Condensation, superfluidity and lasing of coupled light-matter systems.

Jonathan Keeling

University of St Andrews 600 YEARS

SUPA

RETUNE, Heidelberg, June 2012

Outline

1. Introduction to polariton condensation
   - Approaches to modelling
2. Pattern formation
   - Non-equilibrium pattern formation
   - Spontaneous vortex lattices
3. Superfluidity
   - Non-equilibrium condensate spectrum
   - Experiments and aspects of superfluidity
   - Current-current response function
4. Coherence
   - Experiments
   - Power law decay of coherence

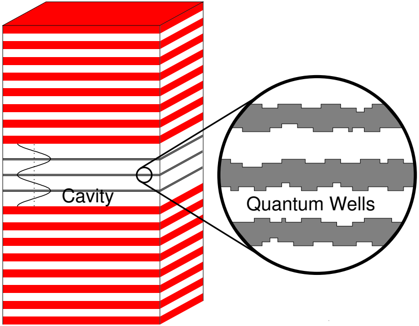
Acknowledgements

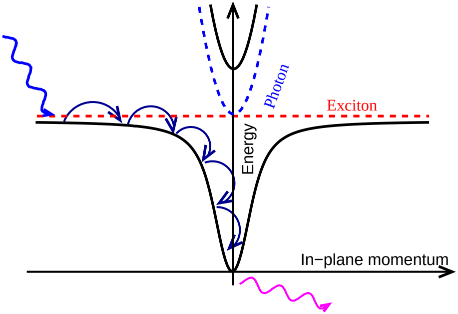
People:

Funding:

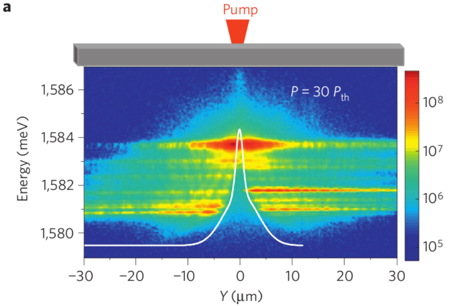
EPSRC
Engineering and Physical Sciences
Research Council

Microcavity polaritons — incoherent pumping

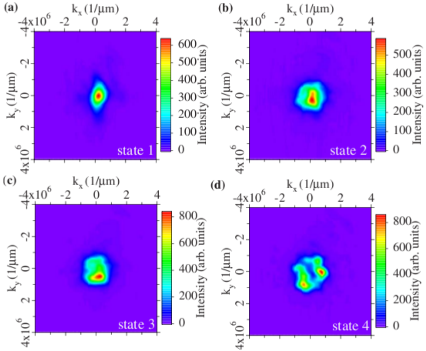




Non-equilibrium features in experiment

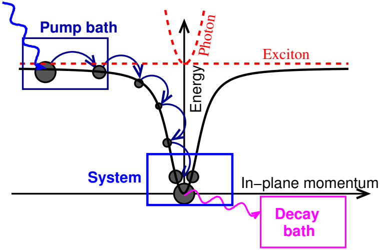


Flow from pumping spot
[Wertz *et al.* Nat. Phys. '10]



$|\psi(\mathbf{k})|^2 \neq |\psi(-\mathbf{k})|^2$:
Broken time-reversal symmetry.
[Krizhanovskii *et al.* PRB '09]

Non-equilibrium approach: Steady state, and fluctuations



$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}},$$

$$H_{\text{sys}} = \sum_k \omega_k \psi_k \psi_k^\dagger + \sum_\alpha g_\alpha (\phi_\alpha^\dagger \psi_k + \text{H.c.}) + H_{\text{ex}}[\phi_\alpha, \phi_\alpha^\dagger]$$

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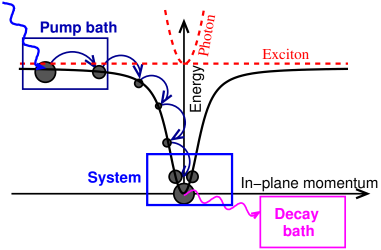

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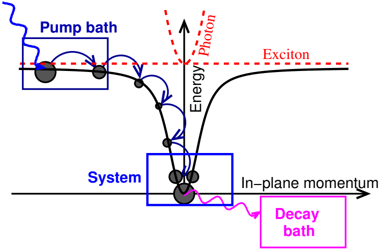

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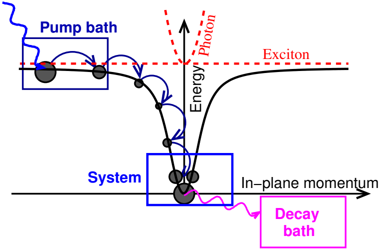

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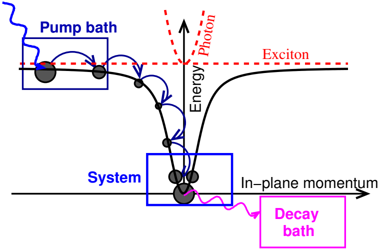

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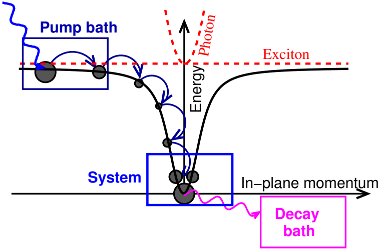

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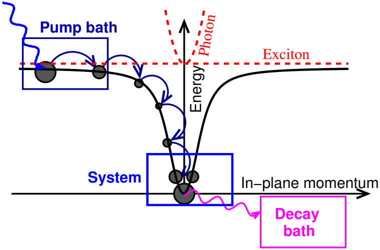

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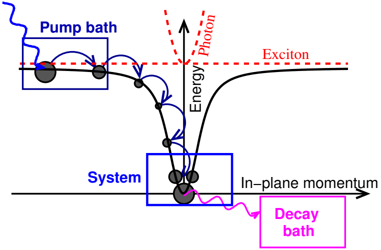

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Pattern formation:

Complex Gross-Pitaevskii equation

Steady state equation:

$$(\mu_s - \omega_0 + i\kappa)\,\psi = \chi(\psi, \mu_s)\psi$$

- Local density limit:

See also [Wouters and Carusotto, PRL '07]

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$$\left(i\partial_t + i\kappa - \left[V(r) - \frac{\nabla^2}{2m}\right]\right)\psi(r) = \chi(\psi(r,t))\psi(r,t)$$

Nonlinear, complex susceptibility (**incoherent** pump)

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Stability of Thomas-Fermi solution

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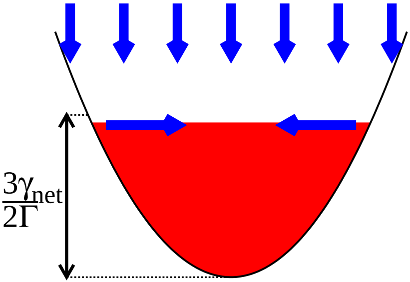

