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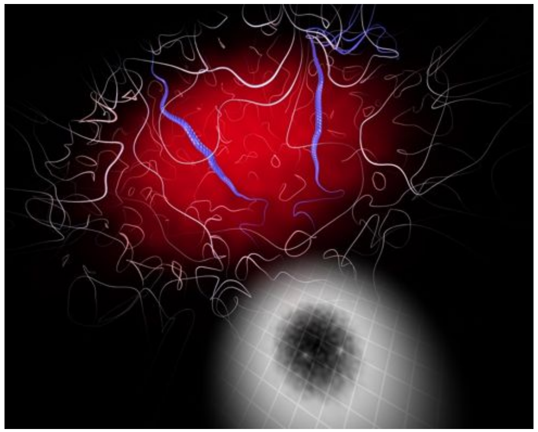
Non-equilibrium Bose gases with c-fields

Matthew Davis

Collaborators: Tod Wright, Mike Garrett, Geoff Lee, Chao Feng, Jacopo Sabbatini, Blair Blakie, Karen Kheruntsyan, Ashton Bradley



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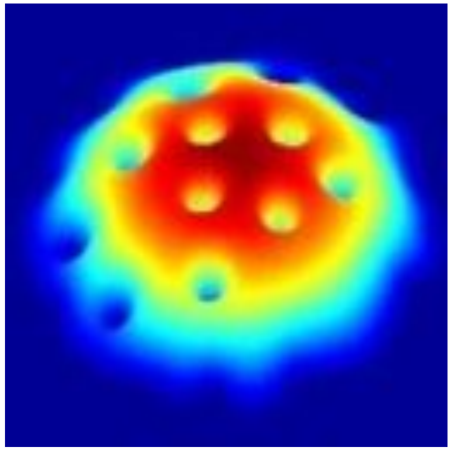

Non-equilibrium with ultra-cold atoms

- Ultra-cold atoms:
  - Simple ingredients, but complex phenomena.
  - “Easy” to manipulate far from equilibrium.
  - Isolated systems – have to equilibrate “on their own.”
- GPE: good for T = 0.
- What about T > 0?

Motivation

- What we want: a practical formalism for non-equilibrium dynamics of Bose gases

- Desirable features:
  - More than a few particles
  - More than one dimension
  - Can handle finite temperature
  - Goes beyond mean field theory
  - Can manage realistic experiment parameters
  - Computations finish on a reasonable time scale

- **Answer: c-field methods: variations of GPE.**

Outline

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- Formalism:
  - stochastic projected Gross-Pitaevskii equation (SPGPE)
- Equilibrium
  - Momentum distribution of trapped 1D Bose gas
  - Anomalous correlations and superfluidity in 2D Bose gas
- Non-equilibrium
  - Condensate formation
  - Classical and quantum Kibble Zurek mechanism with BEC
  - Non-equilibrium steady states
- Conclusions

FORMALISM

Classical fields

Q: Why is the Gross-Pitaevskii equation so successful?

A: The condensate mode is highly occupied, like a laser

$$\langle \hat{n}_0 \rangle = \langle \hat{a}_0^\dagger \hat{a}_0 \rangle \gg 1 \qquad [\hat{a}_0, \hat{a}_0^\dagger] \approx 0$$

Thus a BEC at T=0 can be approximated as classical field

$$\psi(\mathbf{x}) \approx \langle \hat{\psi}(\mathbf{x}) \rangle$$

But: also true for many excited modes at finite temperature

Svistunov, Kagan, Shylapnikov: J. Mosc. Phys. Soc. **1**, 373 (1991); JETP **75**, 387 (1992); PRL **79**, 3331 (1997).

Highly occupied Bose-Einstein ➡ equipartition distribution

$$\langle \hat{n}_k \rangle = \frac{1}{e^{(\epsilon_k - \mu)/k_B T} - 1} \approx \frac{1}{1 + (\epsilon_k - \mu)/k_B T + \ldots - 1} = \frac{k_B T}{\epsilon_k - \mu}$$

See K. Goral *et al.* (2001), A. Sinatra *et al.* (2001), M. J. Davis *et al.* (2001).

Applicability

^{87}Rb atoms, $N = 2 \times 10^6$, $\nu = 100$ Hz

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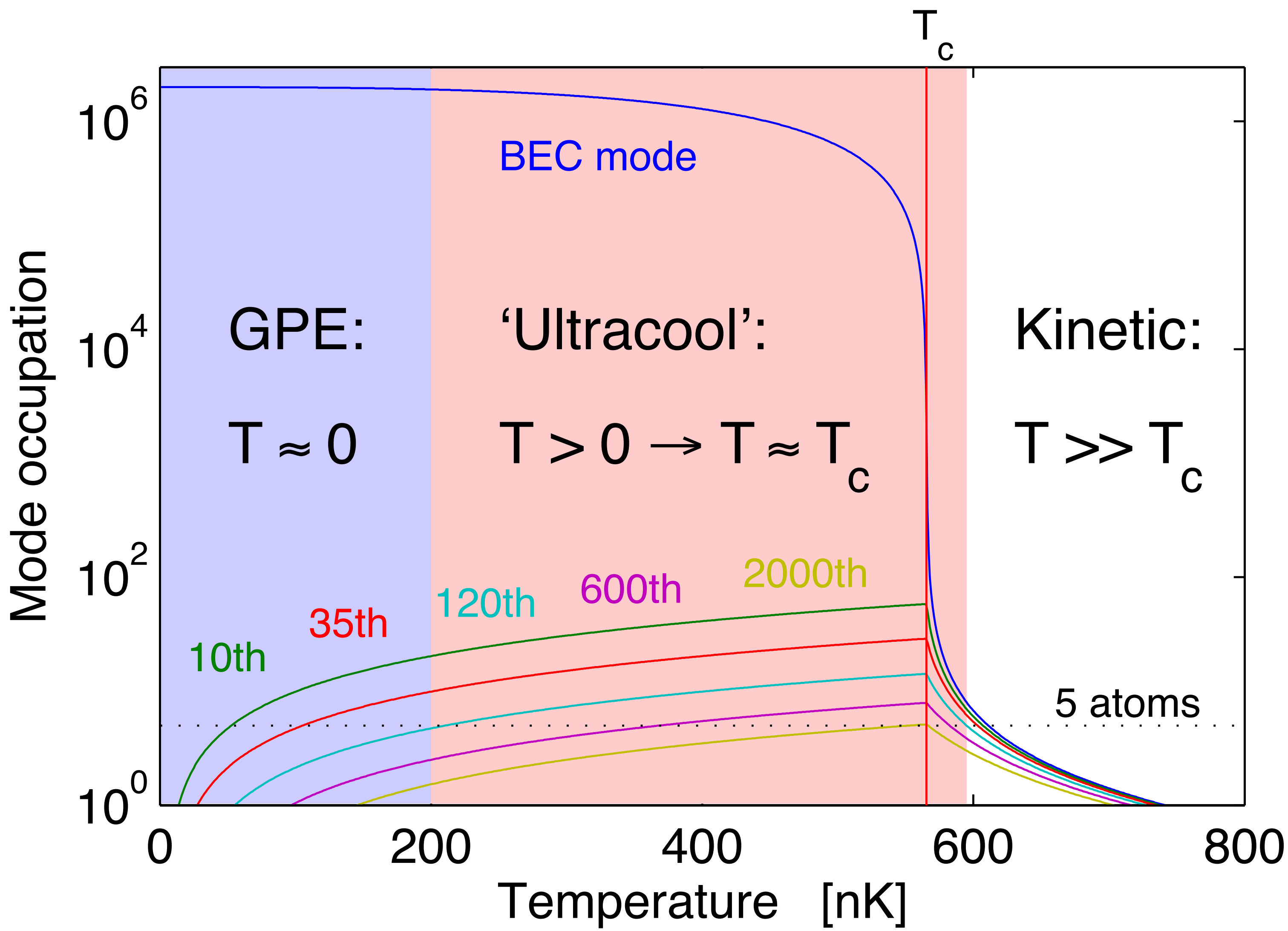
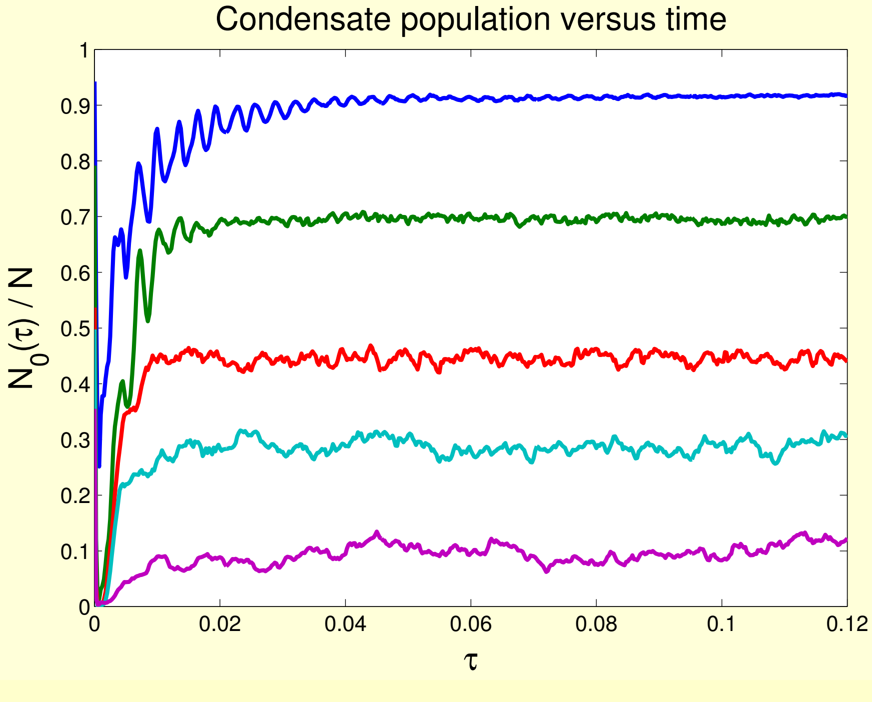
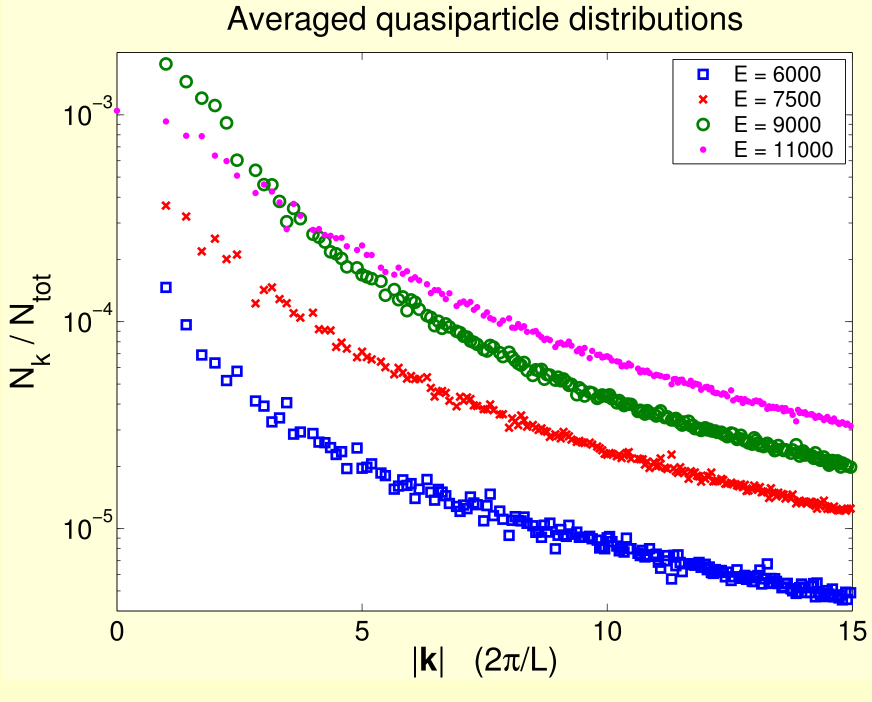


