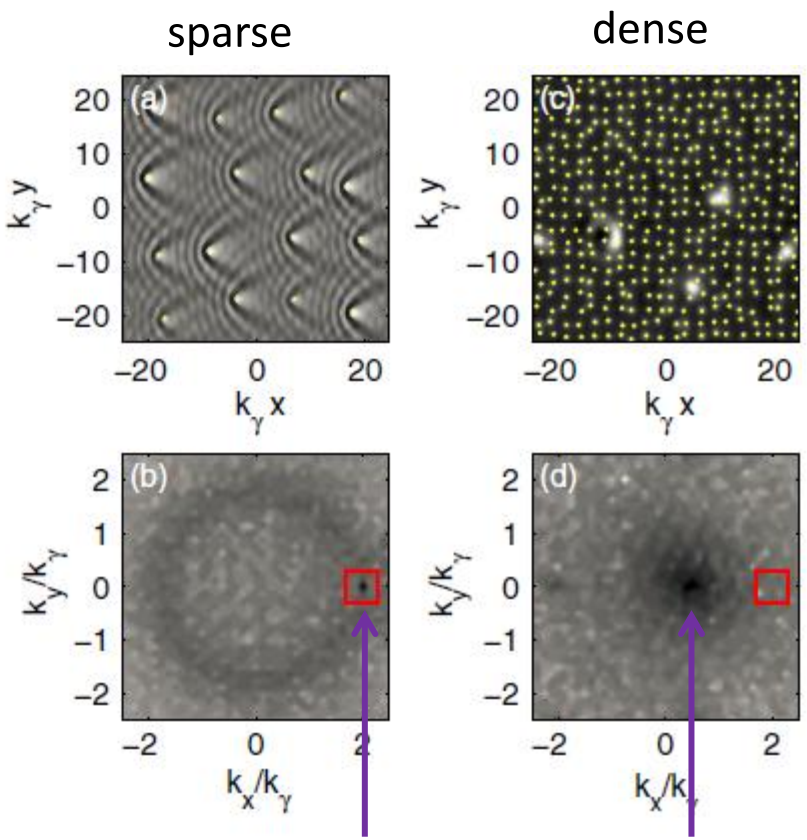
single defect



perturbation of the wave function,
force on defect,
but no decay of superflow

Damping of excitations enhances stability of superflow!

many defects



no decay

decay

long range order

- BEC: U(1) broken state has ODLRO:

$$\left\langle \hat{\psi}^{+}(x)\,\hat{\psi}(x')\right\rangle \to n_c \qquad \text{for} \qquad |x-x'| \to \infty$$

- What about the OPO ?

  description of fluctuation needed → go beyond mean field theory

- out of equilibrium → no exp[-β H] Boltzmann distribution

- dynamical theory of open system :

$$\frac{\partial}{\partial t}\rho = \underbrace{-i[\rho, H]}_{\text{Hamiltonian evolution}} + \underbrace{L[\rho]}_{\text{Lindblad: losses, incoherent gain from reservoir}} \quad \longleftarrow \quad \text{simplification needed!!}$$

quantum → c-field

- map quantum dissipative dynamics to classical stochastic process by means of the Wigner (quasi-) probability distribution $P_W(\varphi)$

  = Truncated Winger approximation

- Observables: e.g. 1-body density matrix

$$\frac{1}{2}\left\langle \hat{\psi}^+(x)\hat{\psi}(x')+\hat{\psi}(x')\hat{\psi}^+(x)\right\rangle = \int\left[d^2\varphi(x)\right]P_W(\varphi)\,\varphi^*(x)\varphi(x')$$

- $P_W(\varphi)$ is sampled by functions that follow the stochastic process

$$d\varphi(x)=\underbrace{\left\{\left[\varepsilon(-i\nabla)+g\left|\varphi(x)\right|^2-i\frac{\gamma}{2}\right]\varphi(x)+F_L\right\}}_{\text{GPE}}\,dt+\underbrace{\sqrt{\frac{4\gamma}{\Delta V}}dW(x,t)}_{\text{noise, related to losses}}$$

$$\left\langle dW^*(x,t)dW(x',t')\right\rangle = 2\,dt\,\delta_{x,x'}\delta_{t,t'}$$

relation to Boltzmann eqn.