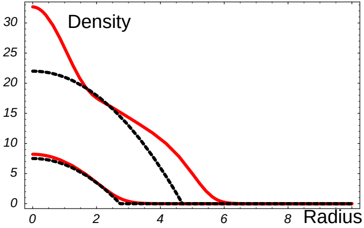

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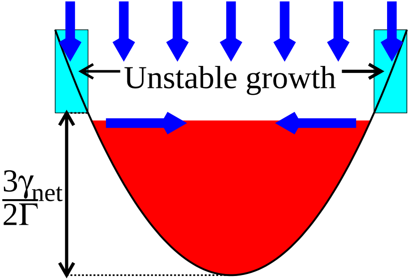

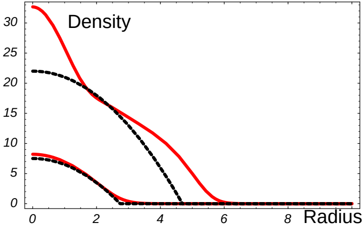

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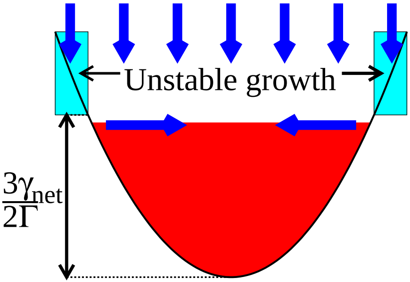

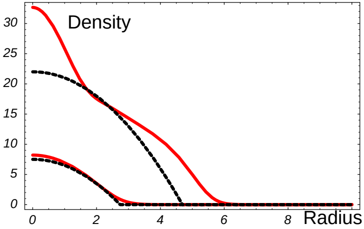

High m modes: $\delta\rho_{n,m} \simeq e^{im\theta} r^m \ldots$

$$\frac{1}{2}\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

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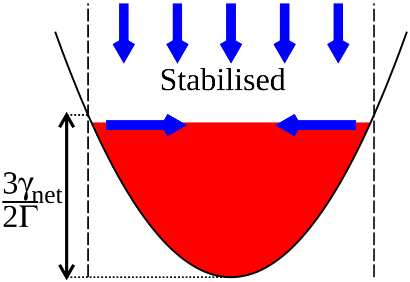

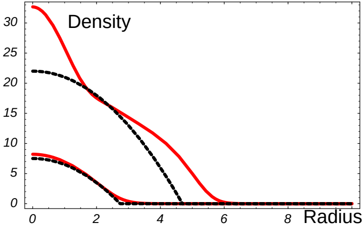

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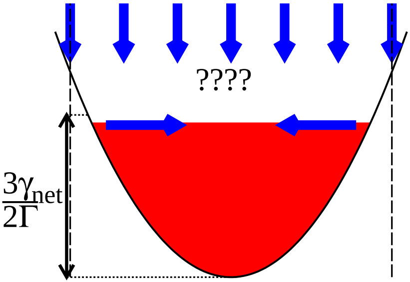

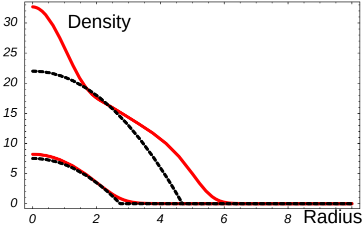

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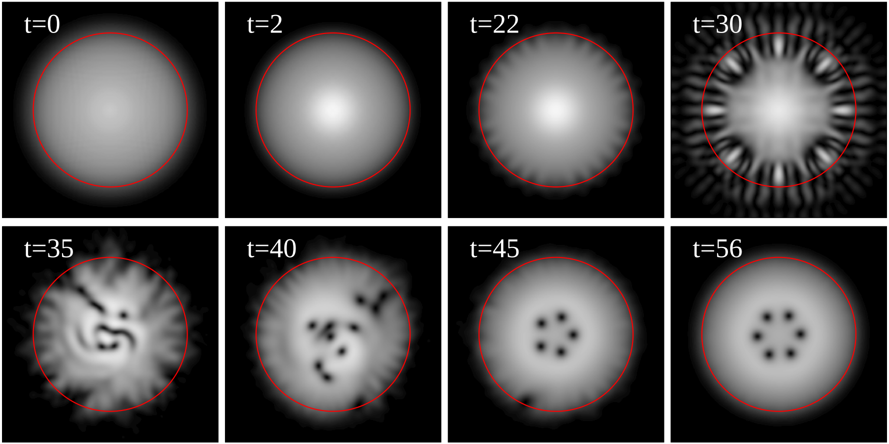

[Keeling & Berloff PRL '08]

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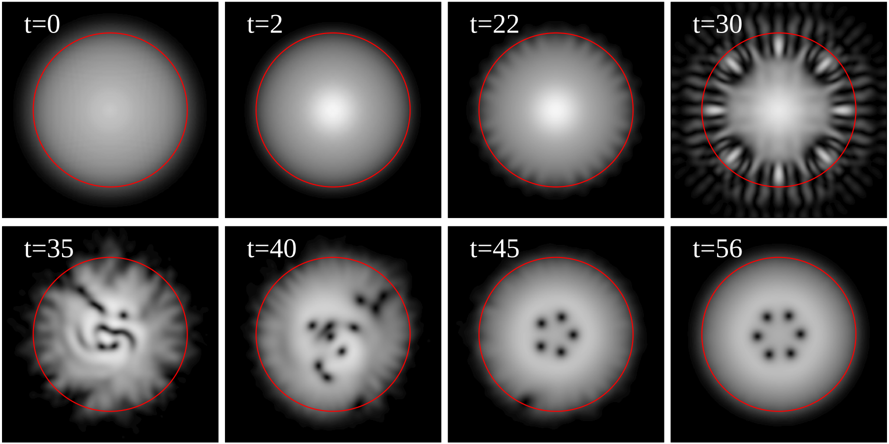

Why? $i\partial_t\psi = (\mu - 2\Omega L_z)\psi$ $\Omega = \omega$, cancels trap.

[Keeling & Berloff PRL '08]

Observability of vortex lattices

- Not seen experimentally (yet?)
- Observation: Fast rotation
- Stability: Disorder, ellipticity
- Relaxation, thermalisation?