P. B. Blakie and M. J. Davis, J. Phys. B, **40**, 2043 (2007)

GPE can describe thermalisation

- Start with randomised initial conditions, fixed energy and fixed particle number
- Evolve with GPE on finite basis





M. J. Davis et al. PRL 87, 160402 (2001); PRA 66, 053618 (2002). Goral et al. PRA 66, 051602 (2002), Sinatra et al. PRL 87, 210404 (2001), Stoof and Bijlsma, J. Low. Temp. Phys 124, 431 (2001).

Formalism

Split field operator into c-field (high occupation) and incoherent regions

Derive master equation for c-field region density operator

Make “high temperature” approximation: $k_B T/\epsilon_k \gg 1$

Derive a Fokker-Planck equation for the Wigner quasi-probability distribution

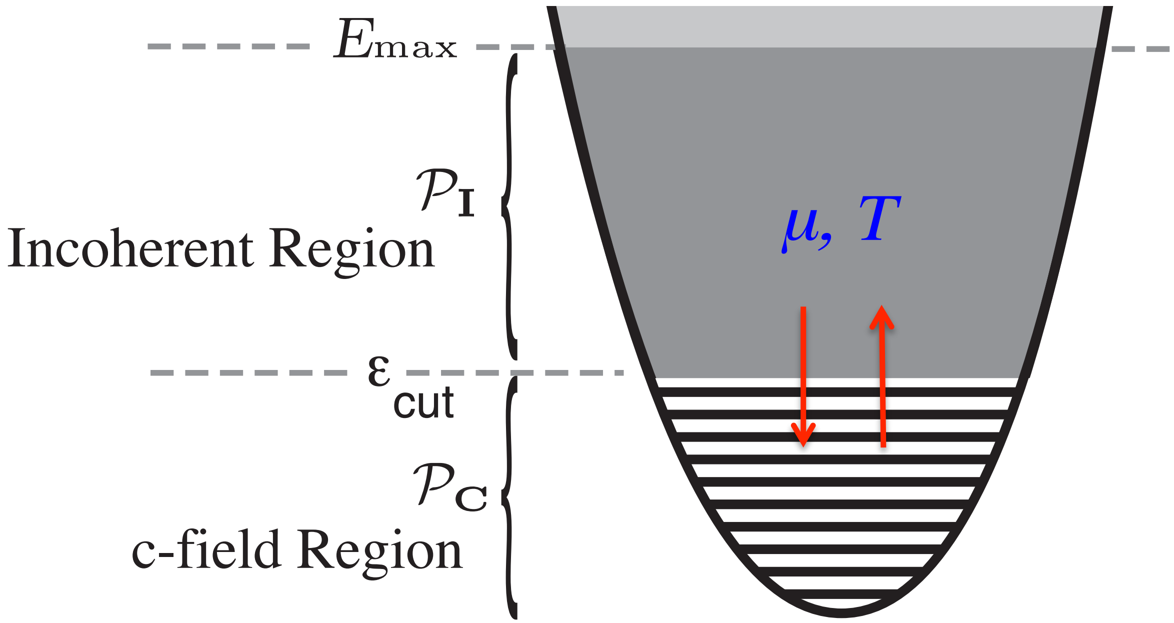
Neglect third order derivatives: probabilistic interpretation as stochastic DEs

Summary:

BEC + low energy excitations are treated as a classical field.

High energy atoms treated as thermal bath:

Chemical potential μ
Temperature T



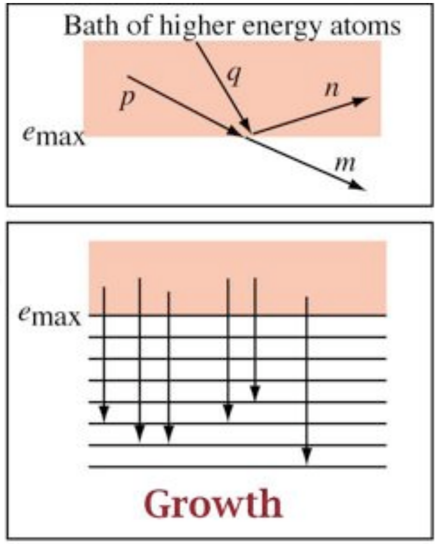
C. W. Gardiner and M. J. Davis, J. Phys. B, **36**, 4731 (2003); P. B. Blakie *et al.*, Adv. Phys. **57**, 363 (2008).

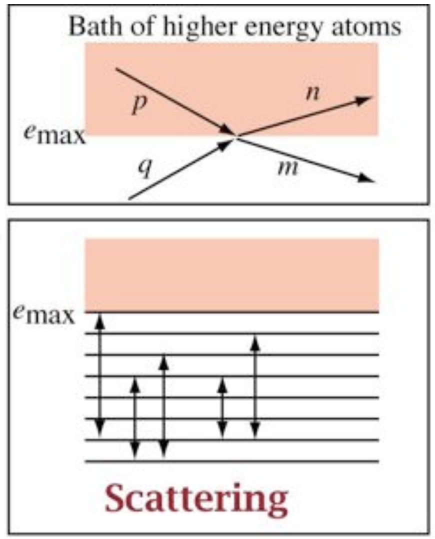
Stochastic Projected GPE (SPGPE)

$$
\begin{aligned}
d\psi(\mathbf{x}) \quad = \quad & \mathcal{P}_C\left\{-\frac{i}{\hbar}L_{\rm GP}\psi(\mathbf{x})\;dt\right.\\
& + \frac{G(\mathbf{x})}{k_BT}(\mu - L_{\rm GP})\psi(\mathbf{x})\,dt + dW_G(\mathbf{x},t)\\
& \left. + \frac{\mathcal{M}}{k_BT}\frac{1}{\sqrt{-\nabla^2}}\mathrm{Re}\left[\psi^*(\mathbf{x})L_{\rm GP}\psi(\mathbf{x})\right]dt + i\psi(\mathbf{x})dW_{\mathcal{M}}\right\}
\end{aligned}
$$

Projector

Collision integrals





GP operator:

$$L_{\rm GP} = -\frac{\hbar^2\nabla^2}{2m} + V_{\rm trap}(\mathbf{x}) + g|\psi(\mathbf{x})|^2\psi(\mathbf{x})$$

Noise correlations:

$$\langle dW_G^*(\mathbf{x},t)dW_G(\mathbf{x}',t)\rangle = 2G(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}')dt$$

$$\langle dW_{\mathcal{M}}(\mathbf{x}',t)dW_{\mathcal{M}}(\mathbf{x},t)\rangle = \frac{2\mathcal{M}}{\sqrt{-\nabla^2}}\delta(\mathbf{x}-\mathbf{x}')dt$$

C. W. Gardiner and M. J. Davis, J. Phys. B, **36**, 4731 (2003); P. B. Blakie *et al.*, Adv. Phys. **57**, 363 (2008).

Simplifications

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- Simple growth SPGPE
  - Drop scattering term (multiplicative noise)
  - Equilibrium is unchanged : grand canonical.

- Projected GPE (PGPE)
  - Keep high occupation condition, but neglect bath coupling.
  - Microcanonical system
  - Valid for equilibrium (detailed balance)

- Truncated Wigner approximation
  - Treat **entire gas** using c-field.
  - Include initial quantum noise – seeds spontaneous processes.
  - Valid for high occupations, short times.

EQUILIBRIUM

Thermodynamics

- PGPE (SPGPE) samples thermodynamic equilibrium through microcanonical (grand canonical) ergodic dynamics
- Nonperturbative in particle interactions
- Independent of computational basis (using projector)
- Incoherent region accounted using Hartree-Fock theory

- Applications:
  - Shift in critical temperature
  - Correlations at finite temperature
  - Low-dimensional physics
    - Quasicondensation
    - Berezinkskii-Kosterlitz-Thouless physics