for homogeneous system:

$$\left\langle \hat{\psi}^+(x)\,\hat{\psi}(x')\right\rangle = \int n(k)e^{ik(x-x')}dk$$

→ information on LRO in momentum distribution, that can be computed with Boltzmann

"RPA" on GPE without noise → Boltzmann equation without "spontaneous scattering"

$$\frac{d}{dt}n(k) = \sum_{k_i}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\left[-n_1n_2n_3 \quad -n_1n_2n_4 \quad +n_3n_4n_1 \quad +n_3n_4n_2\right]$$

"RPA" on GPE with noise → Boltzmann equation with spontaneous scattering + …

$$\frac{d}{dt}n(k) = I_B \qquad + \frac{\pi g^2}{2}\sum_{k_i}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\left[-n_1-n_2+n_3+n_4\right]$$

full usual Boltzmann

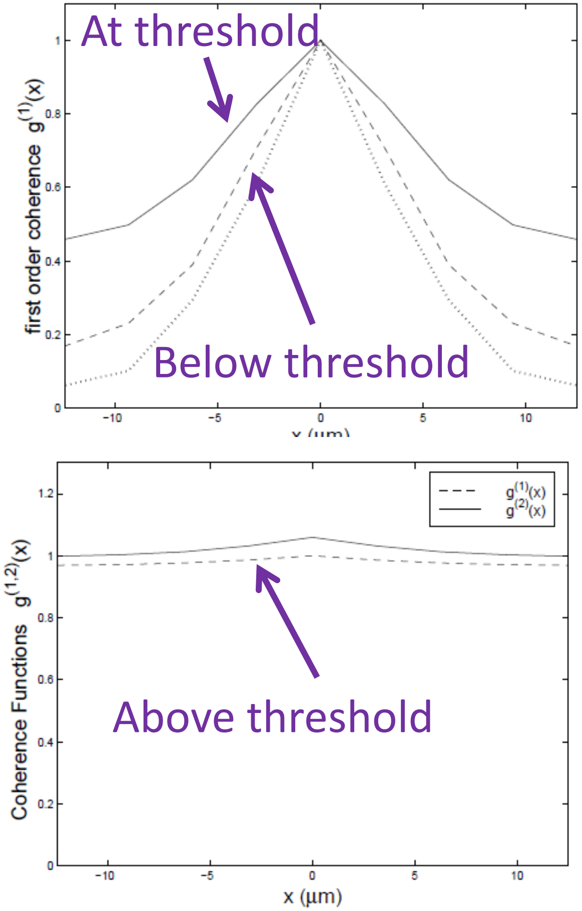
spurious, should be small with respect to I_B or γn

satisfied if $nk_{\max}^2 \gg 1$ or $\gamma \gg gk_{\max}^2$

see e.g. [Y. Kagan in "Bose-Einstein condensation", Griffin, Snoke, Stringari eds. 1994.]