[Borgh *et al.* PRB ’12 in press]

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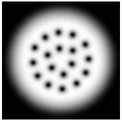

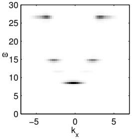

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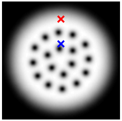

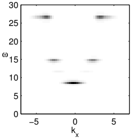

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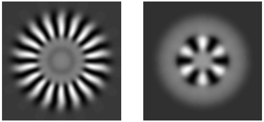

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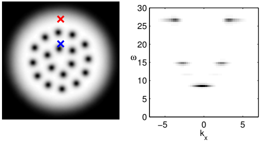

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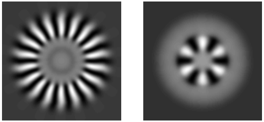

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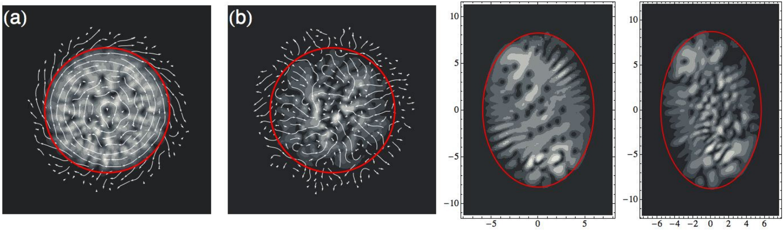

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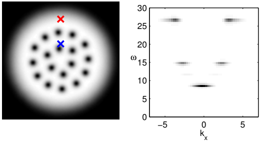
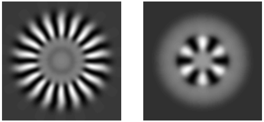
Spectrum: 

Interference: 

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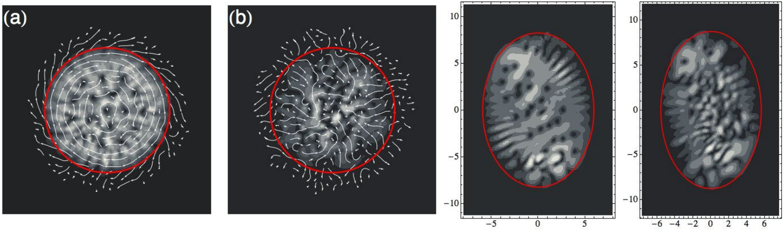

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Superfluidity

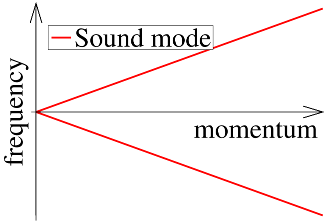
Fluctuations above transition

When condensed

$$\text{Det}\left[D^R(\omega, k)\right]^{-1} = \omega^2 - \xi_k^2$$

With $\xi_k \simeq ck$
Poles:

$$\omega^* = \xi_k$$



- Generic structure of Green's function:

$$[D^R]^{-1} = \begin{pmatrix} \omega + i\gamma_{net} - \epsilon_k - \mu & i\gamma_{net} - \mu \\ -i\gamma_{net} - \mu & -\omega - i\gamma_{net} - \epsilon_k - \mu \end{pmatrix}$$

Fluctuations above transition

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With $\xi_k \simeq ck$
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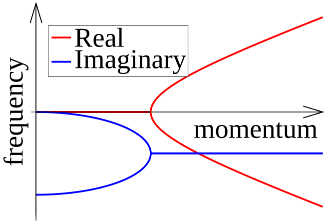
$$\omega^* = -i\gamma_{\text{net}} \pm \sqrt{\xi_k^2 - \gamma_{\text{net}}^2}$$



- Generic structure of Green's function:

$$[D^R]^{-1} = \begin{pmatrix} \omega + i\gamma_{\text{net}} - \epsilon_k - \mu & i\gamma_{\text{net}} - \mu \\ -i\gamma_{\text{net}} - \mu & -\omega - i\gamma_{\text{net}} - \epsilon_k - \mu \end{pmatrix}$$

Fluctuations above transition

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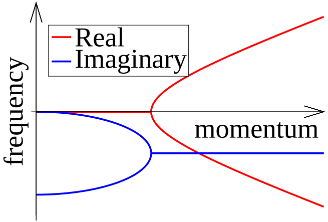

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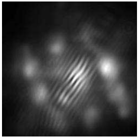
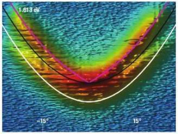
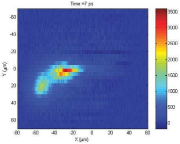
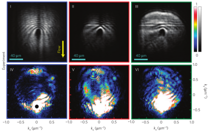
Aspects of superfluidity

	Quantised vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydrodynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✔	✔	✔	✔	✔	✔
Non-interacting Bose-Einstein condensate	✔	✗	✗	✔	✔	✗
Classical irrotational fluid	✗	✔	✗	✔	✔	✔
Incoherently pumped polariton condensates	✔	✗	?	?	✗	?









Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

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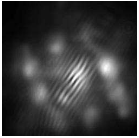
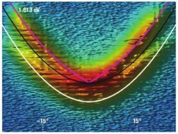
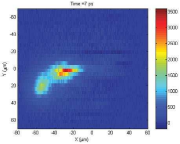
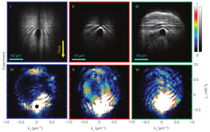




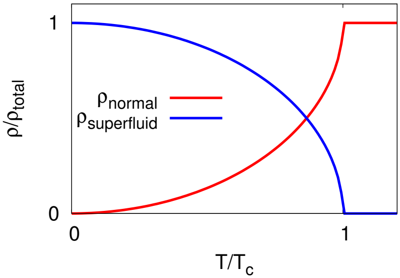




Lagoudakis *et al.* Nat. Phys. '08. Utsunomiya *et al.* Nat. Phys. '08. Amo *et al.* Nature '09; Nat. Phys. '09

Superfluid density

- Two-fluid hydrodynamics



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- To calculate, transverse/longitudinal:

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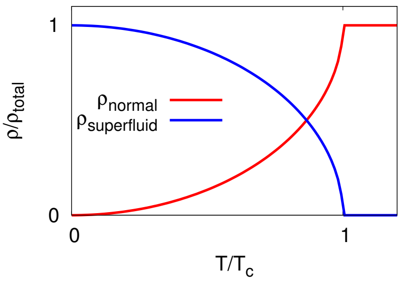

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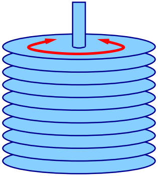

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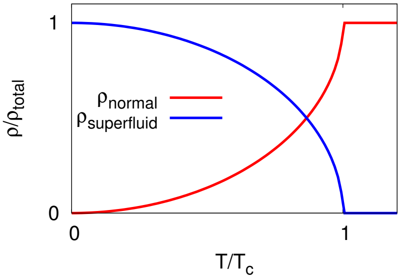

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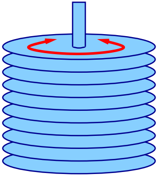

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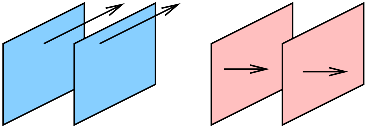

