



# Off-shell dynamical approach for relativistic heavy-ion collisions

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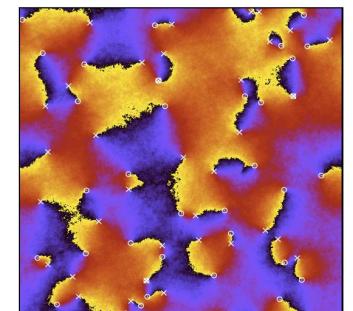
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Relaxation, Turbulence, and Non-Equilibrium Dynamics of Matter Fields

• RETUNE 2012

Heidelberg · Germany · 21 – 24 June 2012





# 2<sup>nd</sup> International Symposium on Non-equilibrium Dynamics & TURIC Network Workshop

25-30 June, 2012, Hersonissos, Crete, Greece

The 2nd International Symposium on Non-equilibrium Dynamics (NeD-2012) and  
the 3d Network I3-HP3 Workshop on Theory of Ultra-Relativistic heavy-Ion Collisions (TURIC-2012)  
will be held together from June 25 to 30, 2012, in Hersonissos, Crete, Greece

#### NeD topics:

- dynamical description of strongly interacting systems
- Kadanoff-Baym equations and solutions
- transport models for strongly interacting systems
- description of phase transitions
- viscous hydrodynamics

#### TURIC topics:

- properties of the quark-gluon plasma before hadronization and the phase transition towards the hadronic world
- transport properties of hard probes in the quark gluon plasma and their traces in final hadronic spectra
- microscopic study of initial thermalization

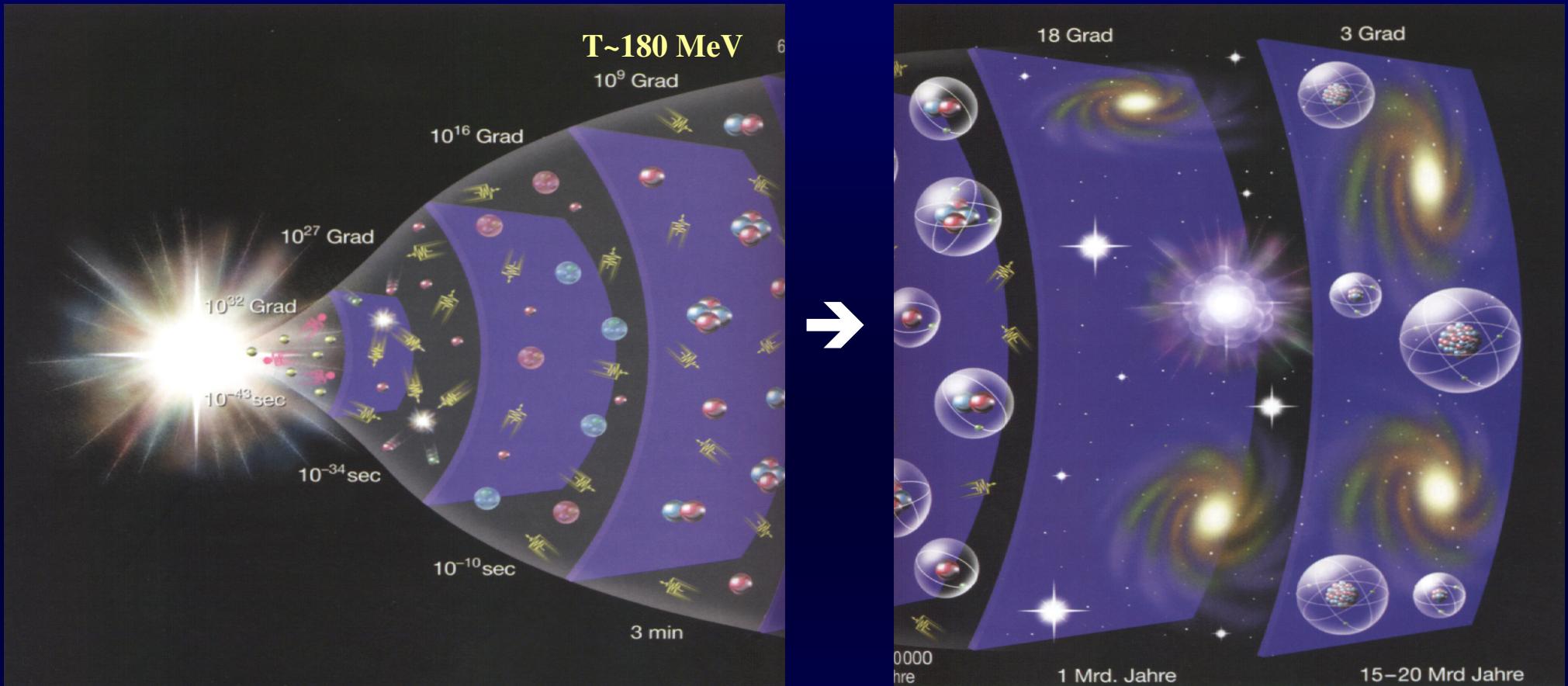
Home-page: <http://fias.uni-frankfurt.de/crete2012/>

Contact: [crete2012@fias.uni-frankfurt.de](mailto:crete2012@fias.uni-frankfurt.de)

**HIC** | FAIR  
Helmholtz International Center



# From Big Bang to Formation of the Universe



<i>time</i>	$10^{-3}$ sec	3 min	300000 years	15 Mrd years
quarks		nucleons		
gluons		deuterons		
photons		$\alpha$ -particles	atoms	our Universe



Can we go back in time ?