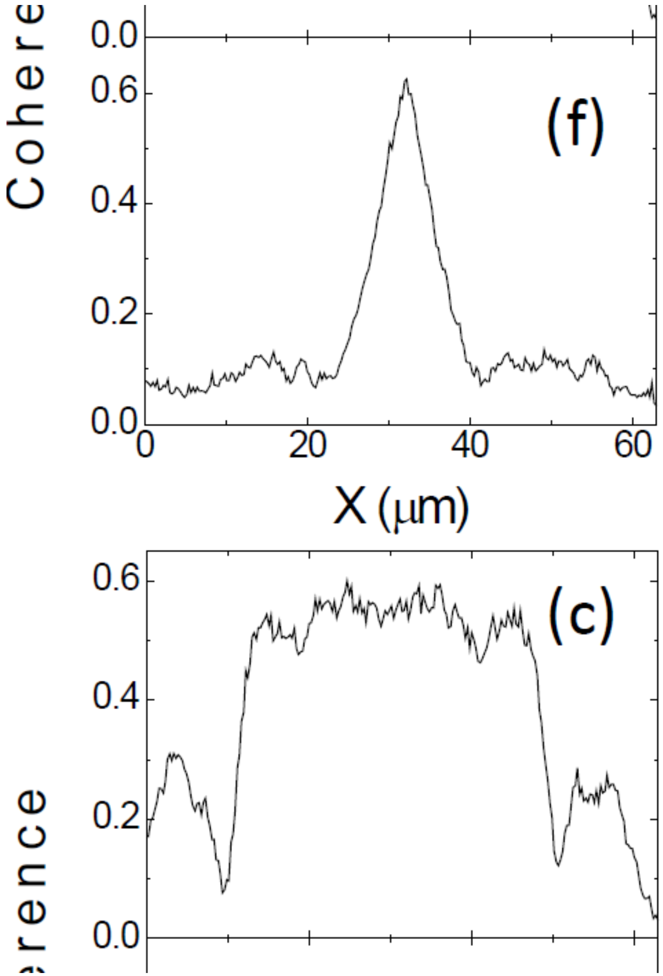
OPO: coherence across threshold

theory



[Carusotto and Ciuti, PRB 2005]

experiment



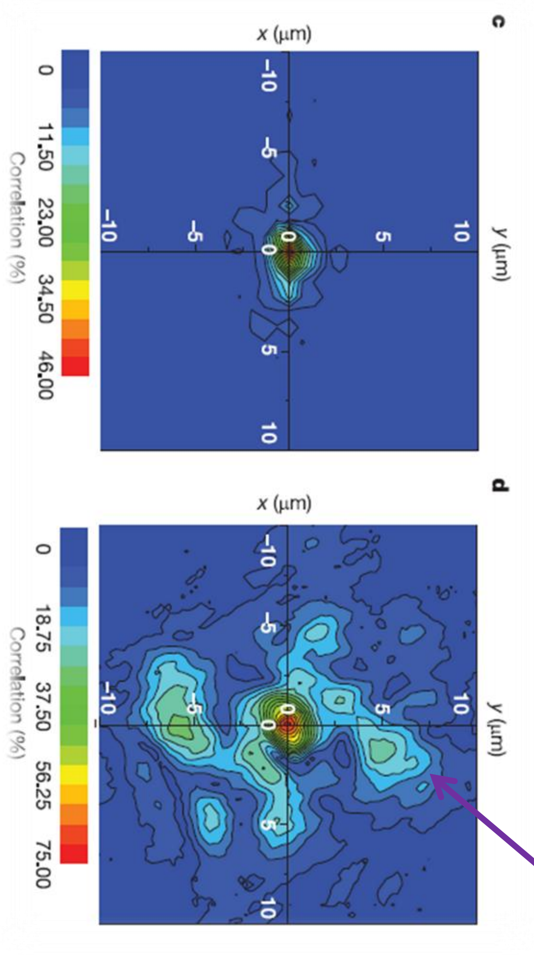
[R. Spano et al, arXiv:1111.4894]

Nonresonant excitation: coherence

experiment

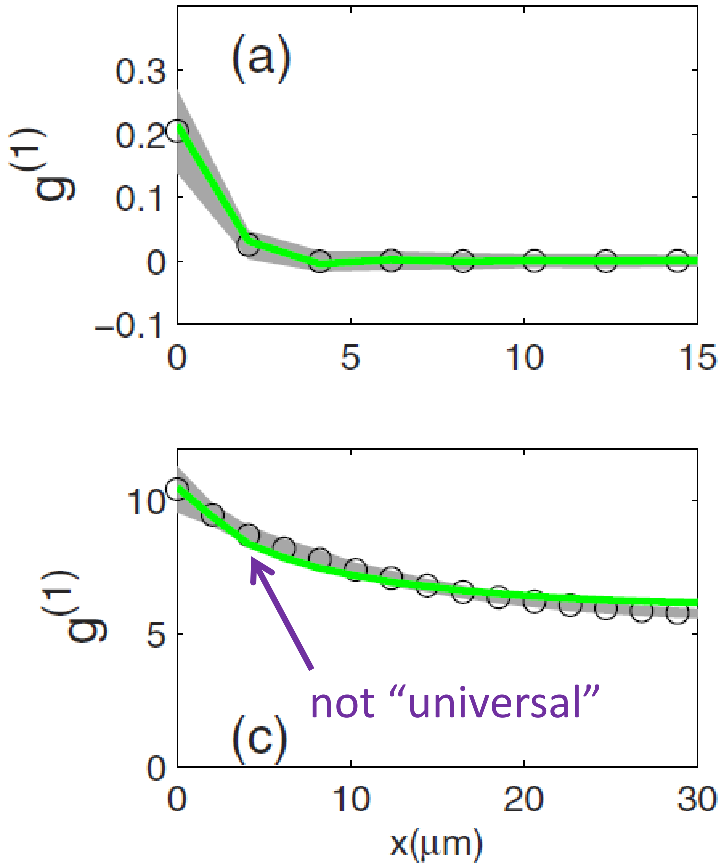
theory (reservoir+Wigner)