Superfluid density

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$$\mathbf{J} = \rho \mathbf{v} = \psi^\dagger i \nabla \psi = |\psi|^2 \nabla \phi$$

$$J_i(\mathbf{q}) = \psi^\dagger_{\mathbf{k}+\mathbf{q}} \frac{2k_i + q_i}{2m} \psi_{\mathbf{k}}$$

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$$H \to H - \sum_q \mathbf{f}(\mathbf{q}) \cdot \mathbf{J}_i(\mathbf{q}) \qquad J_i(\mathbf{q}) = \chi_{ij}(\mathbf{q}) f_j(\mathbf{q})$$

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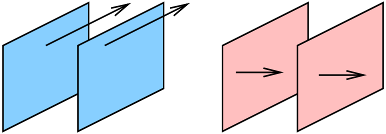

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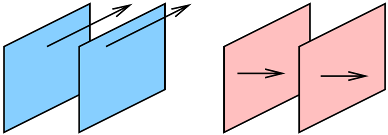

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Non-equilibrium superfluid response

- Superfluid response exists because:

$$= \frac{i\psi_0 q_i}{2m} (1, -1)\, D^R(q, \omega = 0) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{i\psi_0 q_j}{2m}$$

- $D^R(\omega = 0) \propto 1/\epsilon_q$ despite pumping/decay — superfluid response exists.
- Normal density:

$$\rho_N = \int d^d k \epsilon_k \int \frac{d\omega}{2\pi} \mathrm{Tr}\left[\sigma_z D^K \sigma_z (D^R + D^A)\right]$$

- Is affected by pump/decay:
  Does not vanish at $T \to 0$.

[JK PRL '11]

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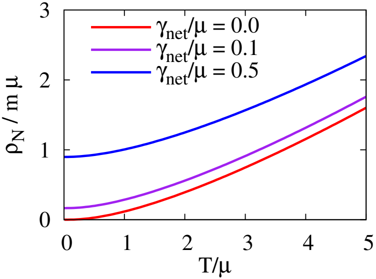


[JK PRL '11]

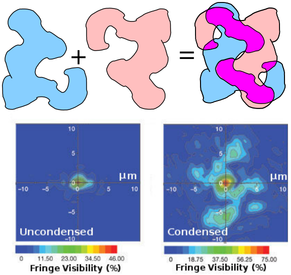
Coherence:

1. Introduction to polariton condensation
   - Approaches to modelling
2. Pattern formation
   - Non-equilibrium pattern formation
   - Spontaneous vortex lattices
3. Superfluidity
   - Non-equilibrium condensate spectrum
   - Experiments and aspects of superfluidity
   - Current-current response function
4. Coherence
   - Experiments
   - Power law decay of coherence

Correlations in a 2D Gas

Correlations:

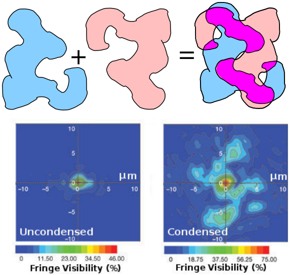
$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$



[Szymańska *et al.* PRL '06; PRB '07] [Wouters and Savona PRB '09]

Correlations in a 2D Gas

Correlations: (in 2D)



$$g_1(\mathbf{r}, \mathbf{r}', t) = \left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0) \right\rangle$$
$$\simeq |\psi_0|^2 \exp\left[-D^<_{\phi\phi}(\mathbf{r}, \mathbf{r}', t) \right]$$

- $D^< = D^K - D^R + D^A$
- Generally, get:

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(0,0) \right\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & t \simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{net} r_0^2) & r \simeq 0 \end{cases} \right]$$

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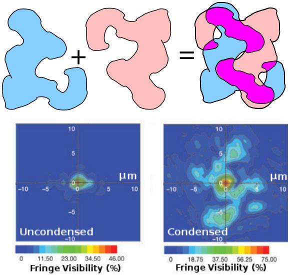

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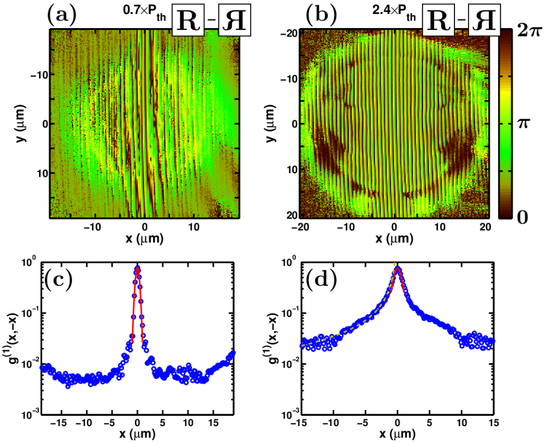
Experimental observation of power-law decay



G. Rompos, Y. Yamamoto *et al.* submitted

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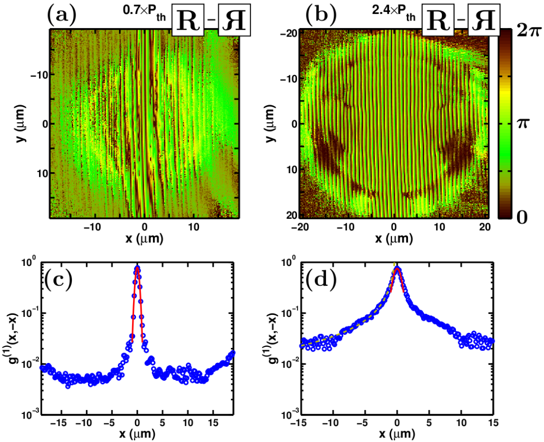

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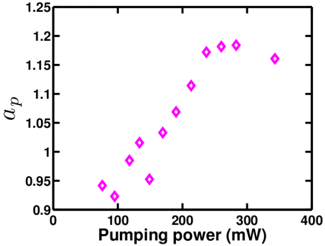
$$g_1(\mathbf{r}, -\mathbf{r}) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$



Exponent in a non-equilibrium 2D gas

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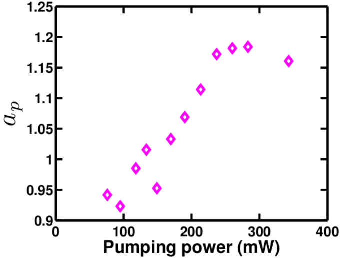
- Experimentally, $a_P \simeq 1.2$
- In equilibrium $a_p = \dfrac{mk_B T}{2\pi\hbar^2 n_s} < \dfrac{1}{4}$ (BKT transition)
- Non-equilibrium theory depends on thermalisation.