Momentum distribution of 1D Bose gas

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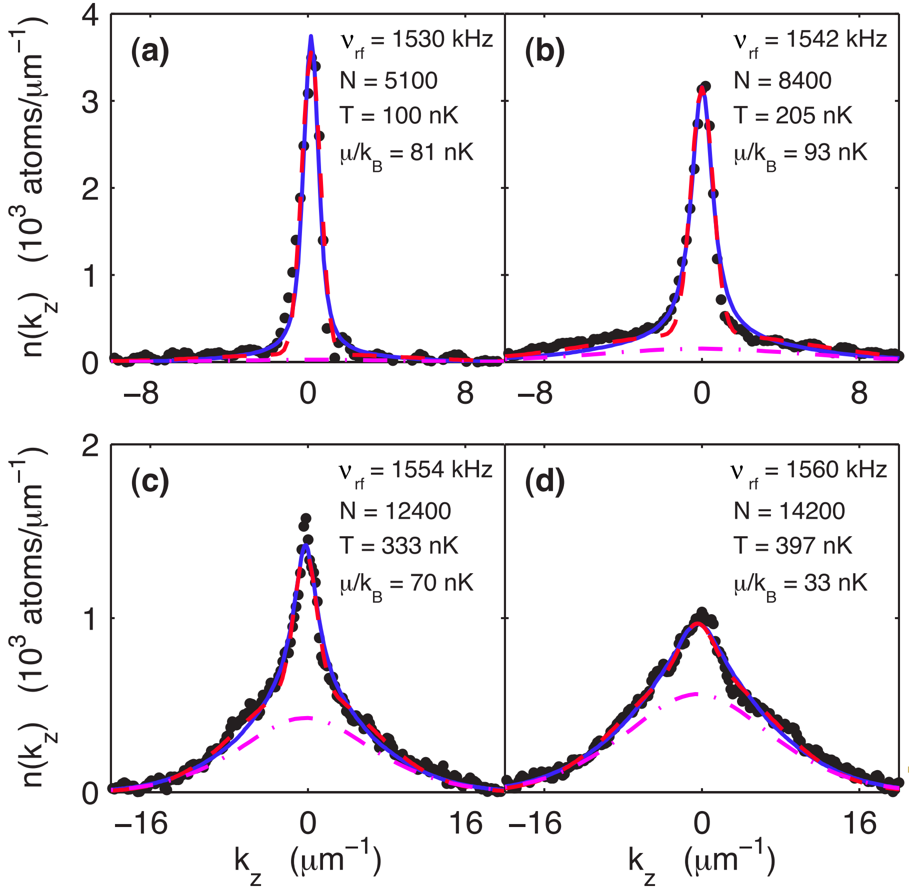
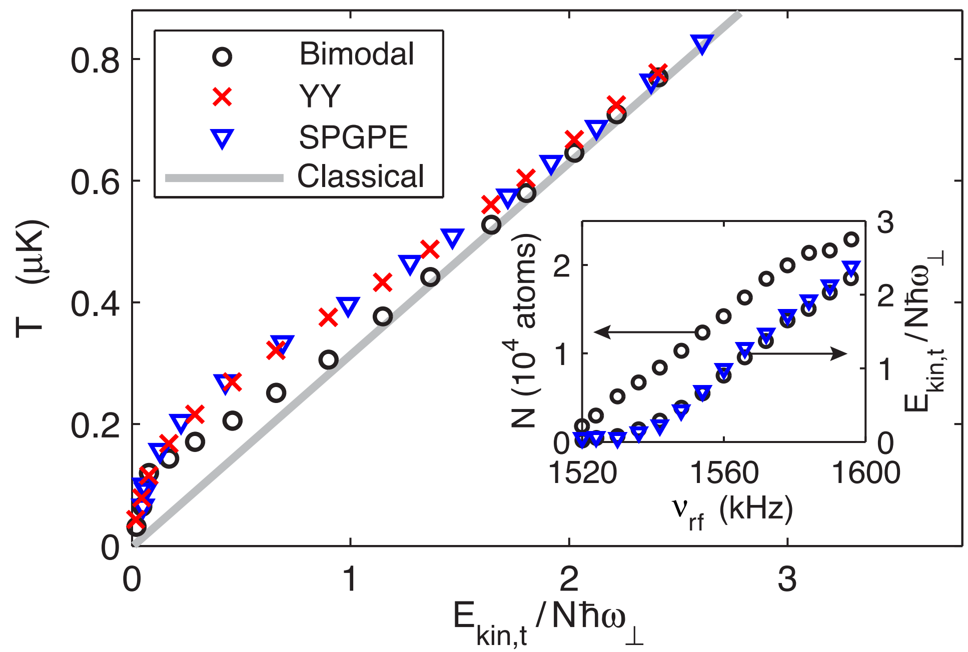
- Measurements by Van Amerongen *et al.*, PRL **100**, 090402 (2008).
- Exact Yang-Yang solutions give density – but no momentum distributions
- We perform trapped SPGPE calculations – fit temperature to data
- Agrees well with Yang-Yang thermometry based on kinetic energy measure





M. J. Davis *et al.*, PRA (2012)

Superfluidity in 2D Bose gases

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Apply PGPE to finite size 2D homogeneous gas

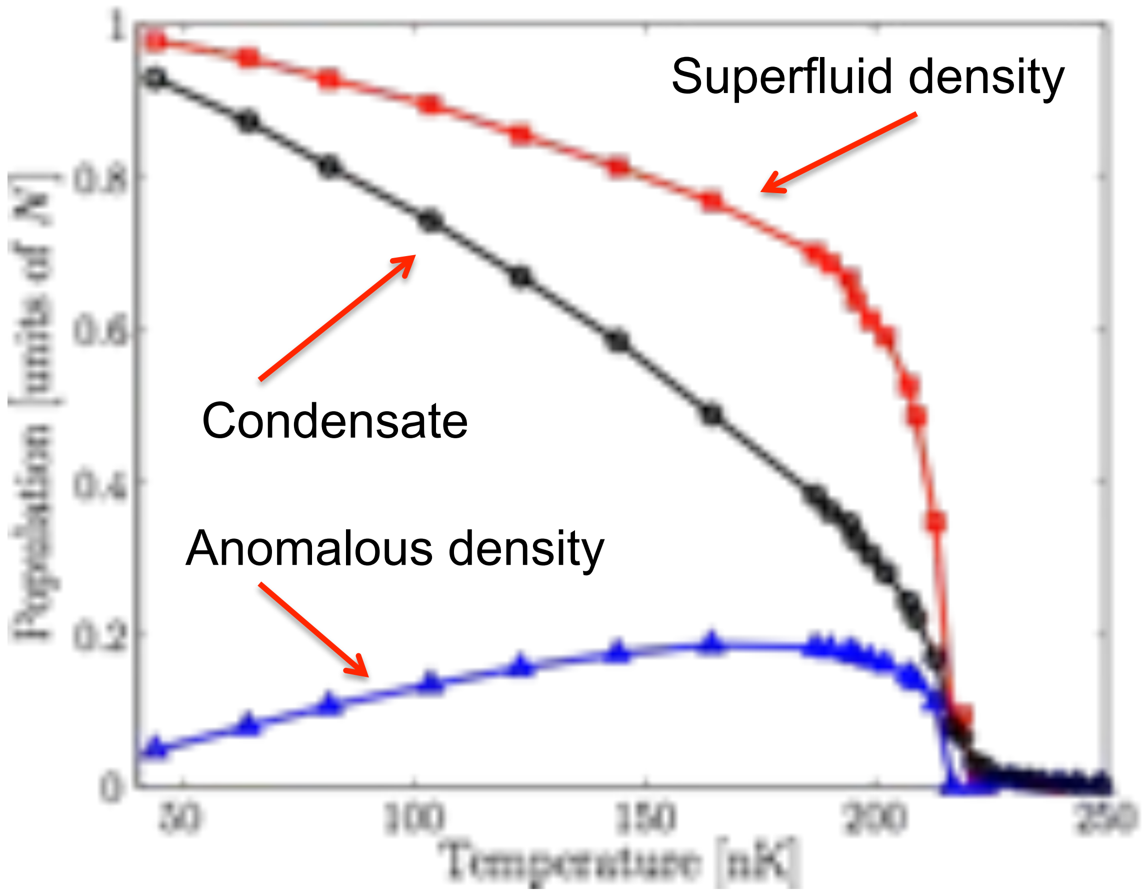
See C. J. Foster *et al.*, Phys. Rev. A **81**, 023623 (2010).

Superfluid density from momentum correlations

Anomalous density drops to zero at same point as superfluid density

Condensate fraction still non-zero at T_c

Also: superfluid density in ring trap from mass current



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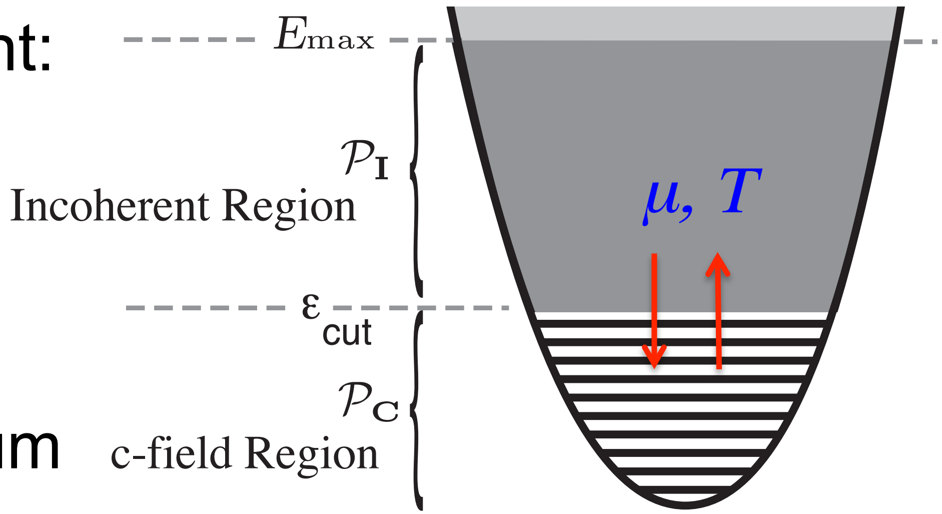
NON-EQUILIBRIUM

Simulating non-equilibrium

- C-field dynamics eventually comes to equilibrium with thermal reservoir
- Thus: manipulate reservoir parameters to e.g. simulate evaporative cooling and condensate formation.

Procedure for BEC formation:

- Begin above BEC critical point: $T > T_c$, $\mu < 0$
- Ramp thermodynamic parameters in time
- Watch the condensate rethermalise to new equilibrium



MOVIE

Condensate formation movie

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Spontaneous vortices in the formation of Bose–Einstein condensates

M. J. Davis and A. S. Bradley
ARC Centre of Excellence for Quantum-Atom Optics, School of Physical Sciences, University of Queensland, Brisbane, QLD 4072, Australia

C. N. Weiler, T. W. Neely, D. R. Scherer and B. P. Anderson
College of Optical Sciences, University of Arizona, Tucson, AZ 85721, USA

Harmonic trap

Instant quench

Initial temperature T_i = 45 nK
Initial chemical potential μ_i = 1.0 $h\nu_r$
Scattering rate γ = 0.005
Trap frequencies (ν_r, ν_z) = (7.8, 15.3) Hz

Trajectory number : 240 / 300

Date : February 2008

Final temperature T_f = 34 nK
Final chemical potential μ_f = 25.0 $h\nu_r$
Cutoff energy E_{cut} = 40 $h\nu_r$