$g^{(1)}(x,-x)$



disorder

[Kasprzak et al. Nature 2006]

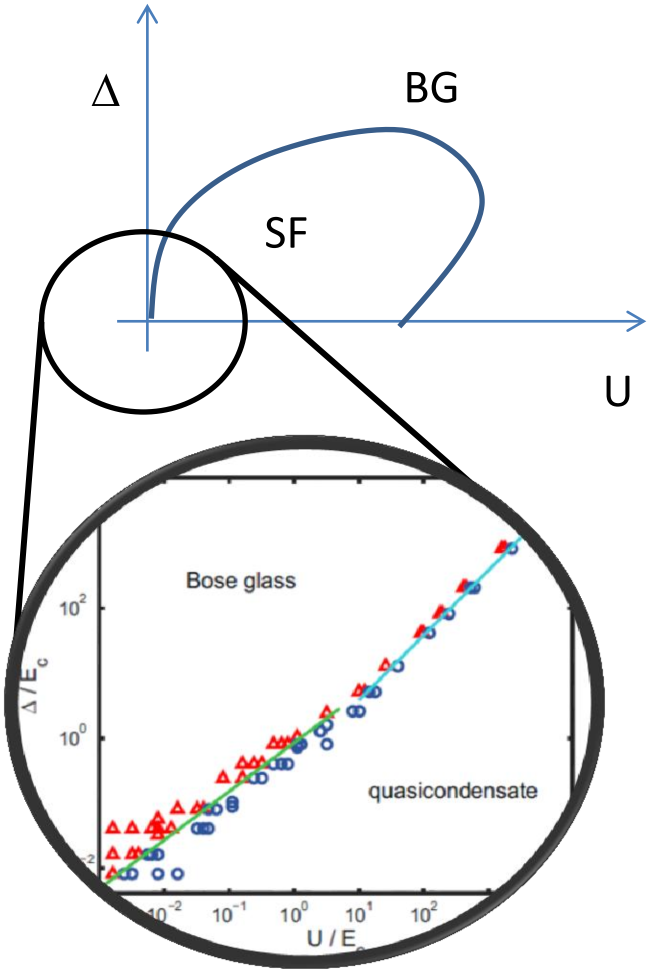


not "universal"

[M.W. and V. Savona, PRB 2009]
[M.H. Szymanska et al PRB 2008]

Disorder: Bose Glass?

equilibrium disordered Bose gas: theory



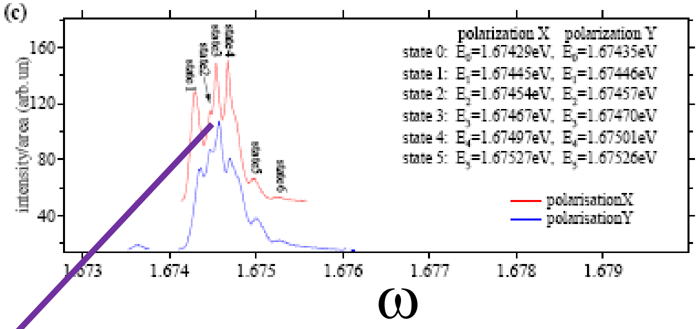
[Fisher et al. PRB 1989]

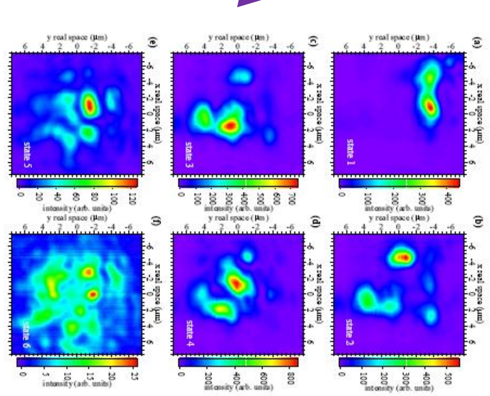
[Fontanesi et al. PRL 2009] see also [Altmann et al. PRB 2010]

superfluidity: qualitatively survives

→ what with Bose glass ??