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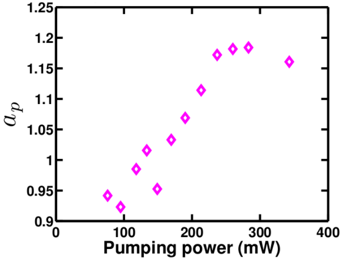

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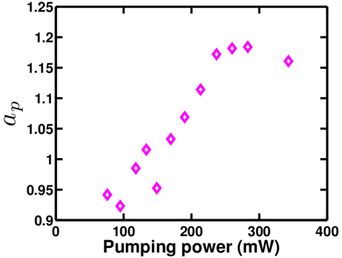

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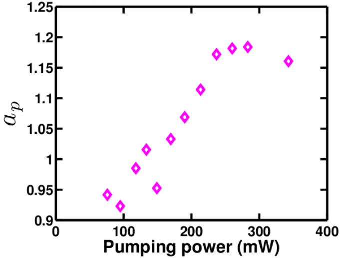

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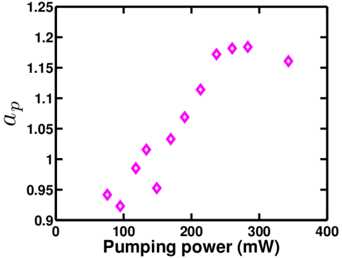
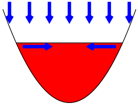
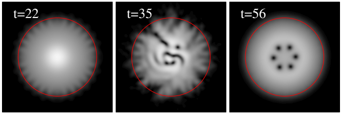


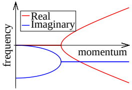
Conclusion

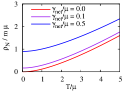
- Instability of Thomas-Fermi and spontaneous rotation



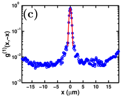


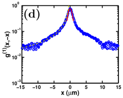
- Survival of superfluid response

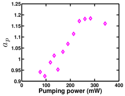




- Power law decay of correlations



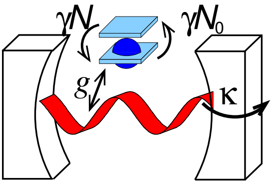




Extra slides

Simple Laser: Maxwell Bloch equations

$$H = \omega_0 \psi^\dagger \psi + \sum_\alpha \epsilon_\alpha S^z_\alpha + \frac{g_{\alpha,\mathbf{k}}}{\sqrt{\mathrm{A}}} \psi S^+_\alpha + \mathrm{H.c.}$$



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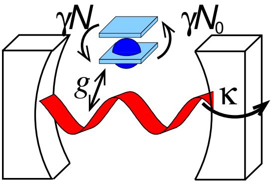

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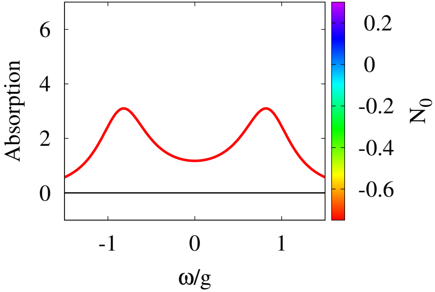

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- Inversion causes collapse before lasing

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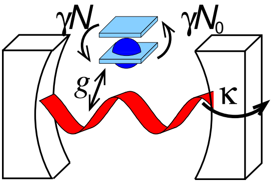

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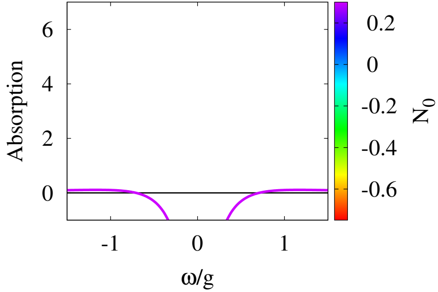

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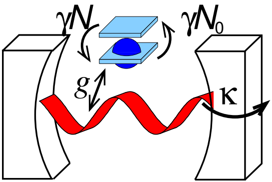

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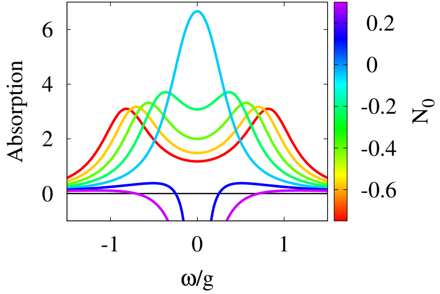

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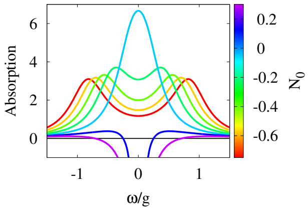
Maxwell-Bloch Equations: Retarded Green's function



- Introduce $D^R(\omega)$:
  Response to perturbation

- Absorption $= -2\Im[D^R(\omega)]$

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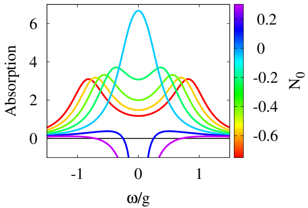

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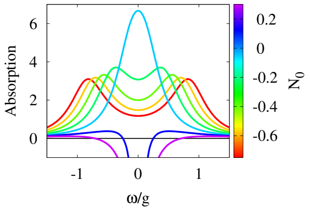

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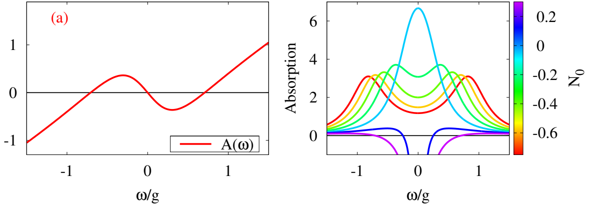

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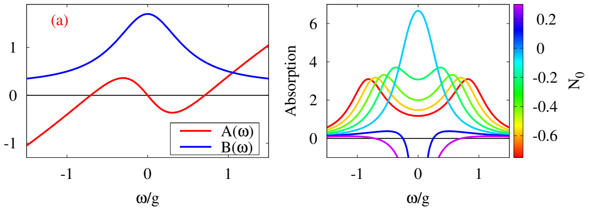

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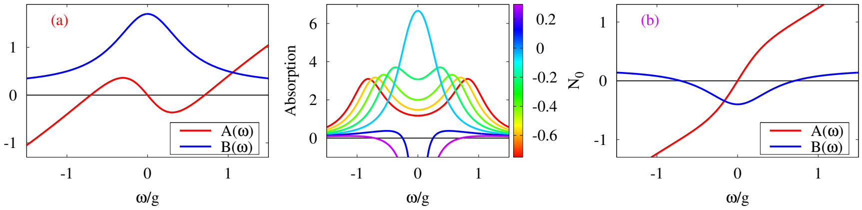