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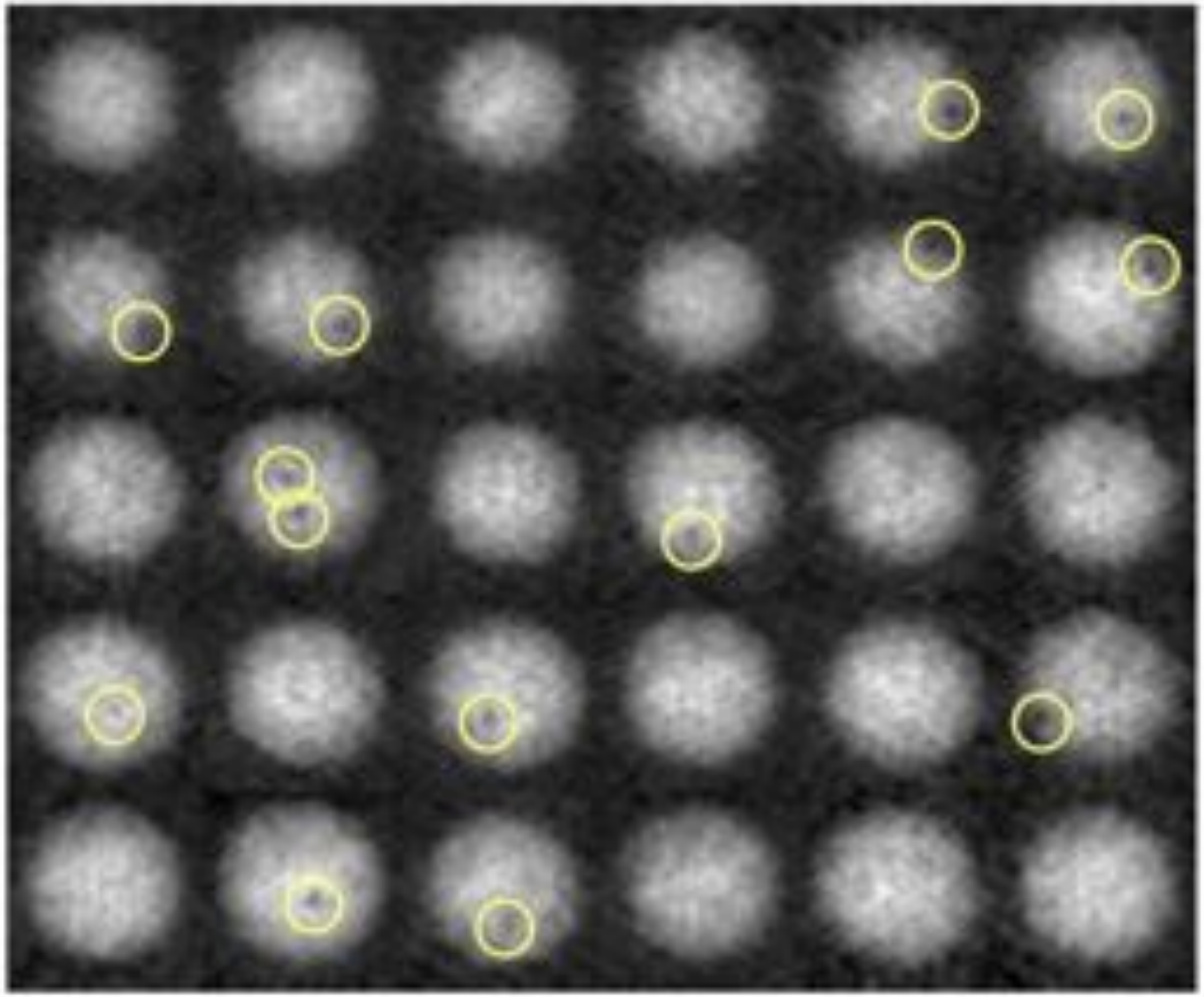
Condensate formation at Arizona



C. N. Weiler *et al.*, Nature **455**, 948 (2008).

Results

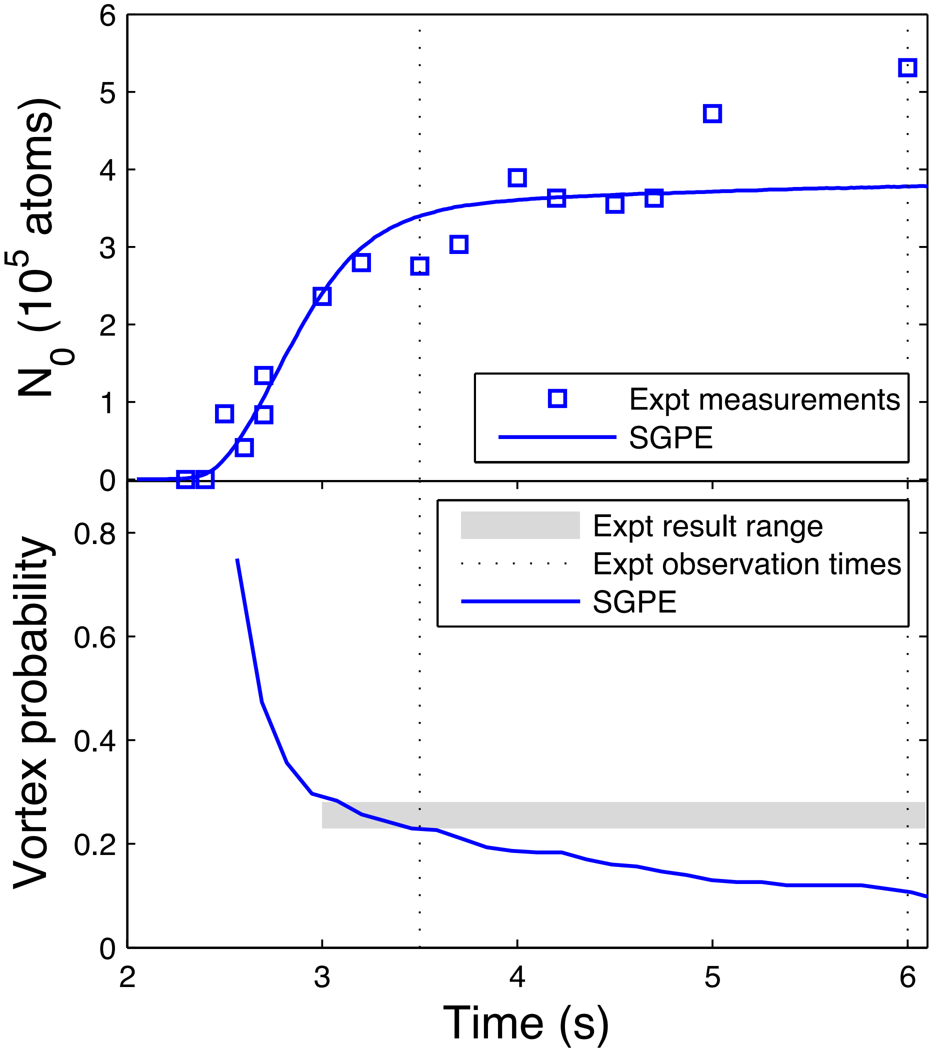
C. N. Weiler *et al.*, Nature **455**, 948 (2008).

The University of Queensland
Australia

Atom numbers, temperatures, taken from experiment

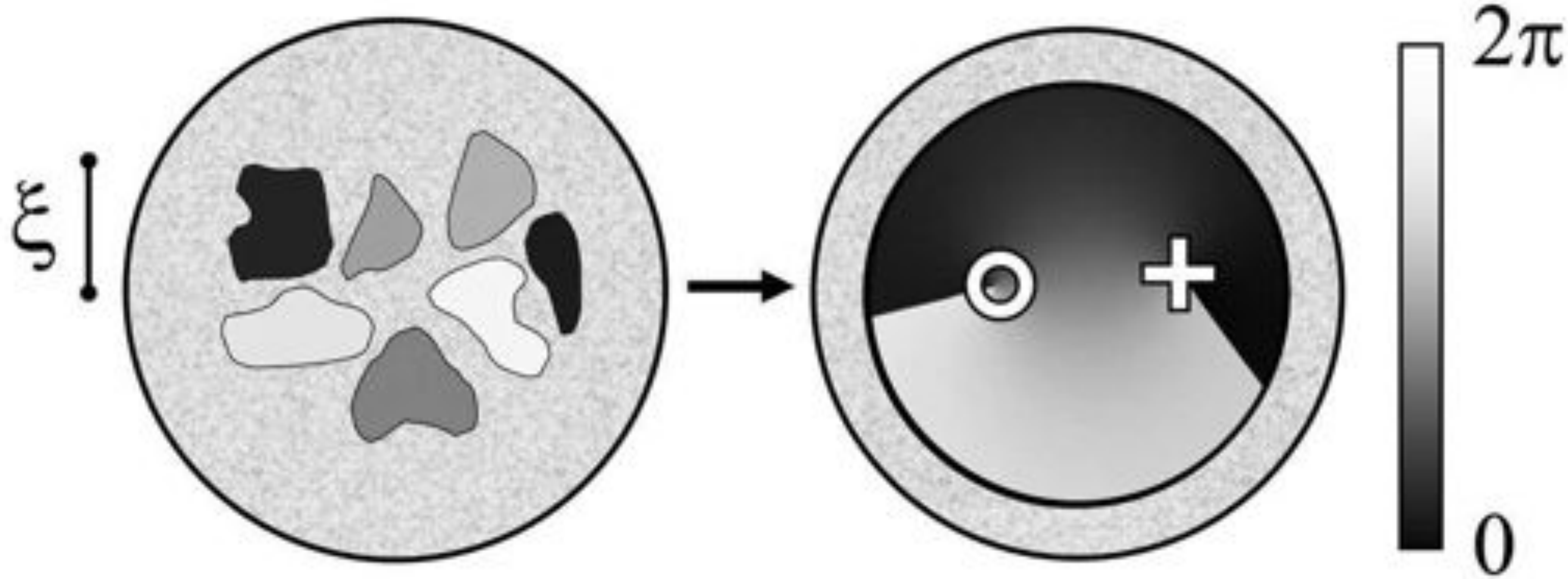
Collision rate chosen to match condensate growth curve

Simulated vortex probability from SPGPE agrees with observations



How do vortices arise?

- Kibble (1976): Causally disconnected regions enter symmetry broken phase independently.
- When these reconnect topological defects can form.



Kibble-Zurek mechanism

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- Real phase transitions occur on a finite time scale.
- Kibble-Zurek mechanism is a universal prescription for estimating the **density of defects** after a quench.
- Idea: control phase transition: $\epsilon = (1 - T/T_c) = t/\tau_Q$
- Equil. correlation length / time diverge at critical point:

$$\xi = \frac{\xi_0}{|\epsilon|^\nu}, \qquad \tau = \frac{\tau_0}{|\epsilon|^{\nu z}}$$

- Speed of transition determines "freeze out" time
  - Faster transition -> earlier freeze out -> smaller correlation volumes -> more defects
  - Defect density:

$$n \propto \frac{1}{\xi_0^d} \left(\frac{\tau_0}{\tau_Q} \right)^{d\nu/(1+\nu z)}$$

Kibble-Zurek mechanism with BEC?