nonequilibrium: experiment





remember:

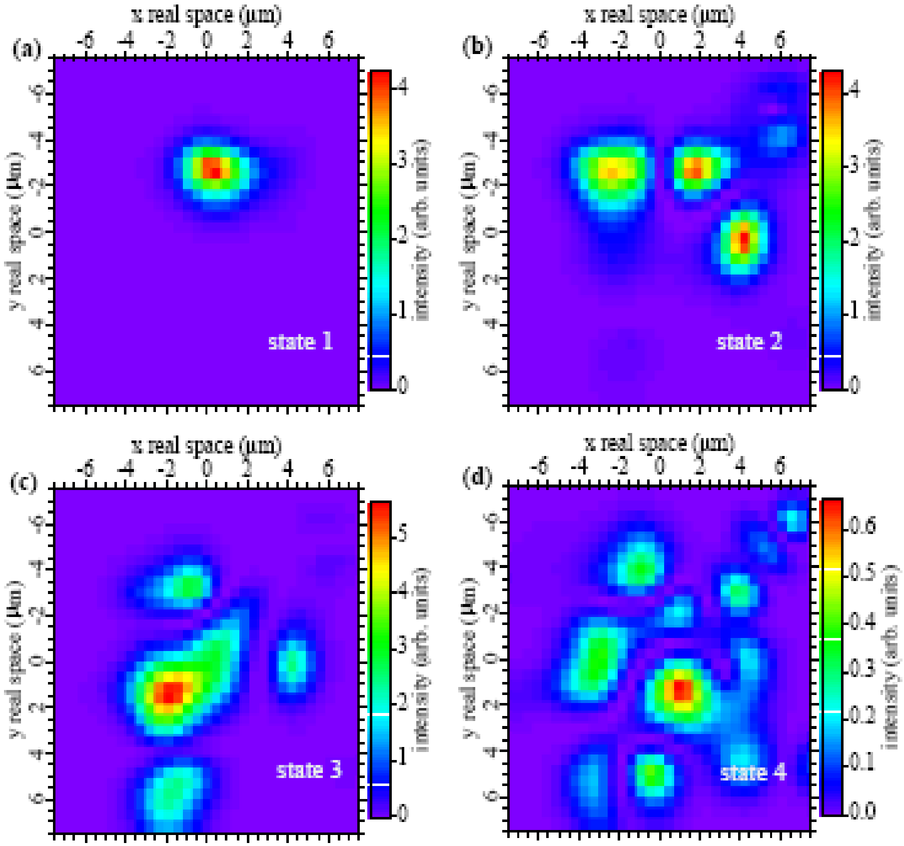
$$\omega \Leftrightarrow \mu$$

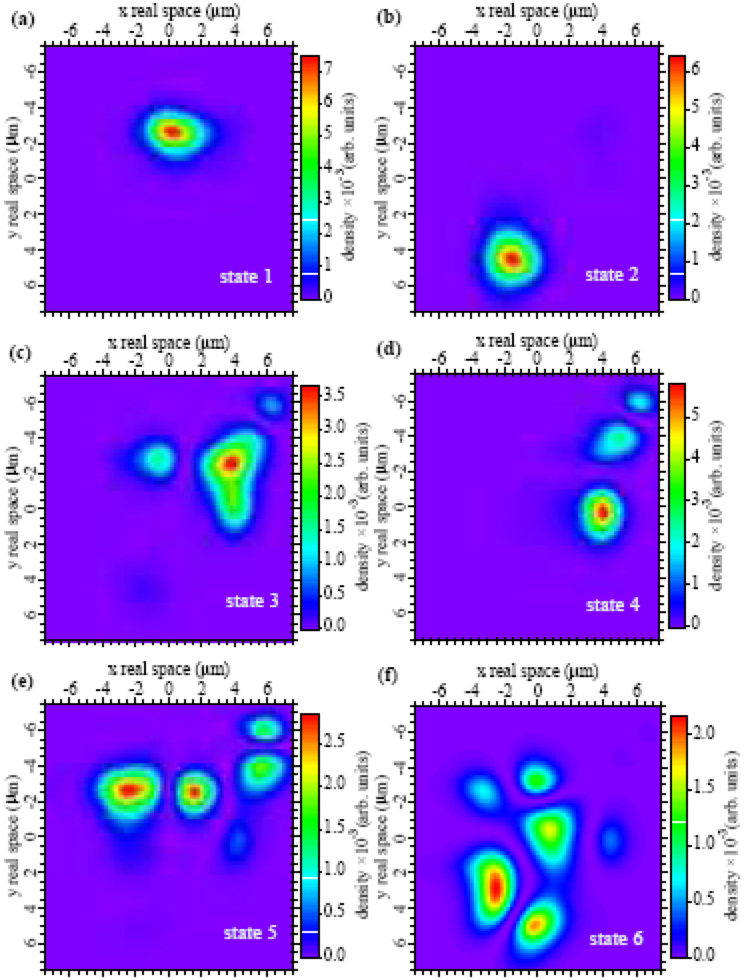
⇒ not a single chemical potential!