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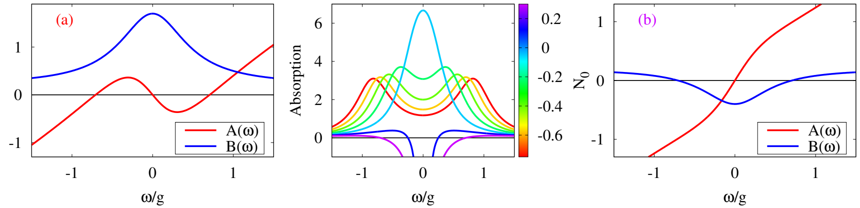
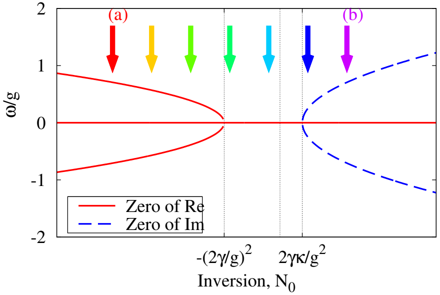
Evolution of poles with Inversion





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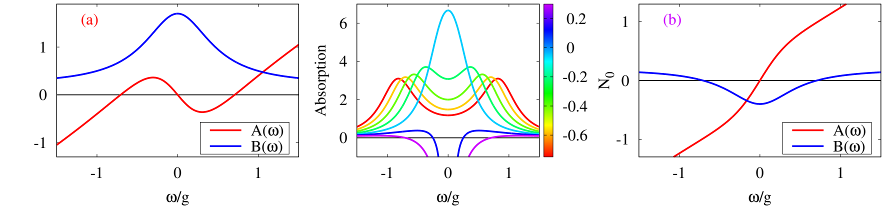
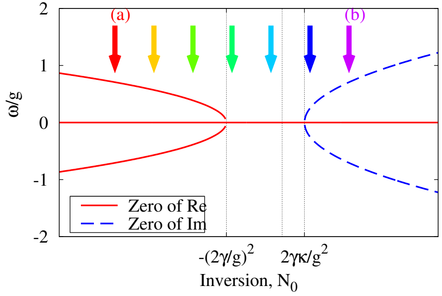
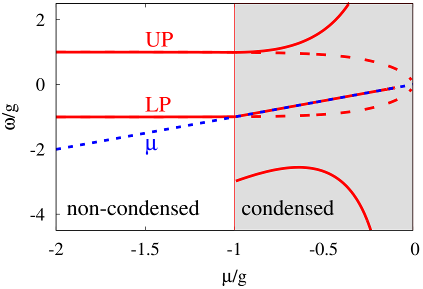


Laser:



Equilibrium:



Luminescence spectrum and Green's functions

$$-2\Im[D^R(\omega)] = \mathrm{DoS}(\omega)$$

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$$\mathrm{DoS}(\omega) = \frac{2B(\omega)}{B(\omega)^2 + A(\omega)^2}$$

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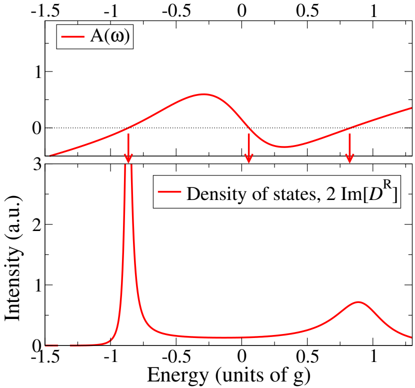

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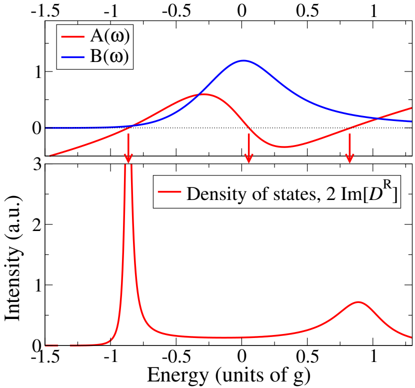

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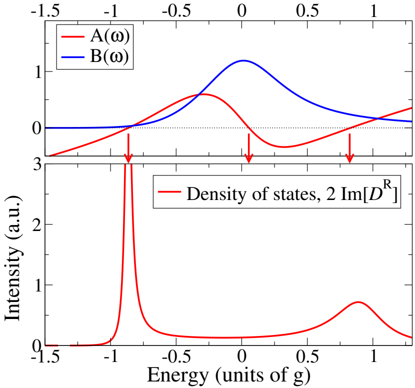

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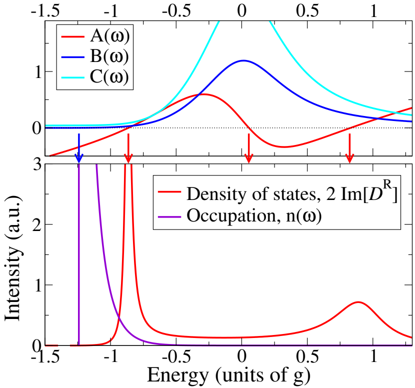
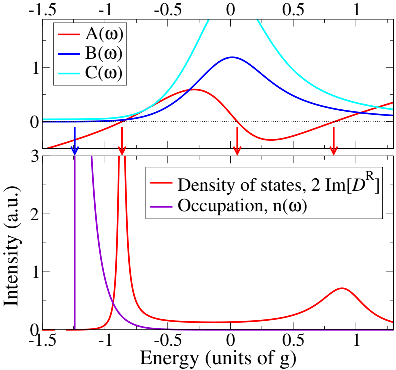


Stability and evolution with pumping



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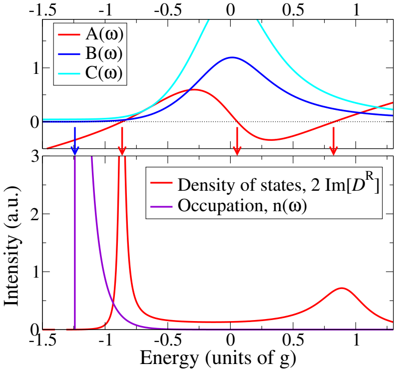

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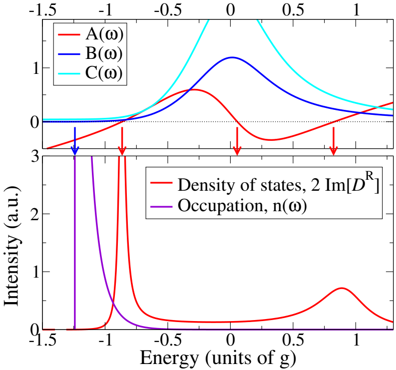

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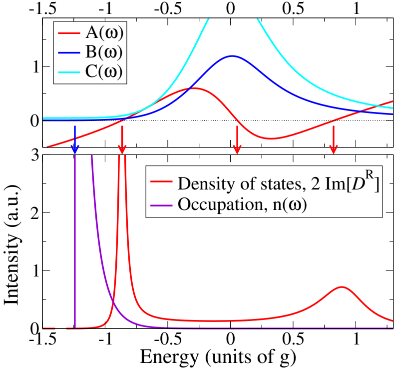