## ... back in time

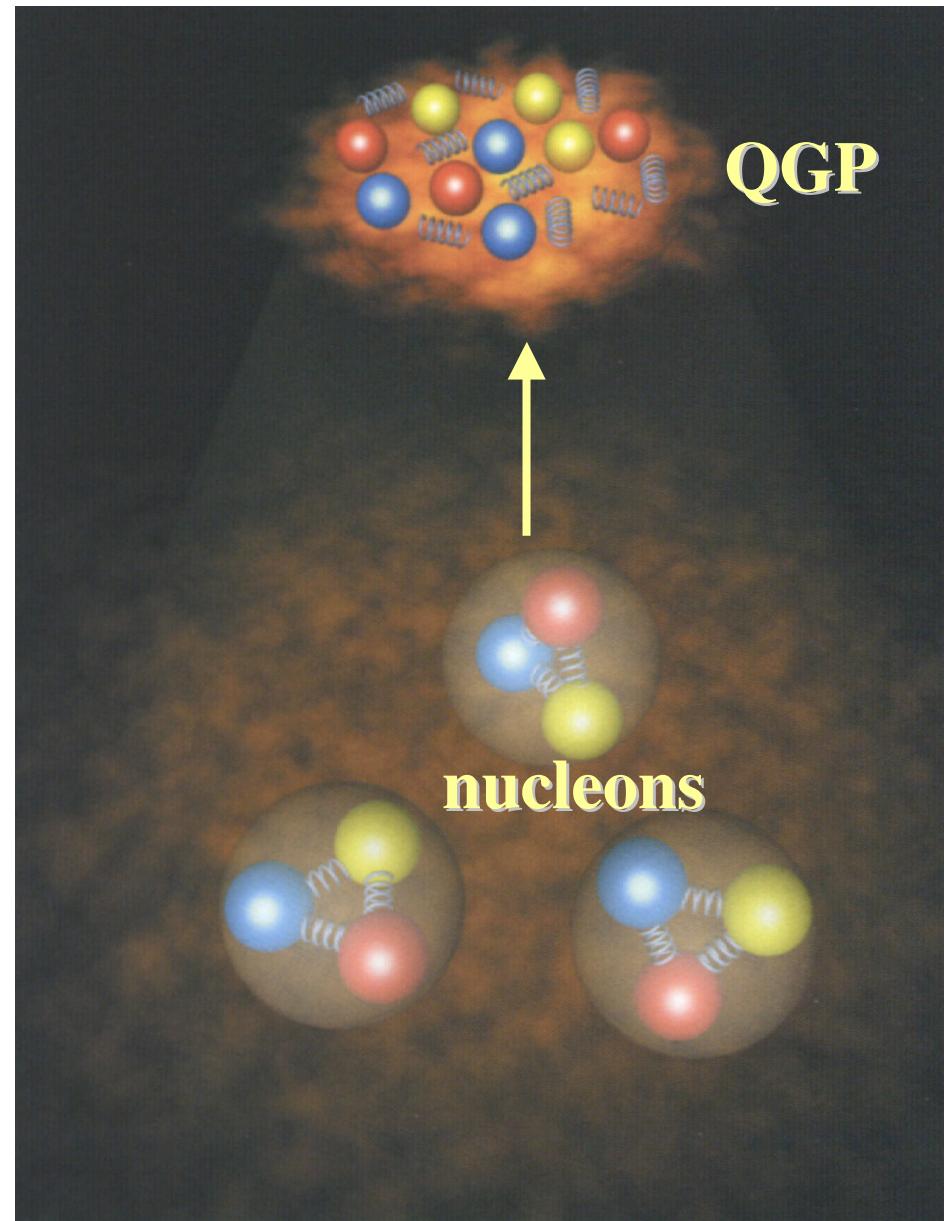
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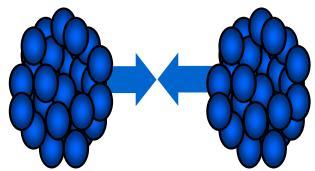
**,Re-create‘ the Big Bang conditions:**  
**matter at high temperature and pressure**

**such that**

**nucleons/mesons decouple to quarks and gluons --**  
**Quark-Gluon-Plasma**

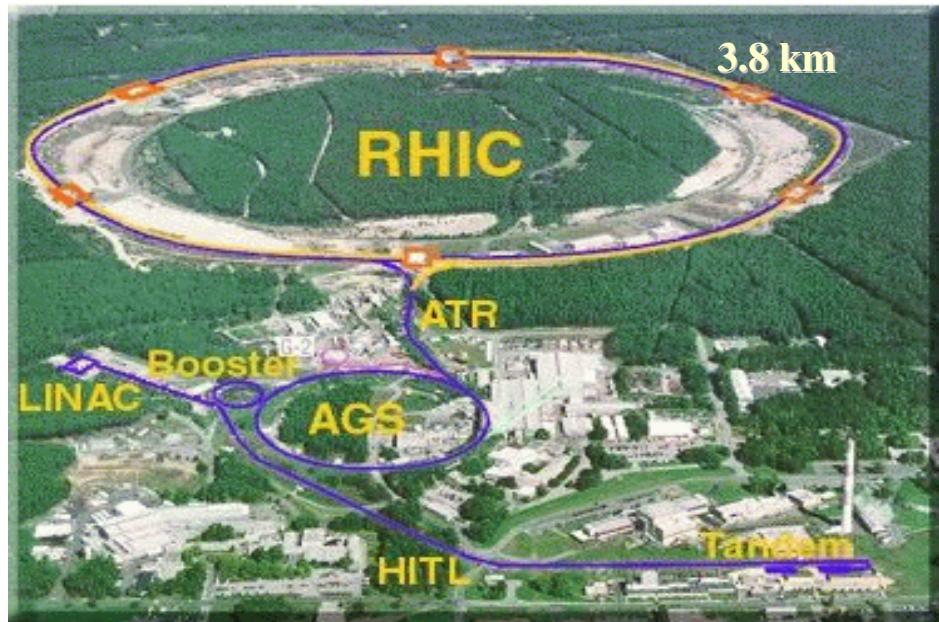
**,Little Bangs‘ in the Laboratory :**  
**Heavy-ion collisions at ultrarelativistic energies**