[Krizhanovskii et al, PRB 2009]

simulated states

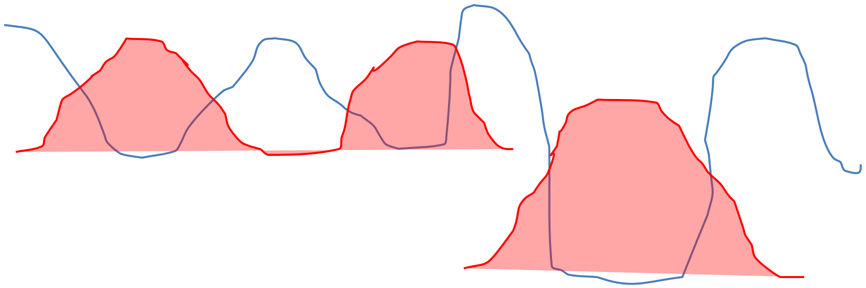
condensate states ↔ **linear eigenstates**





[Krizhanovskii et al, PRB 2009]

physical picture



- gain due to excitons should be saturated everywhere

- lowest energies: eigenstates, higher energies: new states (mode locking)

- single state develops when $U{\sim}\Delta$:

same scaling as in equilibrium for disorder with long correlation length

[Malpuech et al PRL 2006]

- but very different physics: no fluctuations needed for the “incoherent phase” (cf. random lasers)

random lasers see e.g. [D.S. Wiersma Nat. Phys. 2008]

summary

microcavity polaritons challenge us to revisit
well know phenomena (superfluidity, BEC, Bose glass, ….)
out of equilibrium

GPE+ is a good tool to address these questions ,
especially when inhomogeneity is important

two-reservoir model

$$\frac{\partial}{\partial t} n_R = P - \gamma_R n_R - R(n_R)|\psi|^2$$

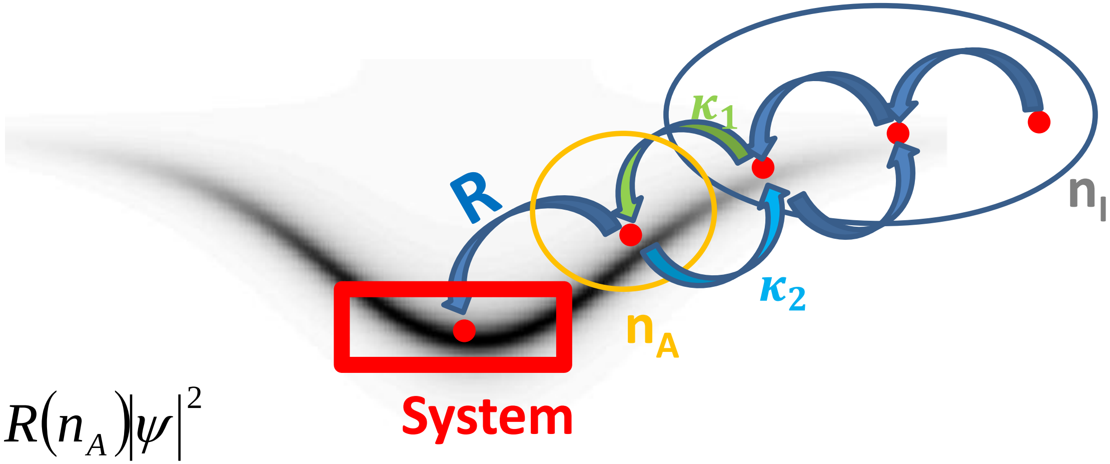
single reservoir time scale

but exciton life time much longer than gain saturation relaxation time scale
→ not good for pulsed excitation

$$\frac{\partial}{\partial t} n_I = P - \gamma_R n_I - \kappa_1 n_A + \kappa_2 n_I$$