- Many experiments on KZM in condensed matter
  - ^{4}He, ^{3}He, liquid crystals, superconducting rings.
- **Lots of simulations show KZ scaling**
  - **but no conclusive agreement between theory and experiment**
- For BEC: number of vortices expected to scale with quench time
- Need to control e.g. temperature or chemical potential
  - Hard in experiment: easier with SPGPE.

$$\epsilon = (1 - T/T_c) = t/\tau_Q$$

Oblate BEC formation (Arizona)

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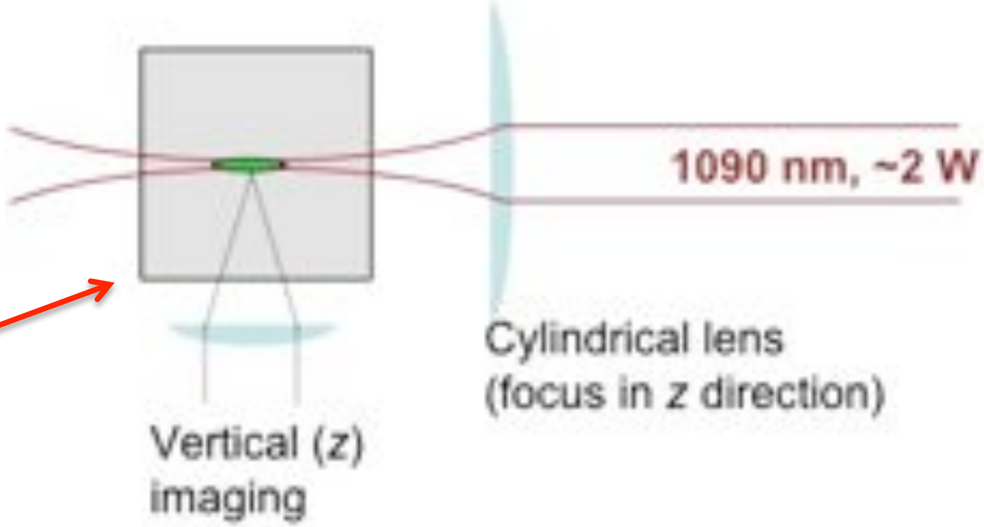
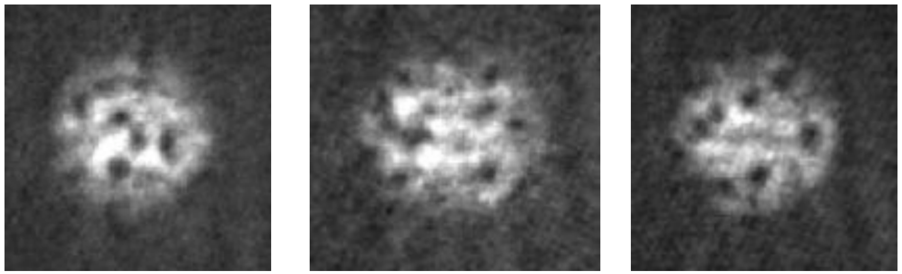
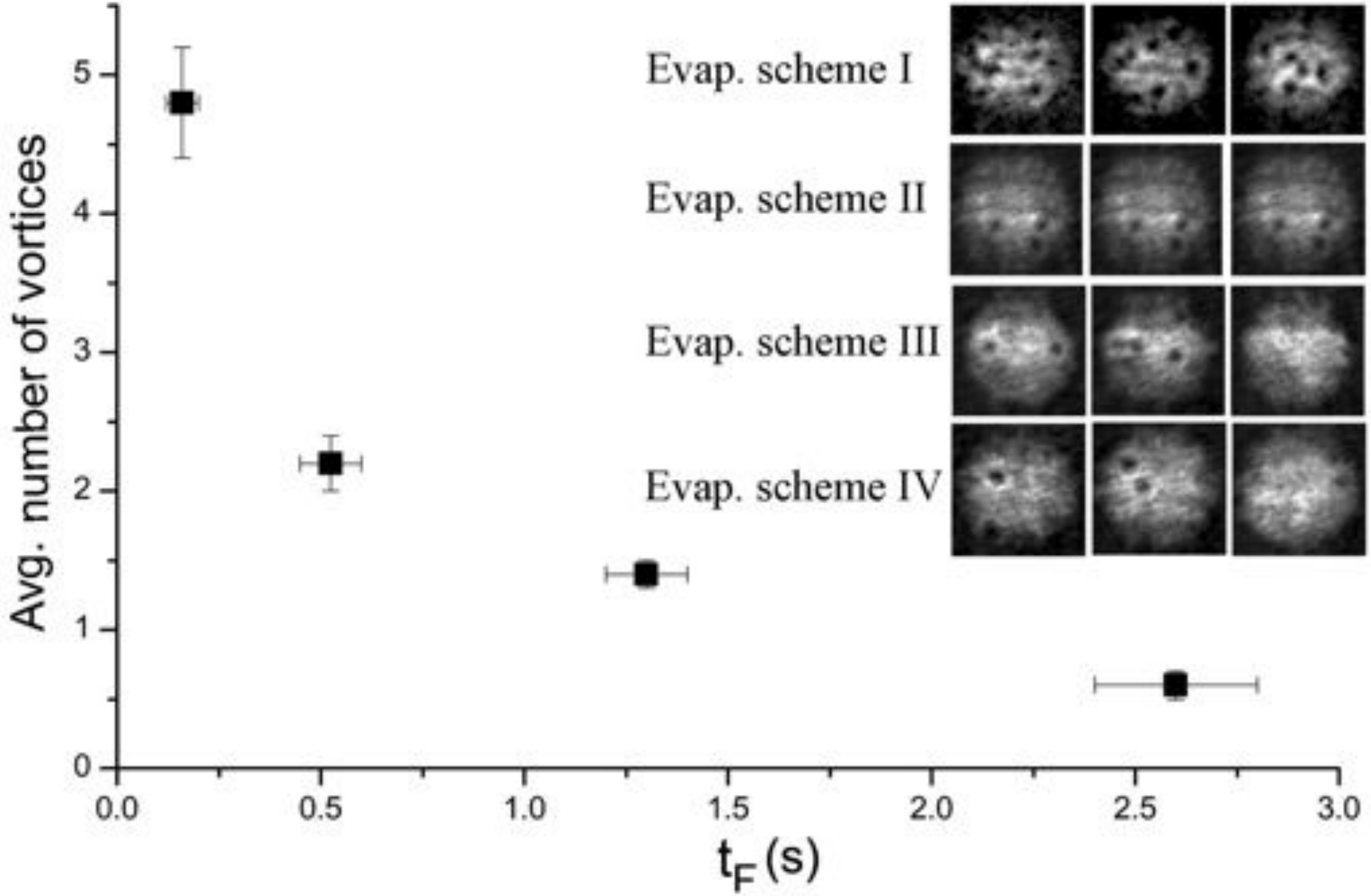
- Rate of quench ~ rate of condensate growth
- How to control this?
  - Compress gas with 2D laser sheet
  - increases collision rate
- May be an way to control a ramp of effective μ
- In expt: see many more vortices





Experimental results (Arizona)