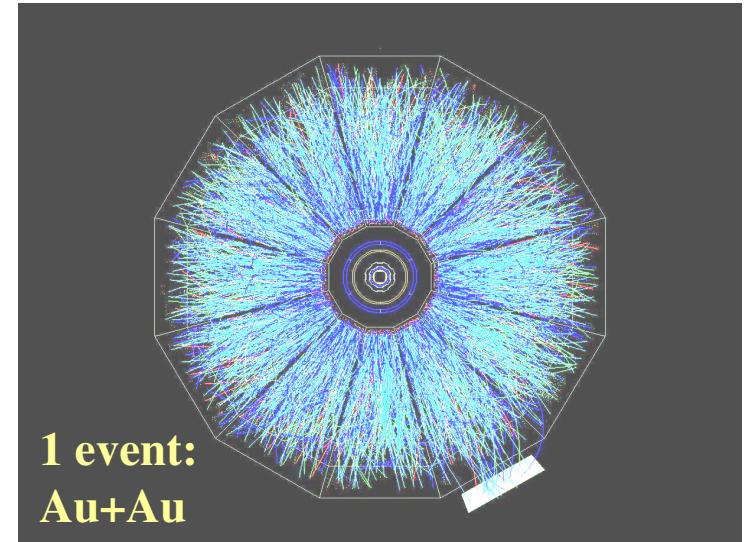
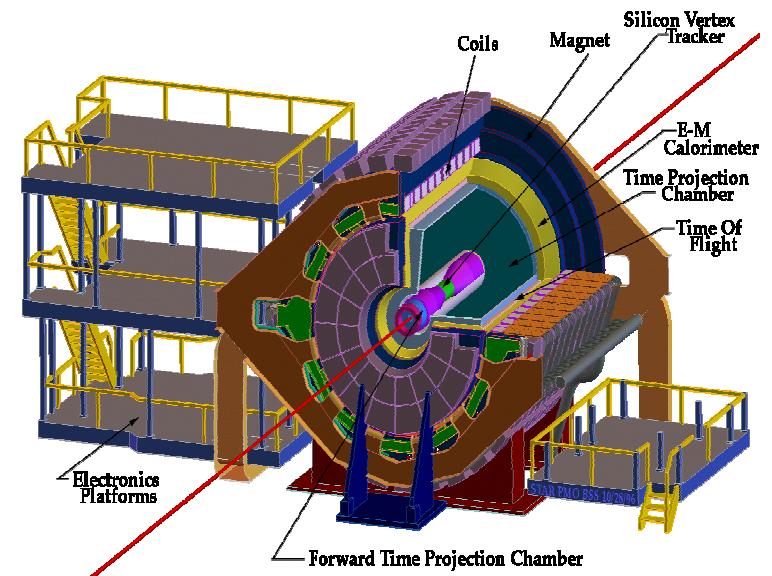
# Heavy-ion accelerators

- Super-Proton-Synchrotron – SPS - (CERN): Pb+Pb at 160 A GeV
- Relativistic-Heavy-Ion-Collider - RHIC - (Brookhaven): Au+Au at 21.3 A TeV



- Large Hadron Collider – LHC - (CERN): Pb+Pb at 574 A TeV
- Future facilities: FAIR (GSI), NICA (Dubna)

## STAR detector at RHIC

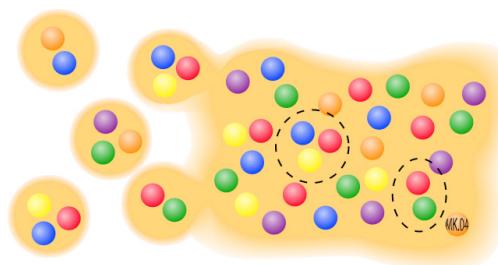


# The QGP in Lattice QCD

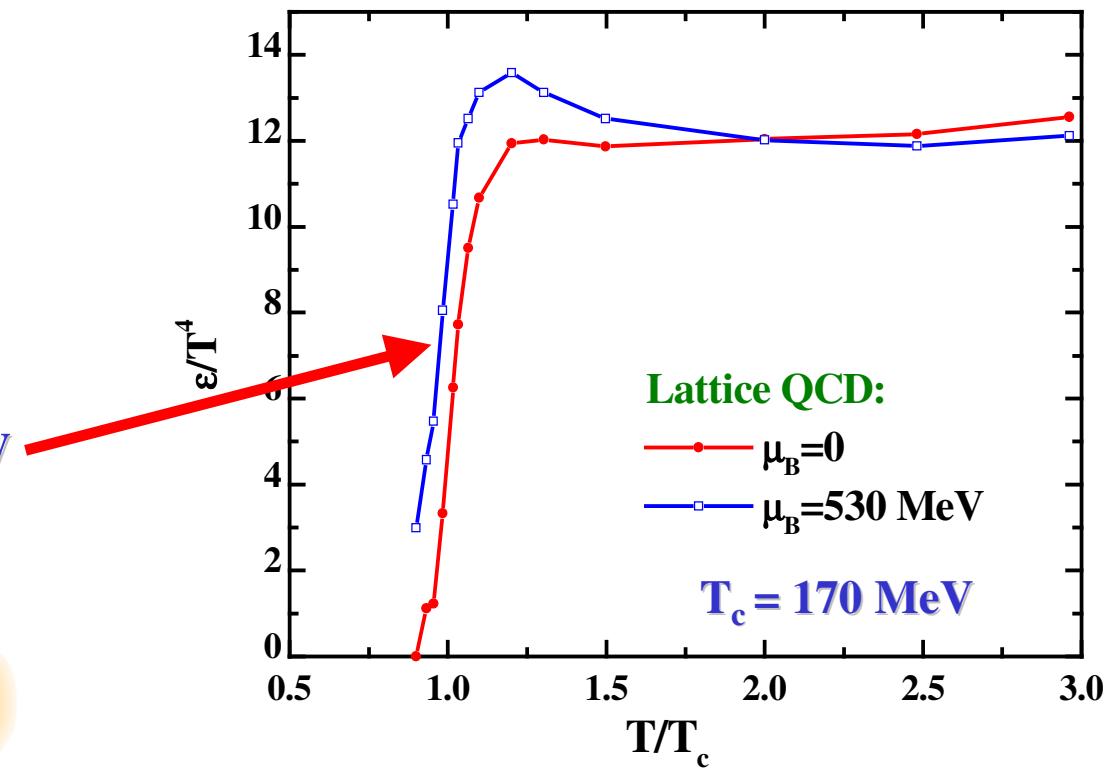
Quantum Chromo Dynamics :

predicts strong increase of  
the energy density  $\epsilon$  at critical  
temperature  $T_c \sim 170$  MeV