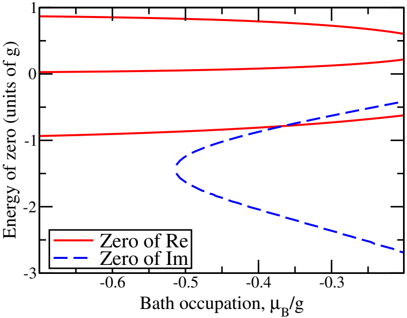

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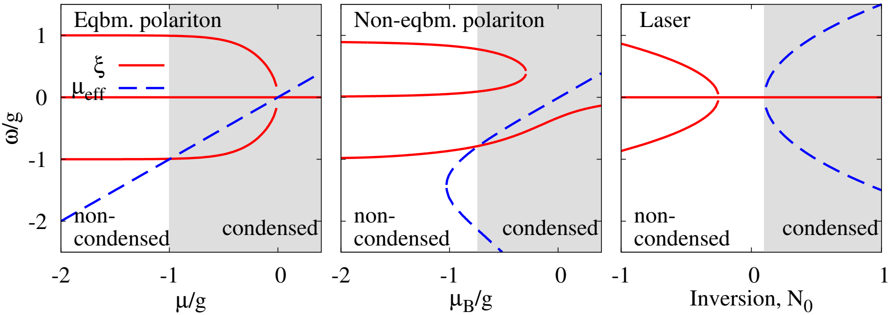
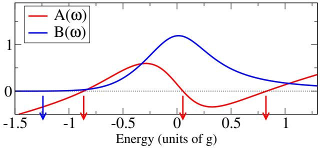
Strong coupling and lasing — low temperature phenomenon



- Laser: Uniformly invert TLS
- Non-equilibrium polaritons: Cold bath
- If $T_B \gg \gamma \rightarrow$ Laser limit



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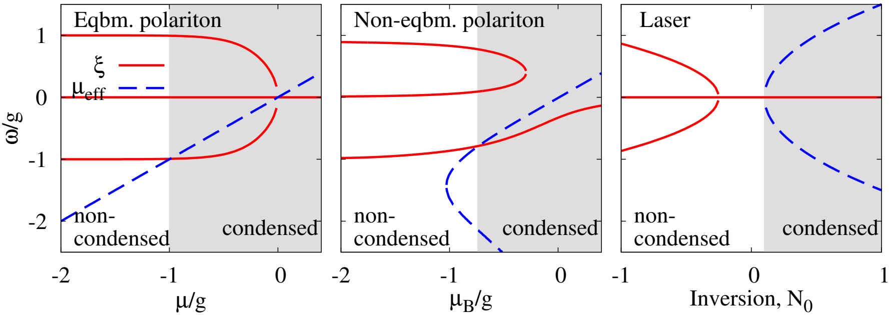

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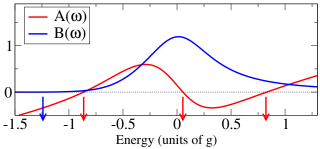

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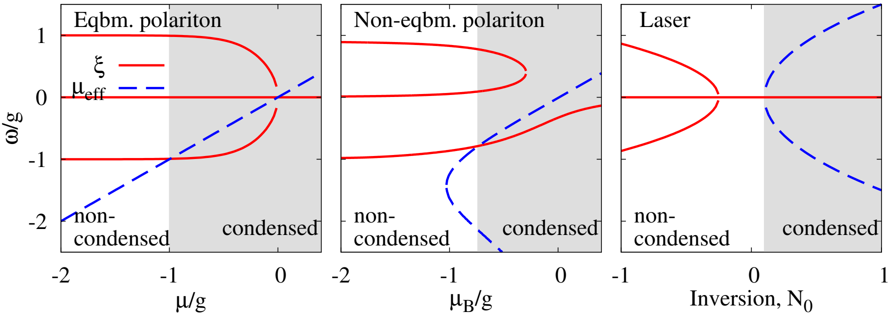

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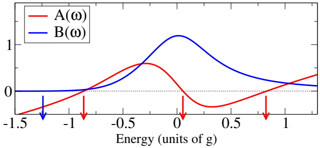

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma \rho)\rho$$

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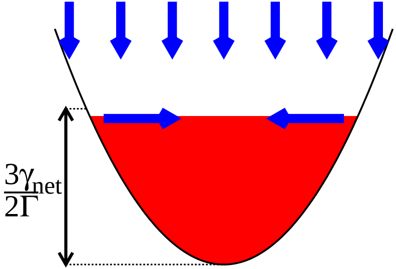

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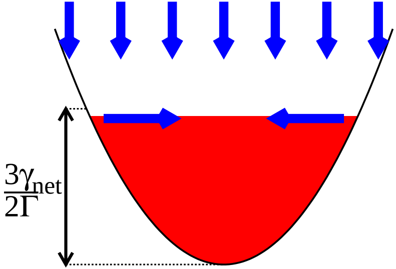

Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$$

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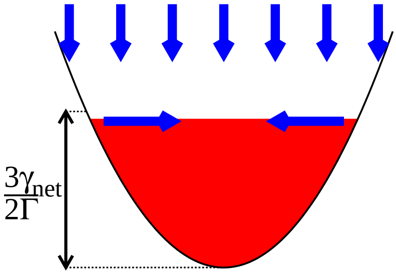

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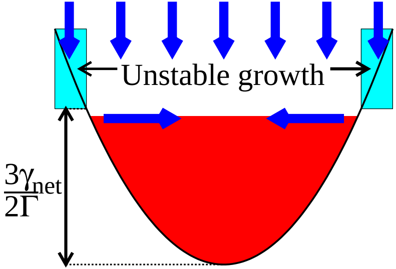
Add weak pumping/decay:

$$\omega_{n,n} \to \omega_{n,m} + i\gamma_{\text{net}}\left[\frac{m(1+2n) + 2n(n+1) - m^2}{2m(1+2n) + 4n(n+1) + m^2}\right]$$

Instability of Thomas-Fermi: details

$$\frac{1}{2}\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = (\gamma_{\text{net}} - \Gamma\rho)\rho$$

$$\partial_t \mathbf{v} + \nabla\left(U\rho + \frac{m\omega^2}{2} r^2 + \frac{m}{2}|\mathbf{v}|^2\right) = 0$$



Normal modes for $\gamma_{\text{net}}, \Gamma \to 0$:

$$\delta\rho_{n,m}(r,\theta,t) = e^{im\theta} h_{n,m}(r) e^{i\omega_{n,m} t}$$

$$\omega_{n,m} = \omega 2 \sqrt{m(1+2n) + 2n(n+1)}$$

Add weak pumping/decay:

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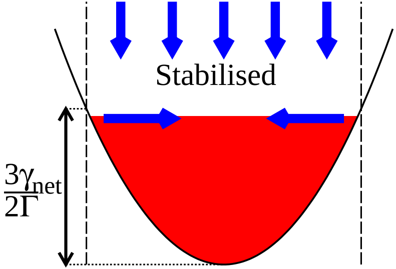

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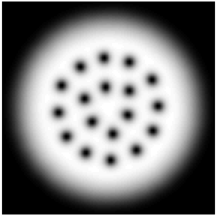
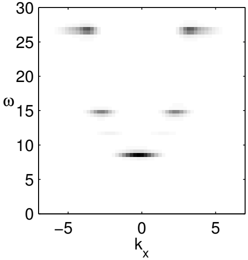
Detecting vortex lattices

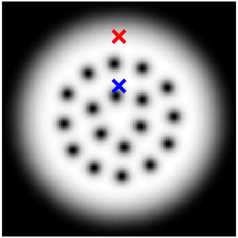
Snapshot

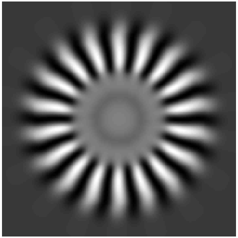


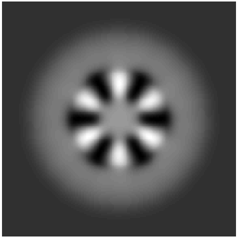
Spectrum:



Defocussed homodyne intereference:







Calculating superfluid response function

- Using Keldysh generating functional

$$\chi_{ij}(q) = -\frac{i}{2}\frac{d^2\mathcal{Z}[f,\theta]}{df_i(q)d\theta_j(-q)}, \qquad \mathcal{Z}[f,\theta] = \int \mathcal{D}\psi \exp(iS[f,\theta])$$

- f, θ couple as force/response current.

$$S[f,\theta] = S + \sum_{k,q} \begin{pmatrix} \bar{\psi}_{cl} & \bar{\psi}_q \end{pmatrix}_{k+q} \begin{pmatrix} \theta_i & f_i + \theta_i \\ f_i - \theta_i & -\theta_i \end{pmatrix}_q \frac{2k_i + q_i}{2m} \begin{pmatrix} \psi_{cl} \\ \psi_q \end{pmatrix}_k$$

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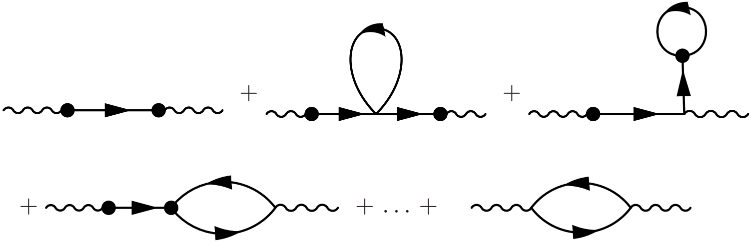

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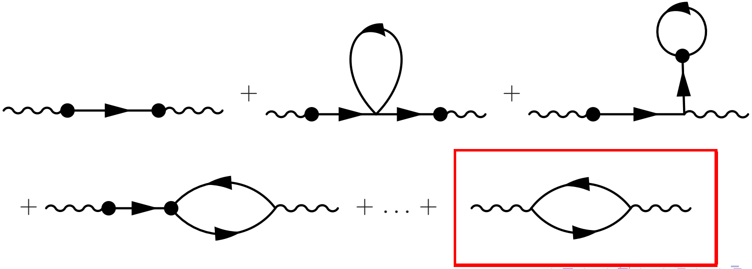
- Saddle point + fluctuations: Only one diagram for χ_N

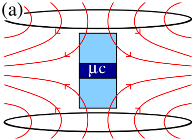


Measuring superfluid density

1. Effect rotating frame

Polariton polarization: $(\psi_{\circlearrowleft}, \psi_{\circlearrowright})$

$$H = \lambda \begin{pmatrix} \ell^2 & r^2 e^{2i\phi} \\ r^2 e^{-2i\phi} & -\ell^2 \end{pmatrix}$$



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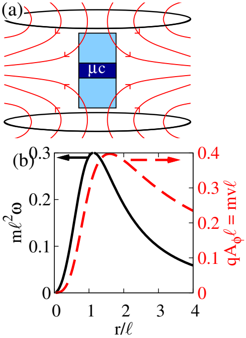
Ground state Berry phase:

$$q\mathbf{A}_{\text{eff}} = m\omega \times \mathbf{r} = \frac{\hat{\phi}}{r}\left[1 - \frac{\ell^2}{\sqrt{r^4 + \ell^4}}\right]$$



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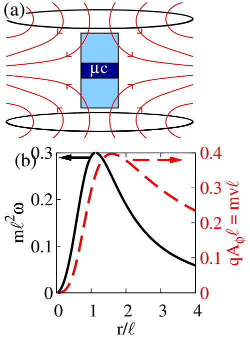
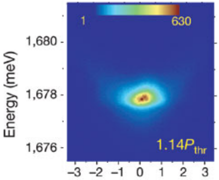


2. Measure resulting current

Energy shift of normal state:

$\Delta E = (1/2)mv^2 = 0.08/m\ell^2 \simeq 0.1\text{meV}$



Finite size effects: Single mode vs many mode

$$\left\langle \psi^\dagger(\mathbf{r}, t)\psi(\mathbf{r}', 0)\right\rangle \simeq |\psi_0|^2 \exp\left[-D^{<}_{\phi\phi}(\mathbf{r}, \mathbf{r}', t)\right]$$

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$D^<_{\phi\phi}(\mathbf{r},\mathbf{r}',t)$ from sum of phase modes. Study $ct \gg r$ limit:

$$D^<_{\phi\phi}(\mathbf{r},\mathbf{r},t) \propto \sum_n^{n_{max}} \int \frac{d\omega}{2\pi} \frac{|\varphi_n(\mathbf{r})|^2(1-e^{i\omega t})}{\left|(\omega + i\gamma_{\text{net}})^2 + \gamma_{\text{net}}^2 - \xi_n^2\right|^2}$$

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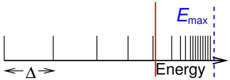
$$\Delta\xi \ll \sqrt{\frac{\gamma_{\text{net}}}{t}} \ll E_{\text{max}}$$



$$D^<_{\phi\phi} \sim 1 + \ln\left(E_{\text{max}} \sqrt{\frac{t}{\gamma_{\text{net}}}}\right)$$

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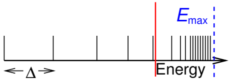

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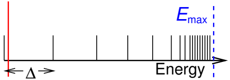

$$D^<_{\phi\phi} \sim \left(\frac{\pi C}{2\gamma_{\text{net}}}\right)\left(\frac{t}{2\gamma_{\text{net}}}\right)$$

(Recovers Schawlow-Townes laser linewidth)