SPGPE simulations

Vary ramp of chemical potential μ

Quench time: τ_Q

Count number of vortices

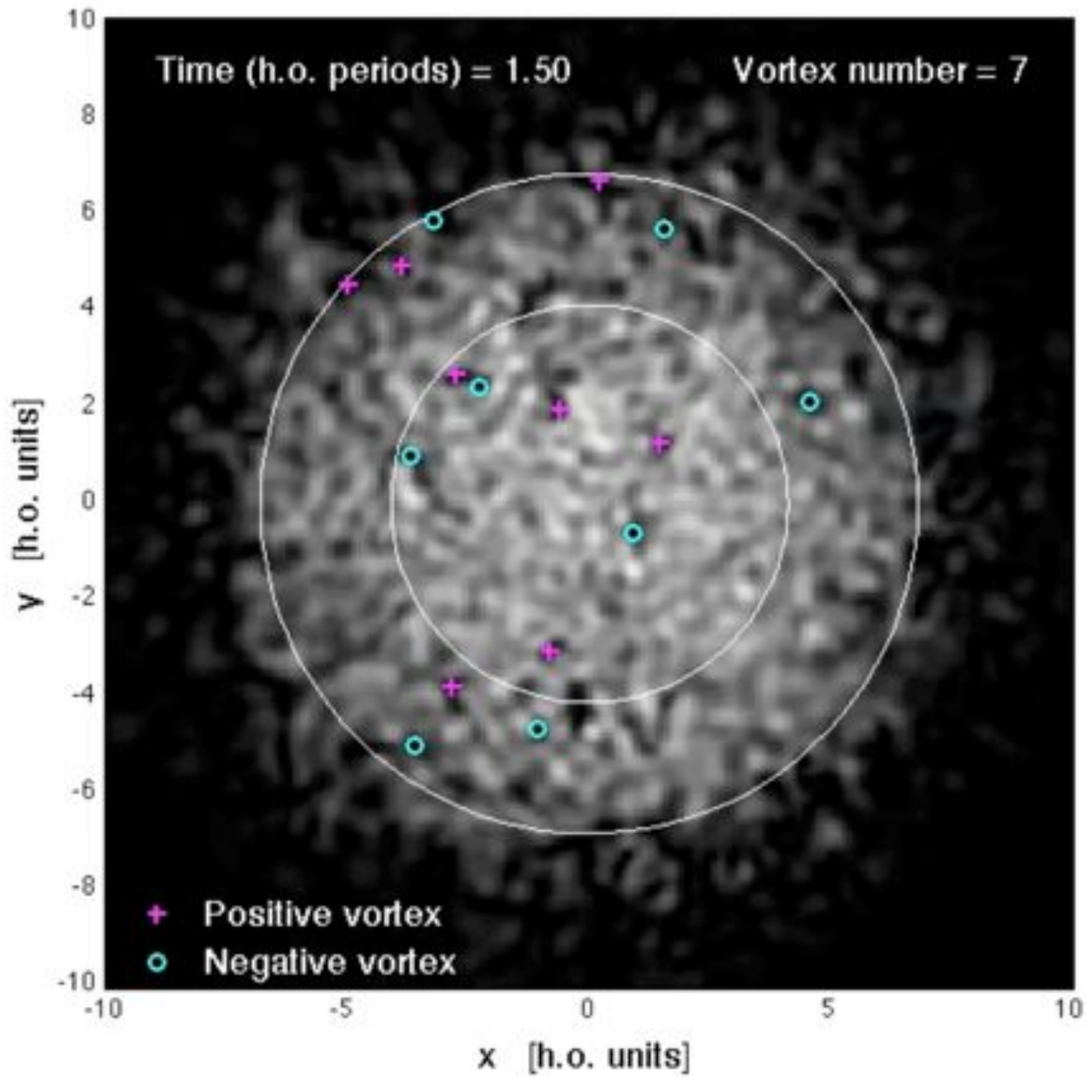
Parameters match Arizona lab

$(\nu_r, \nu_z) = (7.8, 88)\ \mathrm{Hz}$

White circles indicate

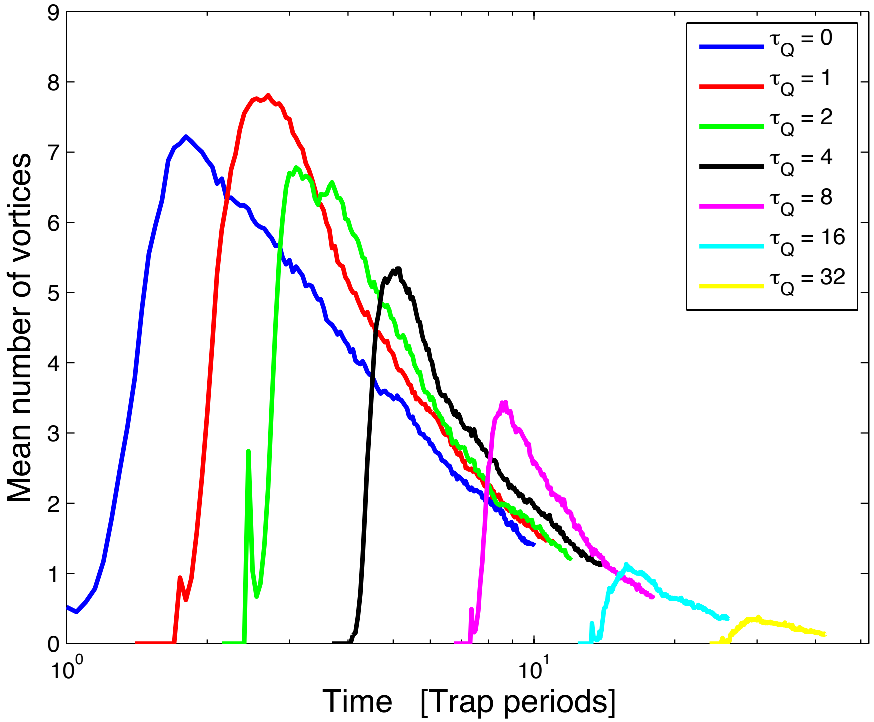
$g^{(2)}(x,x) = (1.1, 1.3)$

MOVIE

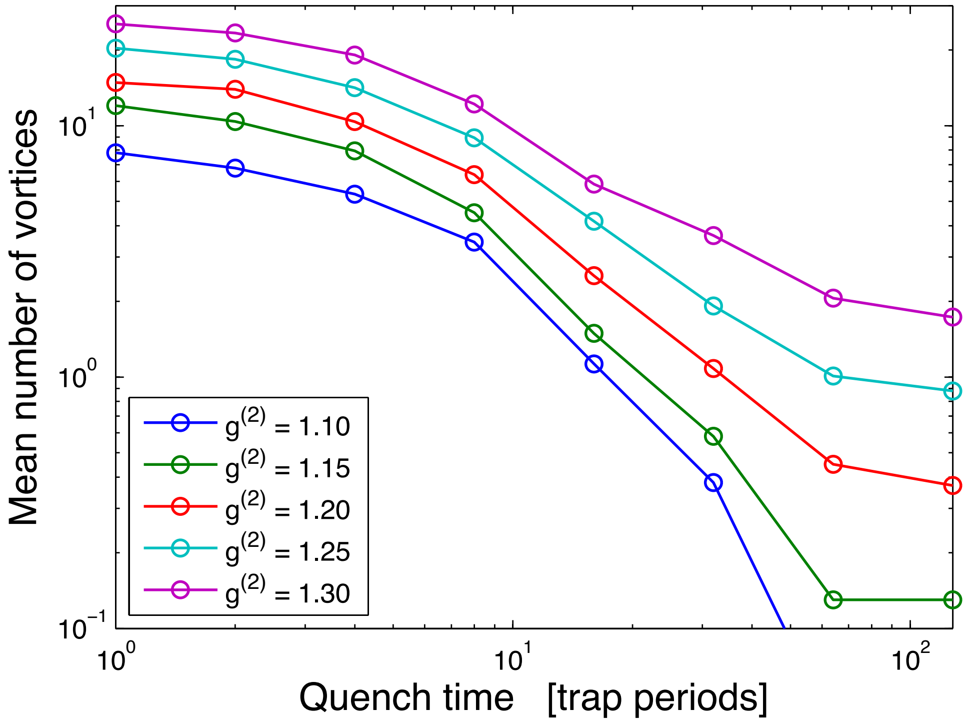


Preliminary results

Vortex number versus time



Scaling of peak vortex number



- No simple scaling apparent so far
- Expected to be universal: A. del Campo *et al.*, New J. Phys. **13**, 083022 (2011).

Quantum phase transitions

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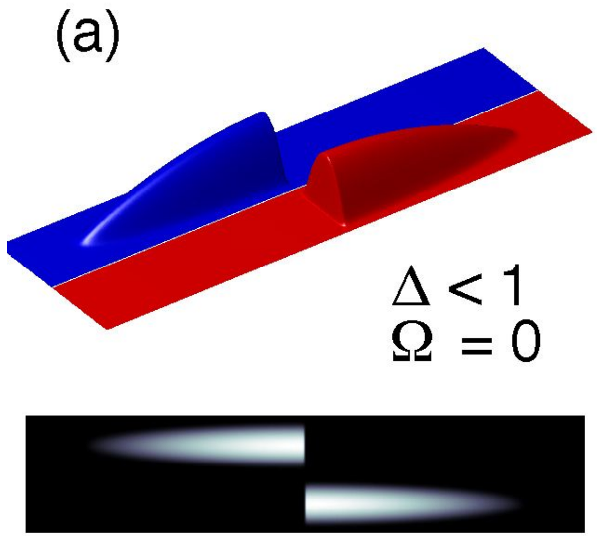
- KZ theory can also be applied to **quantum** phase transitions.
- Ramp Hamiltonian parameter across a critical point.
- Binary BEC: **immiscible** if

$$\Delta = \frac{g_{11}g_{22}}{g_{12}^2} < 1$$



- Can be made **miscible** by internal Josephson coupling above critical value $\Omega_{\rm cr}$:

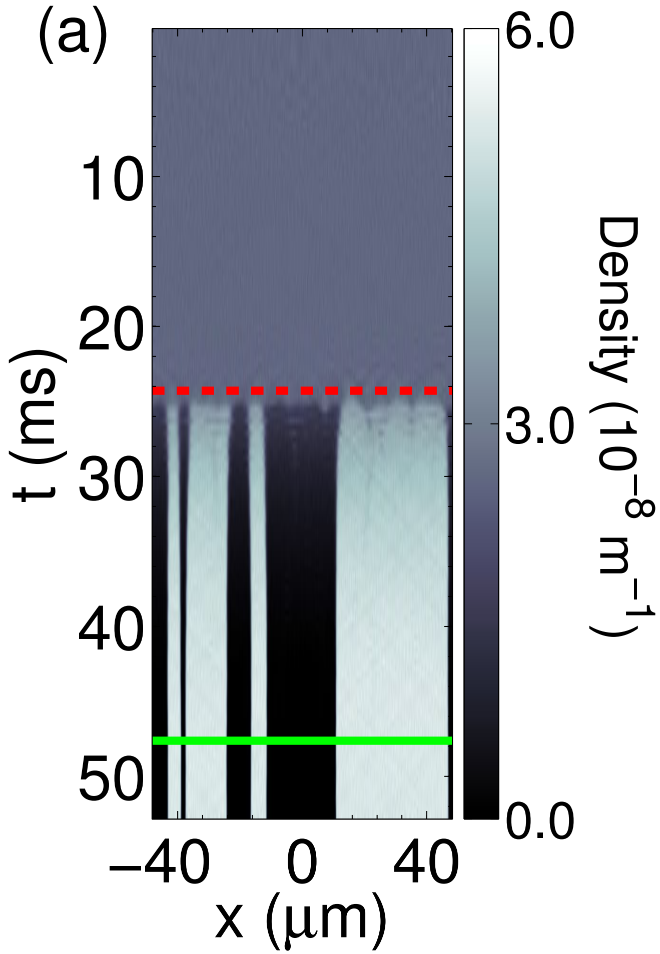
$$-\hbar\Omega(t)[\hat{\psi}_1^\dagger(x)\hat{\psi}_2(x) + \hat{\psi}_2^\dagger(x)\hat{\psi}_1(x)]$$

Procedure

- Load stationary dressed state at large coupling.
- Control parameter:

$$\Omega(t) = \max\left[0, 2\Omega_{\rm cr}\left(1 - \frac{t}{\tau_{\rm Q}}\right)\right]$$

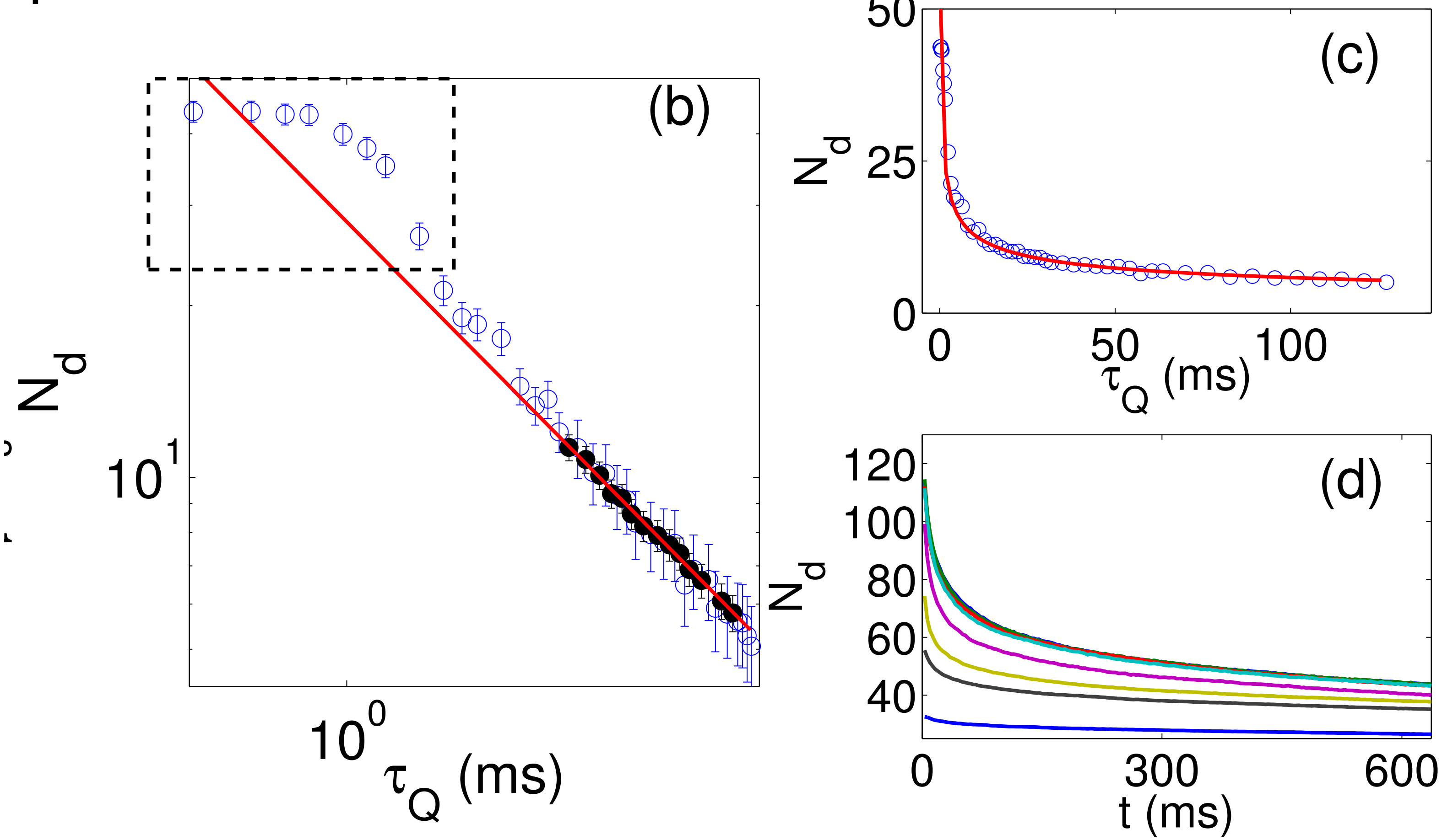
- Use truncated Wigner method
- Quantum noise in initial state seeds **domain formation.**
- Simulate many trajectories
- Count mean number of domains.