⇒ Possible phase transition from  
hadronic to partonic matter  
(quarks, gluons) at critical energy  
density  $\epsilon_c \sim 0.5$  GeV/fm<sup>3</sup>



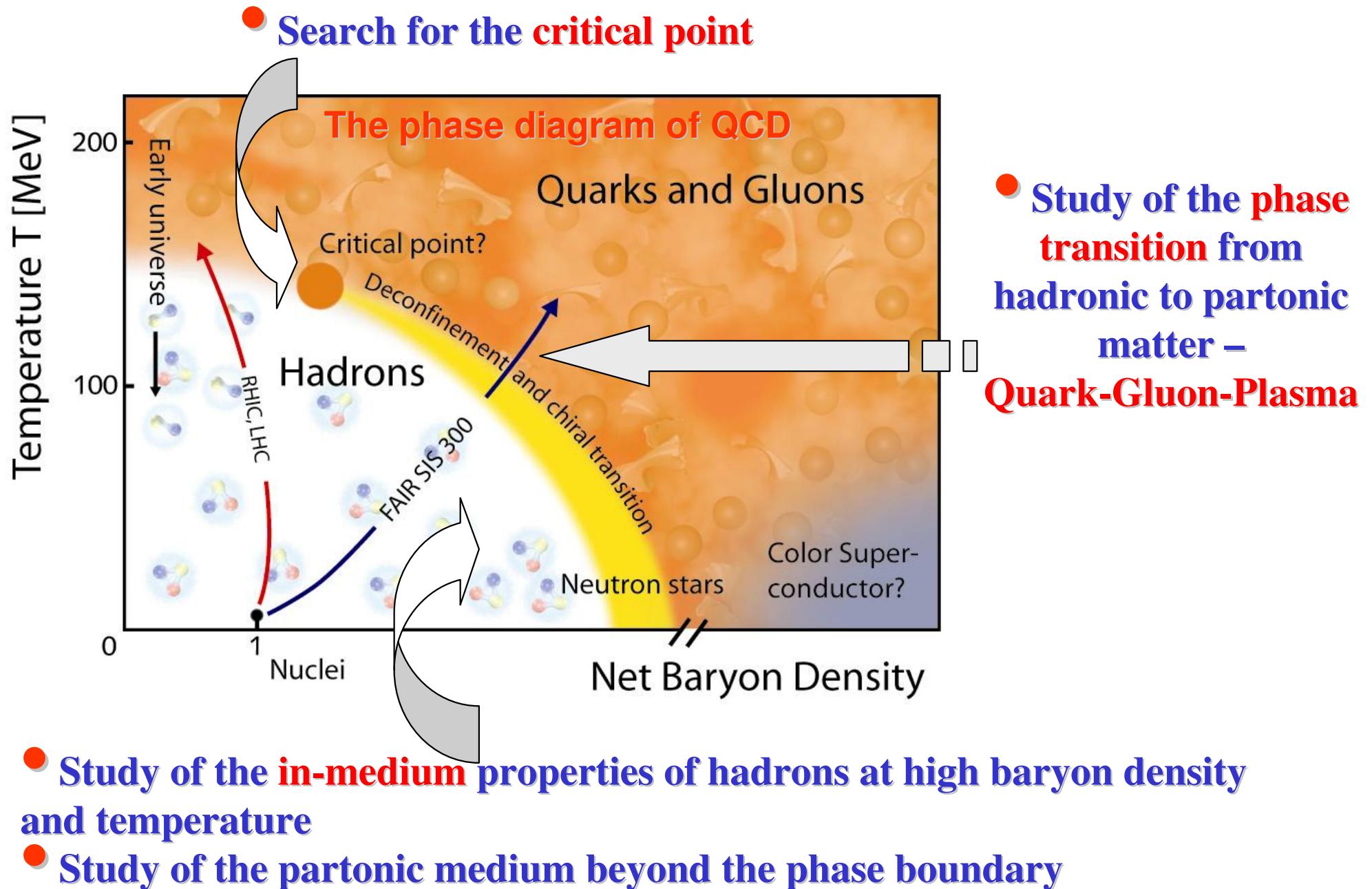
Lattice QCD:  
energy density versus temperature