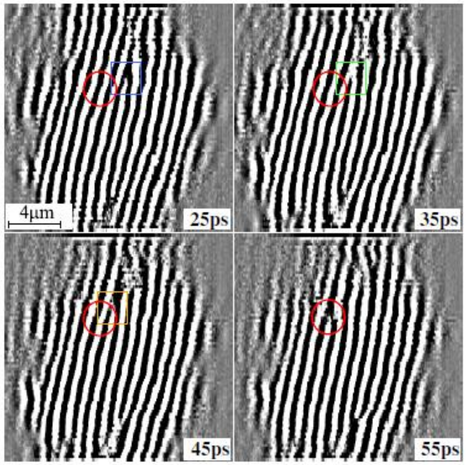
exciton life time   gain time scale

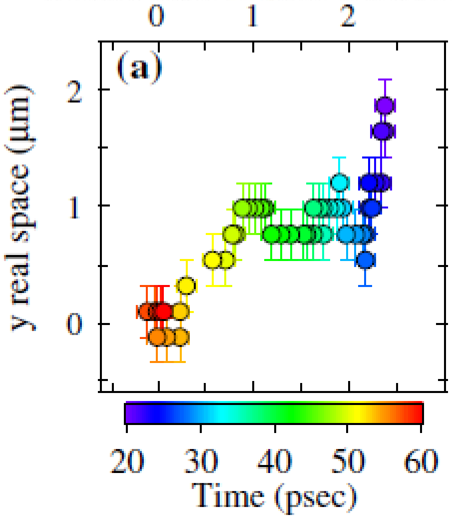
$$\frac{\partial}{\partial t} n_A = -\gamma_R n_A + \kappa_1 n_I - \kappa_2 n_A - R(n_A)|\psi|^2$$



application: vortex dynamics

experiment

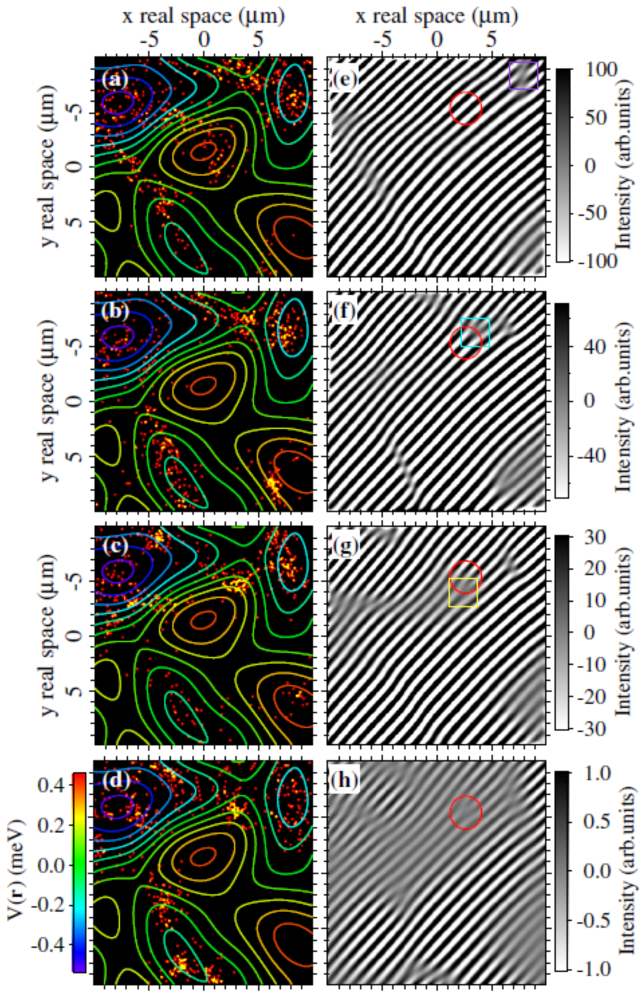




simulation

vortex positions

averaged interferrogram



[K.G. Lagoudakis et al. PRL 2010]