J. Sabbatini, W. H. Zurek, M. J. Davis, Phys. Rev. Lett. **107**, 230402 (2011).

Theoretical results

- For 1D homogenous gas - scaling law agrees with KZ prediction



Experimental results (preliminary)

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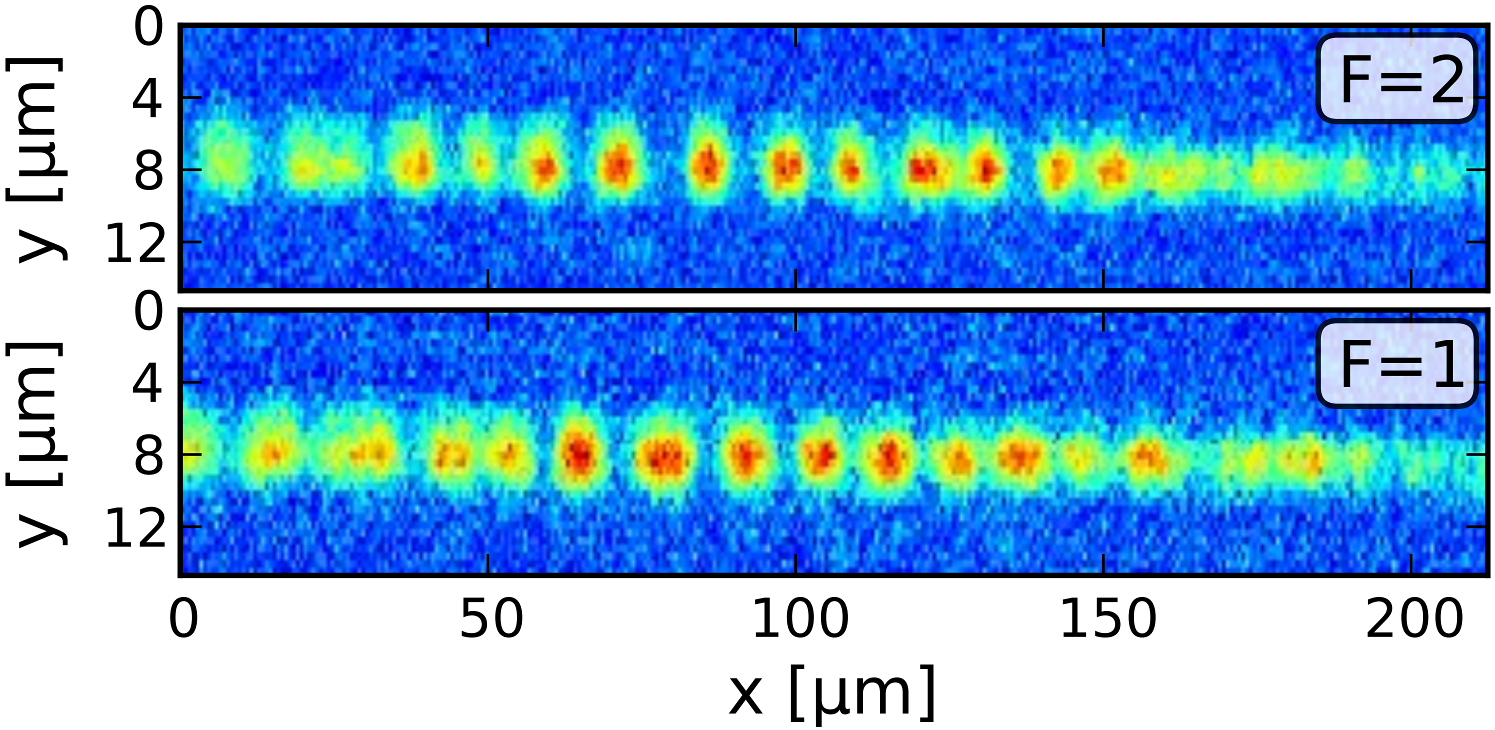
- Oberthaler group, Heidelberg.
- Tune Δ = 0.78 using Feshbach resonance for g_{12}



Experimental results (preliminary)

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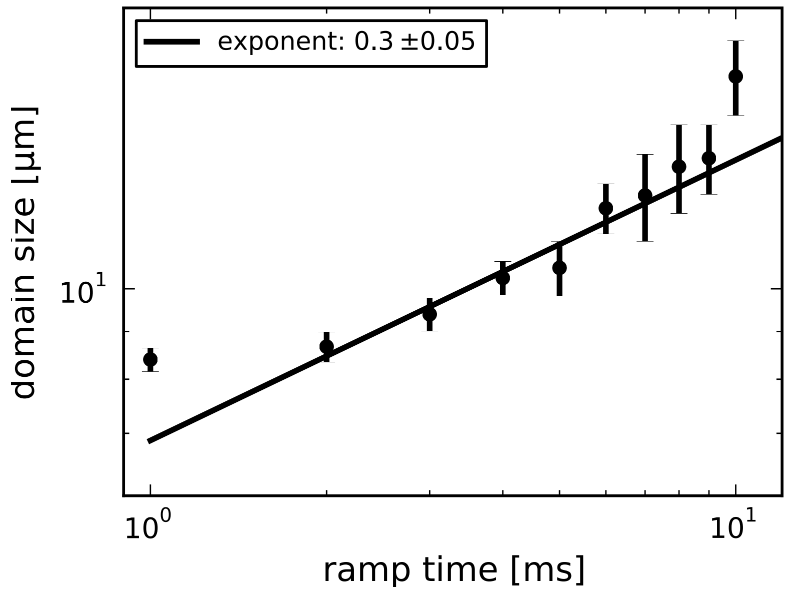
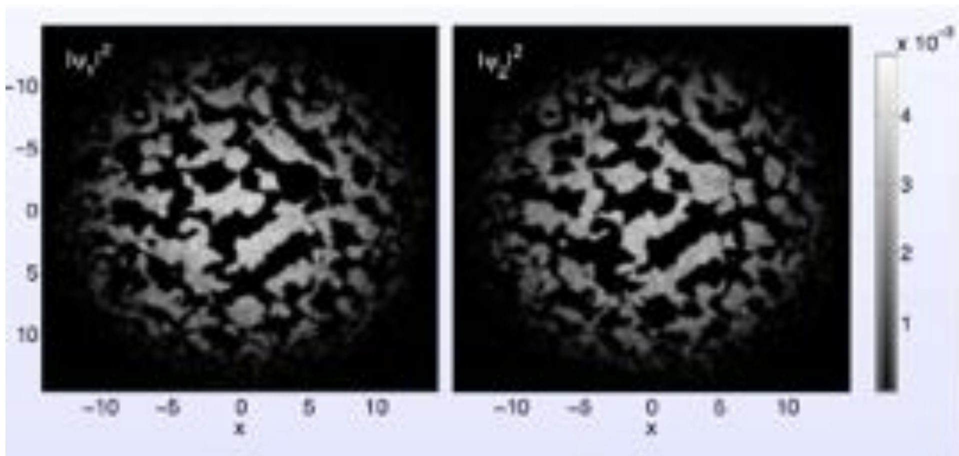
- Appear to see a scaling law consistent with KZ prediction
- Have also investigated time of defect seeding
- Currently attemping to measure critical exponent directly.
- But several complications:
  - Inelastic losses
  - Harmonically trapped
  - Long domain formation time – growth of most unstable Bogoliubov mode.

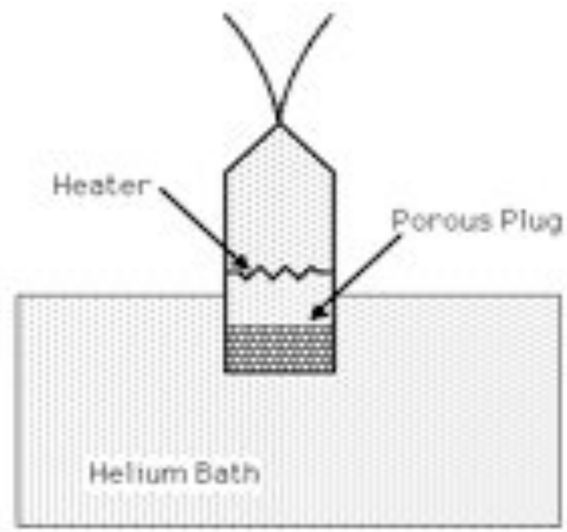


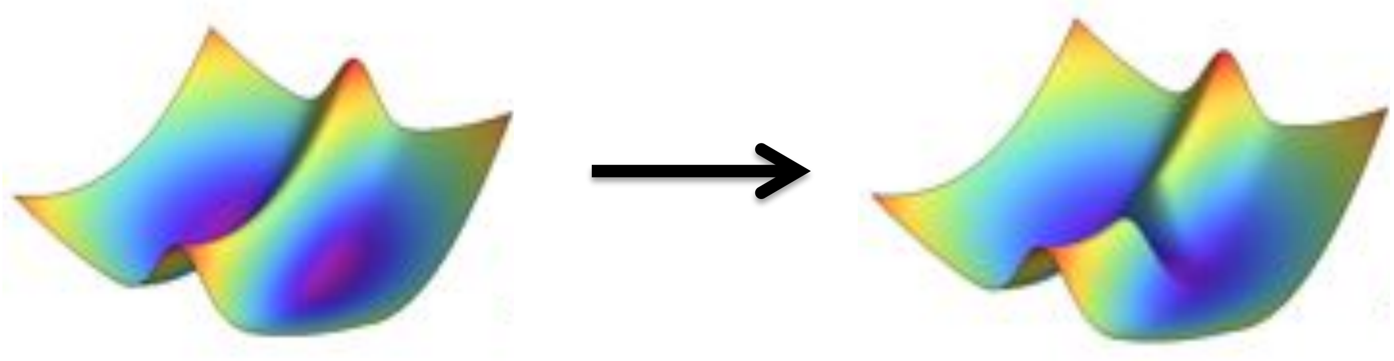


OTHER NON-EQUILIBRIUM

Thermo-mechanical effect

- Thermo-mechanical effect: Karpuik *et al.*, arXiv:1012.2225.
- Connect a hot and a cold BEC through a narrow channel.
- Observe superfluid flow from hot to cold.