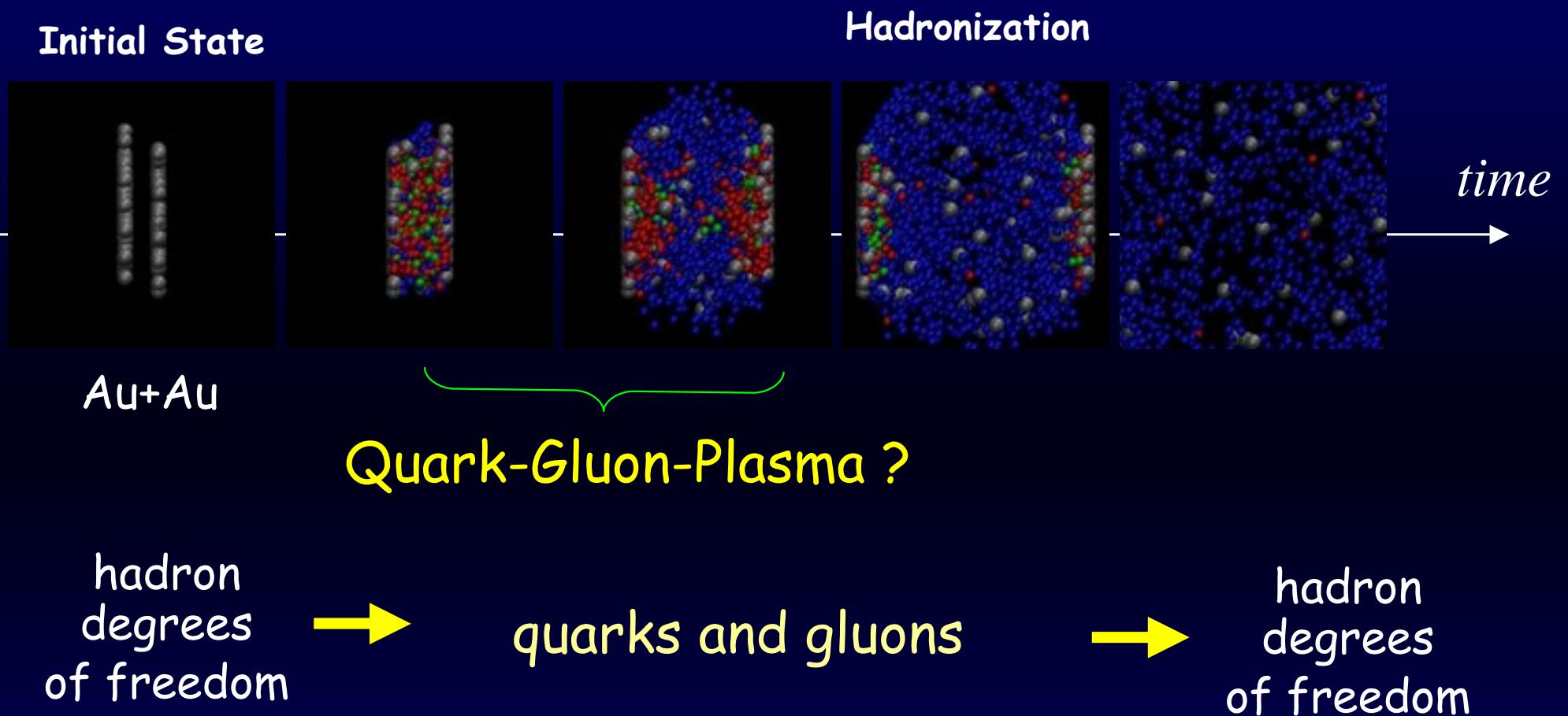
Z. Fodor et al., PLB 568 (2003) 73

Critical conditions -  $\epsilon_c \sim 0.5$  GeV/fm<sup>3</sup>,  $T_c \sim 170$  MeV - can be reached  
in heavy-ion experiments at bombarding energies  $> 5$  GeV/A

# The holy grail:



# ‘Little Bangs’ in the Laboratory



**How can we prove that an equilibrium QGP has been created in central heavy-ion collisions ?!**

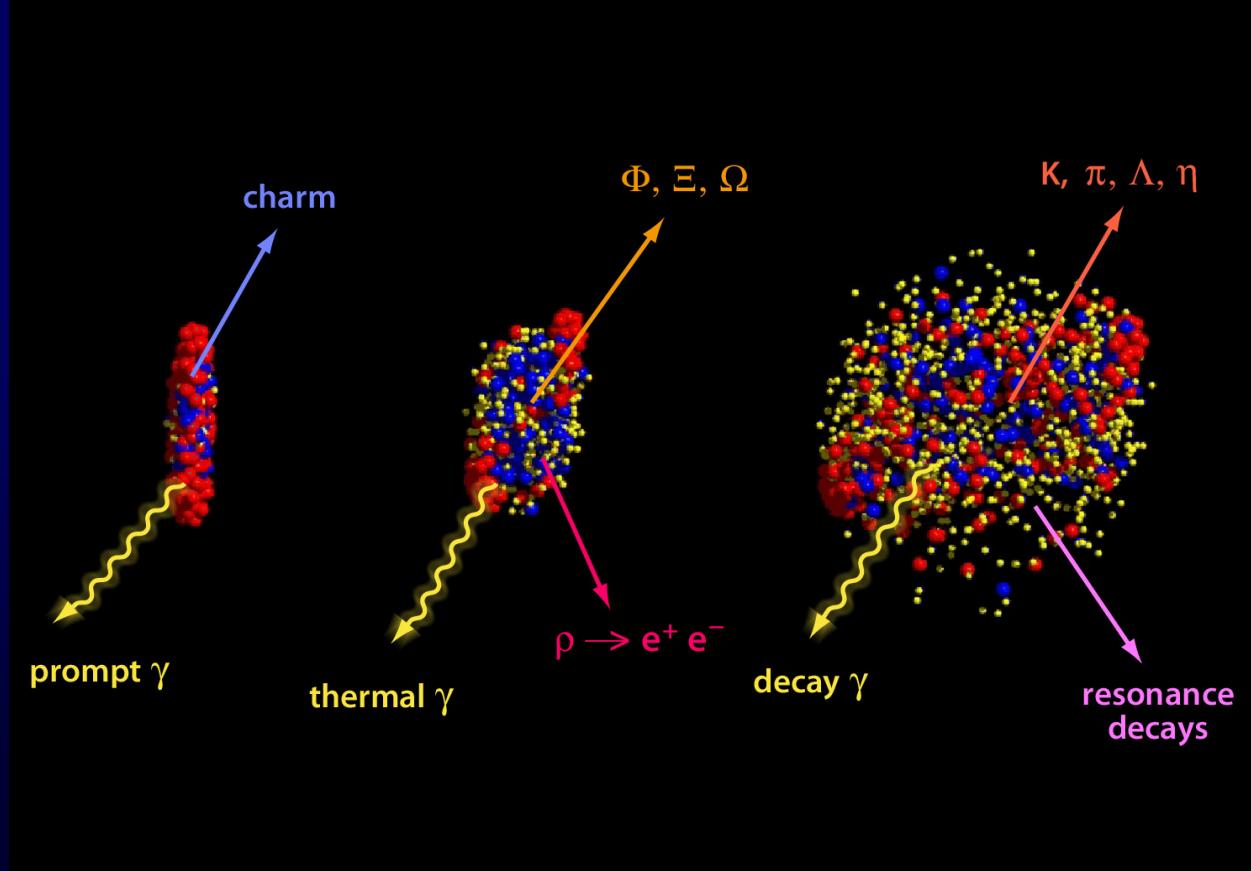
## Signals of the phase transition:

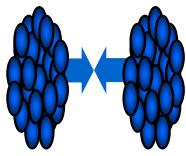
- Multi-strange particle enhancement in A+A
- Charm suppression
- Collective flow ( $v_1, v_2$ )
- Thermal dileptons
- Jet quenching and angular correlations
- High  $p_T$  suppression of hadrons
- Nonstatistical event by event fluctuations and correlations
- ...

**Experiment:** measures final hadrons and leptons

How to learn about physics from data?

Compare with theory!





# Basic models for heavy-ion collisions

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## ● Statistical models:

basic assumption: system is described by a (grand) canonical ensemble of non-interacting fermions and bosons in **thermal and chemical equilibrium**

[ -: no dynamics]

## ● Ideal hydrodynamical models:

basic assumption: conservation laws + equation of state; assumption of local thermal and chemical equilibrium

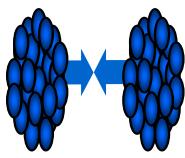
[ -: - simplified dynamics]

## ● Transport models:

based on transport theory of relativistic quantum many-body systems - off-shell Kadanoff-Baym equations for the Green-functions  $S_h^<(x,p)$  in phase-space representation. Actual solutions: Monte Carlo simulations with a large number of test-particles

[+ : full dynamics | -: very complicated]

→ Microscopic transport models provide a unique **dynamical description of nonequilibrium effects in heavy-ion collisions**



# Dynamics of heavy-ion collisions → complicated many-body problem!

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Appropriate way to solve the many-body problem including all quantum mechanical features →

**Kadanoff-Baym equations for Green functions  $S^<$**  (from 1962)

$$\hat{S}_{0x}^{-1} S_{xy}^< = \Sigma_{xz}^{ret} \odot S_{zy}^< + \Sigma_{xz}^< \odot S_{zy}^{adv}$$

$\hat{S}_{0x}^{-1}$  denotes the (negative) Klein-Gordon differential operator      e.g. for bosons     $\hat{S}_{0x}^{-1} = -(\partial_x^\mu \partial_\mu^x + M_0^2)$

" $\odot$ " implies an integration over the intermediate spacetime coordinates from  $-\infty$  to  $\infty$ .

**Greens functions  $S$  / self-energies  $\Sigma$  :**

**retarded (ret),  
advanced (adv)  
(anti-)causal (a,c )**

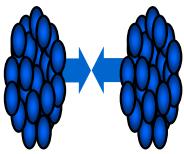
$$i S_{xy}^c = i S_{xy}^{++} = < T^c \{\Phi(x) \Phi^\dagger(y)\} >, \quad i S_{xy}^< = i S_{xy}^{+-} = \eta < \{\Phi^\dagger(y) \Phi(x)\} >, \\ i S_{xy}^> = i S_{xy}^{-+} = < \{\Phi(x) \Phi^\dagger(y)\} >, \quad i S_{xy}^a = i S_{xy}^{--} = < T^a \{\Phi(x) \Phi^\dagger(y)\} >.$$

$$S_{xy}^{ret} = S_{xy}^c - S_{xy}^< = S_{xy}^> - S_{xy}^a, \quad S_{xy}^{adv} = S_{xy}^c - S_{xy}^> = S_{xy}^< - S_{xy}^a \quad \eta = +1 \text{ for bosons and } \eta = -1 \text{ for fermions.} \\ T^c (T^a) \text{ represent the (anti-)time-ordering operators.}$$

➤ do Wigner transformation     $F_{XP} = \int d^4(x-y) e^{iP_\mu(x^\mu-y^\mu)} F_{xy}$

➤ consider only contribution up to first order in the gradients

= a standard approximation of kinetic theory which is justified if the gradients in the mean spacial coordinate  $X$  are small



## ‘On-shell’ transport models

**Basic concept of the ‘on-shell’ transport models (VUU, BUU, QMD etc.):**

- 1) **Transport equations** = first order gradient expansion of the Wigner transformed Kadanoff-Baym equations
- 2) **quasiparticle approximation:**  $A(x,p) = 2 \pi \delta(p^2 - M^2)$

- for each particle species  $i$  ( $i = N, R, Y, \pi, \rho, K, \dots$ ) the **phase-space density**  $f_i$  follows the **transport equations**

$$\left( \frac{\partial}{\partial t} + \left( \nabla_{\vec{p}} U \right) \nabla_{\vec{r}} - \left( \nabla_{\vec{r}} U \right) \nabla_{\vec{p}} \right) f_i(\vec{r}, \vec{p}, t) = I_{coll}(f_1, f_2, \dots, f_M)$$

- with **collision terms**  $I_{coll}$  describing elastic and inelastic **hadronic reactions**: baryon-baryon, meson-baryon, meson-meson, formation and decay of **baryonic and mesonic resonances**, **string formation and decay** (for inclusive particle production:  
 $BB \rightarrow X, mB \rightarrow X, X = \text{many particles}$ )
- with **propagation** of particles in self-generated **mean-field potential**  
 $U(p, \rho) \sim \text{Re}(\Sigma^{\text{ret}})/2p_0$
- Numerical realization – solution of classical equations of motion + **Monte-Carlo simulations** for test-particle interactions