Superfluid convection

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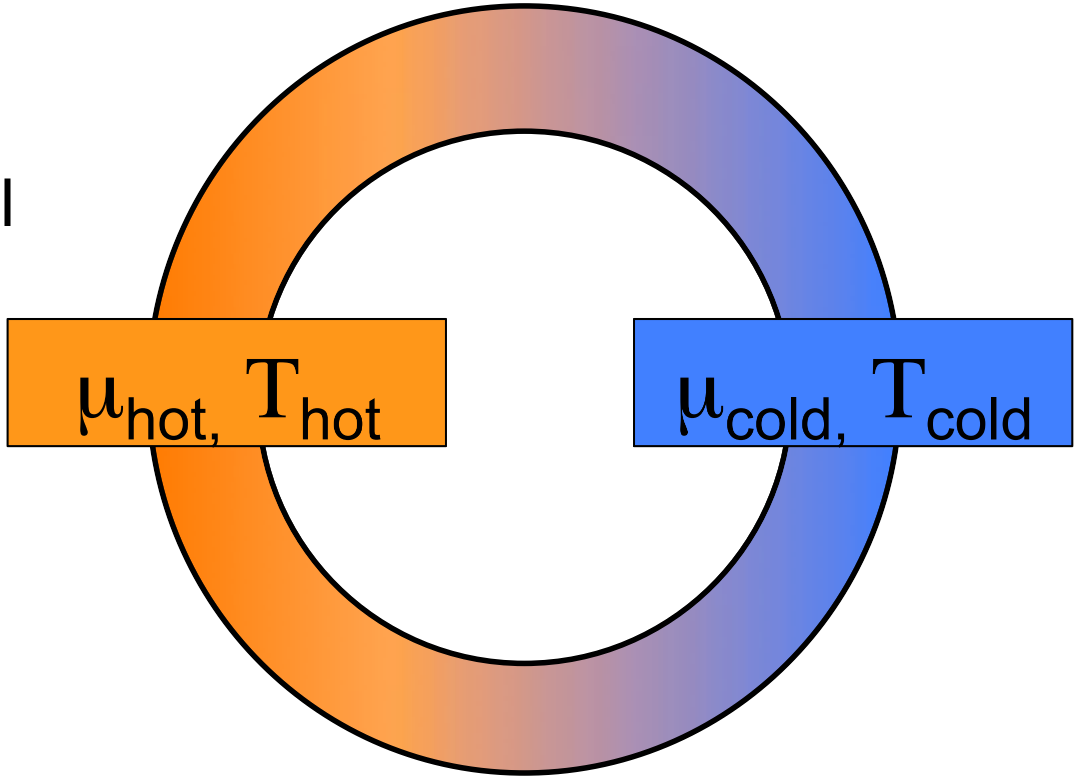
- Lukas Gilz, James R. Anglin, Phys. Rev. Lett. **107**, 090601 (2011)
- 1D Bose-Hubbard model
- Collisionless Bogoliubov approach using kinetic theory
- Observe superfluid convection from cold to hot reservoir.

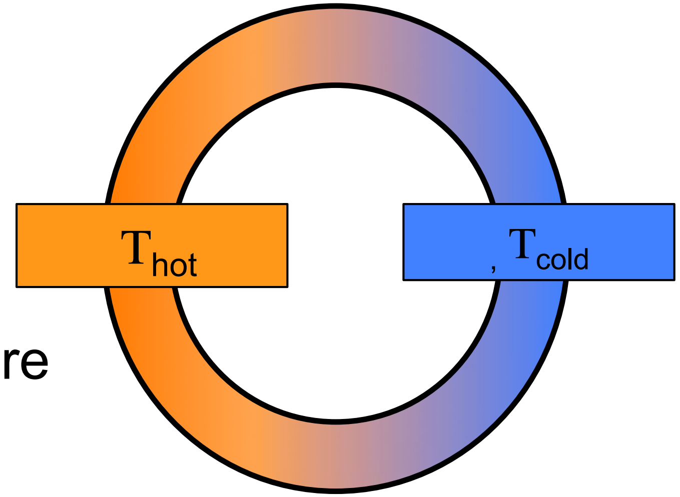


Non-equilibrium steady states

- Make SGPE reservoir spatially dependent: $\mu(x)$, $T(x)$, $G(x)$.
- Classical field must find steady state between two reservoirs.

$$d\psi(\mathbf{x}) = \mathcal{P}_C\left\{-\frac{i}{\hbar}L_{\rm GP}\psi(\mathbf{x})\,dt + \frac{G(\mathbf{x})}{k_B T(\mathbf{x})}(\mu(\mathbf{x}) - L_{\rm GP})\psi(\mathbf{x})\,dt + dW_G(\mathbf{x},t)\right\}$$

- Two species system
  - One species acts as thermal reservoir.
  - Second species is system.
- Opportunities for
  - Superfluid turbulence at finite temperature
  - Thermal counterflow
  - Taylor-Couette flow (rotating cylinders)
  - Non-equilibrium phase diagrams



Conclusions

- C-fields collectively describe
  - Microcanoncial projected GPE (PGPE)
  - Grand canonical stochastic projected GPE (SPGPE)
  - Truncated Wigner approximation (TWA)
- Equilibrium thermodynamics
  - Ergodicity -> can calculate equilibrium properties
- Non-equilibrium dynamics
  - Manipulate reservoir: both temporally and spatially.
  - Condensate formation
  - Kibble Zurek mechanism – classical and quantum.
  - Steady state flows
  - Polariton condensates – see Michel Wouters talk.

UQ Quantum Gases - theory

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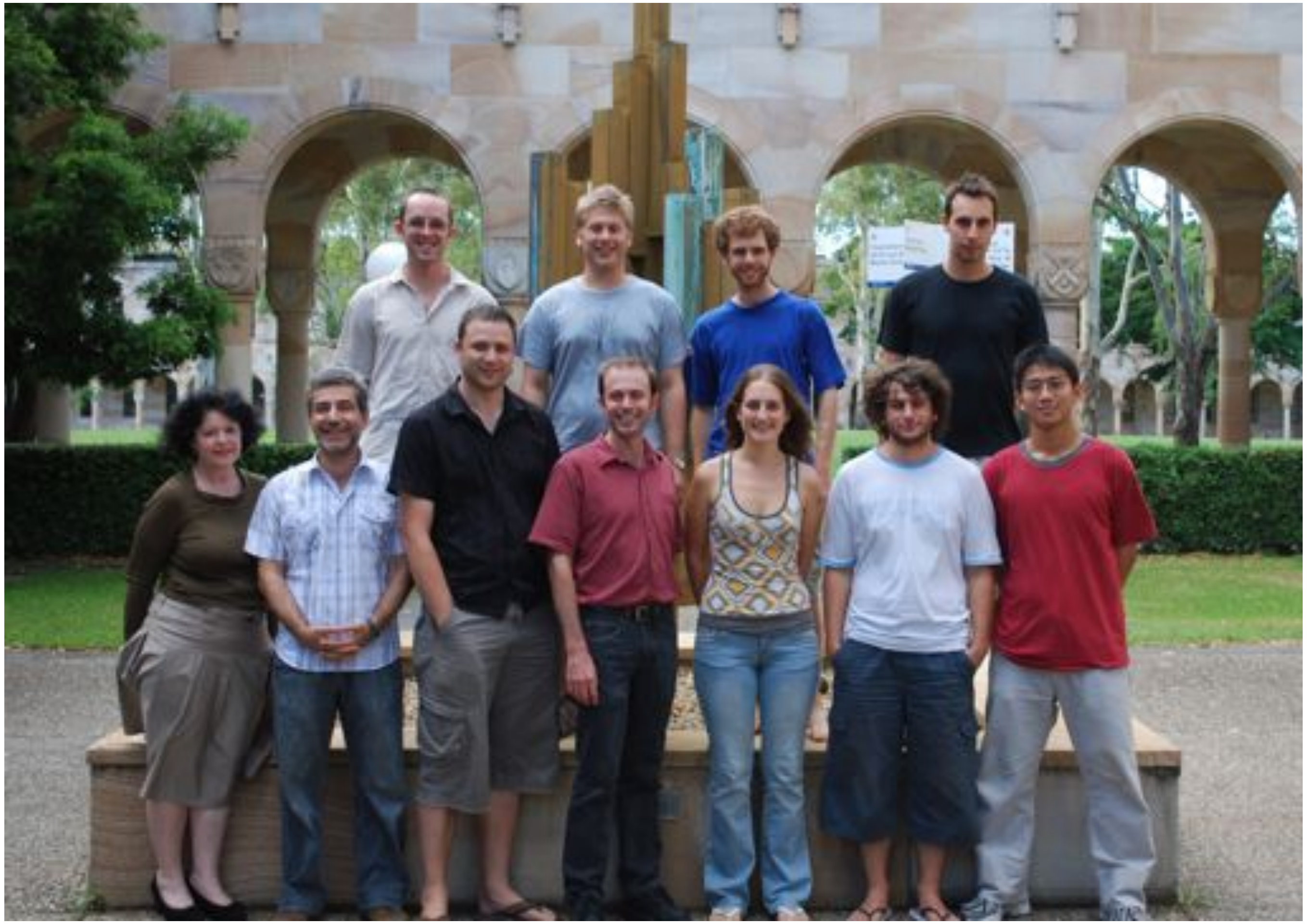

Looking for PhDs – theory and experiment