# Study of in-medium effects within transport approaches

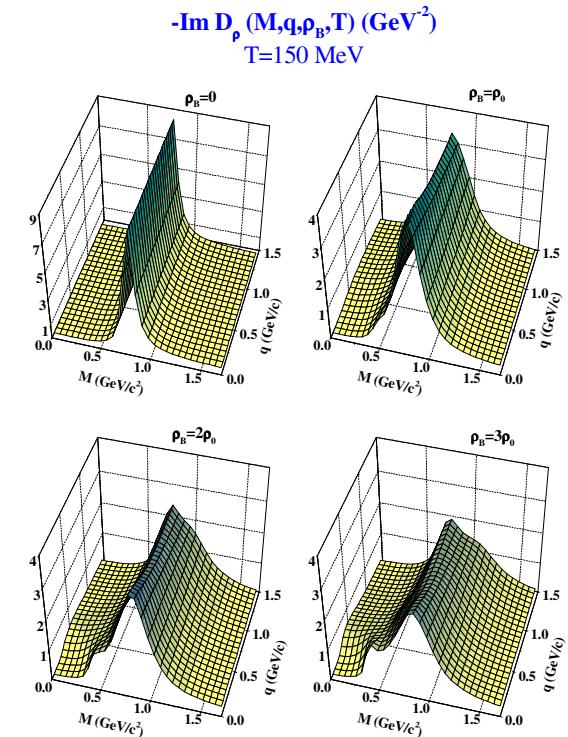
- **Semi-classical on-shell transport models** work very well in describing interactions of point-like particles and **narrow resonances** !

- **In-medium models** - chiral perturbation theory, chiral SU(3) model, coupled-channel G-matrix approach, chiral coupled-channel effective field theory etc. predict **changes of the particle properties** in the hot and dense medium, e.g. **strong broadening of the spectral functions**

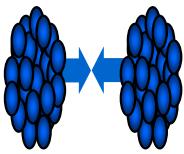
- **Problem : How to treat short-lived (broad) resonances in semi-classical transport models?**

Semi-classical approaches: **on-shell transport models** based on quasi-particle approximation  $A(X,P) = 2 \pi \delta(P^2 - M^2)$

R. Rapp:  $\rho$  meson spectral function



- Accounting for **in-medium effects** with medium-dependent spectral functions requires **off-shell transport models** beyond quasi-particle approximation !  
→ back to Kadanoff-Baym equations



# From Kadanoff-Baym equations to transport equations

After the first order gradient expansion of the Wigner transformed **Kadanoff-Baym equations** and separation into the real and imaginary parts one gets:

## Generalized transport equations:

$$\frac{\diamond \{ P^2 - M_0^2 - Re\Sigma_{XP}^{ret} \} \{ S_{XP}^< \}}{\text{drift term}} + \frac{\diamond \{ \Sigma_{XP}^< \} \{ ReS_{XP}^{ret} \}}{\text{Vlasov term}} = \frac{i}{2} \left[ \Sigma_{XP}^> S_{XP}^< - \Sigma_{XP}^< S_{XP}^> \right] \quad \text{backflow term} \quad \text{collision term} = \text{,loss' term} - \text{,gain' term}$$

**Backflow term** incorporates the **off-shell** behavior in the particle propagation

! vanishes in the quasiparticle limit  $A_{XP} = 2 \pi \delta(p^2 - M^2)$

→ **'on-shell'** transport models (VUU, BUU, QMD, IQMD, UrQMD etc.)

**Greens function  $S^<$**  characterizes the **number of particles (N)** and their properties

(A – spectral function ):  $iS_{XP}^< = A_{XP} N_{XP}$

The imaginary part of the retarded propagator is given by normalized **spectral function**:

$$A_{XP} = i [ S_{XP}^{ret} - S_{XP}^{adv} ] = -2 Im S_{XP}^{ret}, \quad \int \frac{dP_0^2}{4\pi} A_{XP} = 1$$

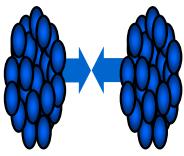
For bosons in first order in gradient expansion:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

$\Gamma_{XP}$  – **width of spectral function** = reaction rate of  
particle (at phase-space position XP)

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond \{ F_1 \} \{ F_2 \} := \frac{1}{2} \left( \frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$



# General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

Employ **testparticle Ansatz** for the real valued quantity  $i S_{XP}^<$  -

$$F_{XP} = A_{XP} N_{XP} = i S_{XP}^< \sim \sum_{i=1}^N \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

**General testparticle off-shell equations of motion:**

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ 2\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma_{(i)}^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \vec{\nabla}_{X_i} Re\Sigma_i^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \frac{\partial Re\Sigma_{(i)}^{ret}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with  $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[ \frac{\partial}{\partial \epsilon_i} Re\Sigma_{(i)}^{ret} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma_{(i)}^{ret}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$



# The baseline concepts of HSD

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## HSD – Hadron-String-Dynamics transport approach:

- for each particle species  $i$  ( $i = N, R, Y, \pi, \rho, K, \dots$ ) the phase-space density  $f_i$  follows the generalized transport equations

with collision terms  $I_{\text{coll}}$  describing:

- elastic and inelastic hadronic reactions:

baryon-baryon, meson-baryon, meson-meson

$BB \leftrightarrow B'B'$ ,  $BB \leftrightarrow B'B'm$

$mB \leftrightarrow m'B'$ ,  $mB \leftrightarrow B'$

- formation and decay of

baryonic and mesonic resonances

Baryons:

and strings - excited color singlet states ( $qq - q$ ) or ( $q - q\bar{q}$ ) -

$B=(p, n, \Delta(1232),$

(for inclusive particle production:  $BB \rightarrow X$ ,  $mB \rightarrow X$ ,  $X = \text{many particles}$ )

$N(1440), N(1535), \dots$

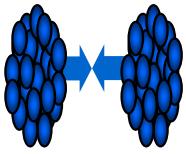
Mesons:

$m=(\pi, \eta, \rho, \omega, \phi, \dots)$

- implementation of detailed balance on the level of  $1 \leftrightarrow 2$

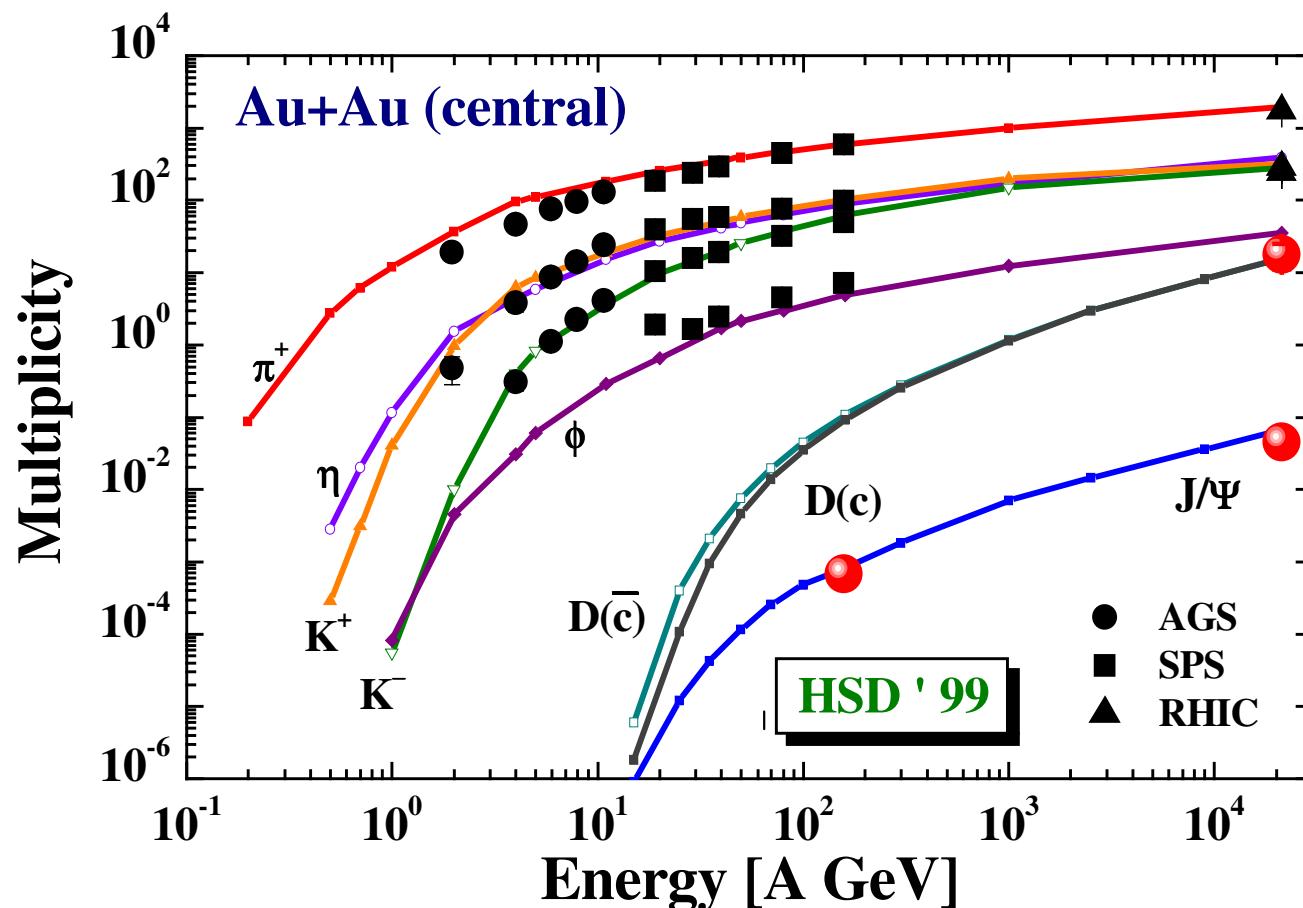
and  $2 \leftrightarrow 2$  reactions (+  $2 \leftrightarrow n$  multi-particle reactions in HSD !)

- off-shell dynamics for short-lived states



## HSD – a microscopic model for heavy-ion reactions

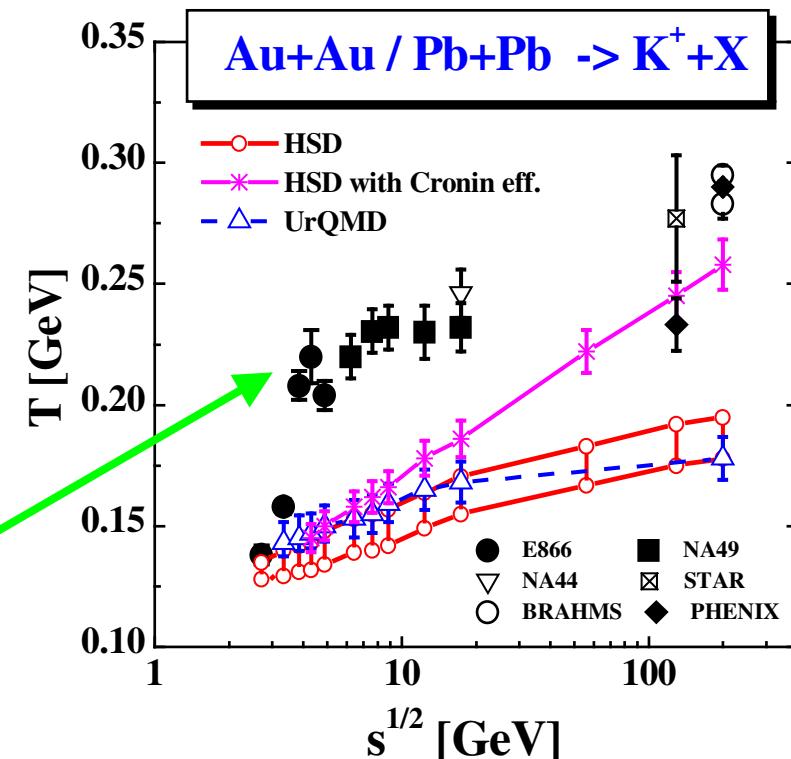
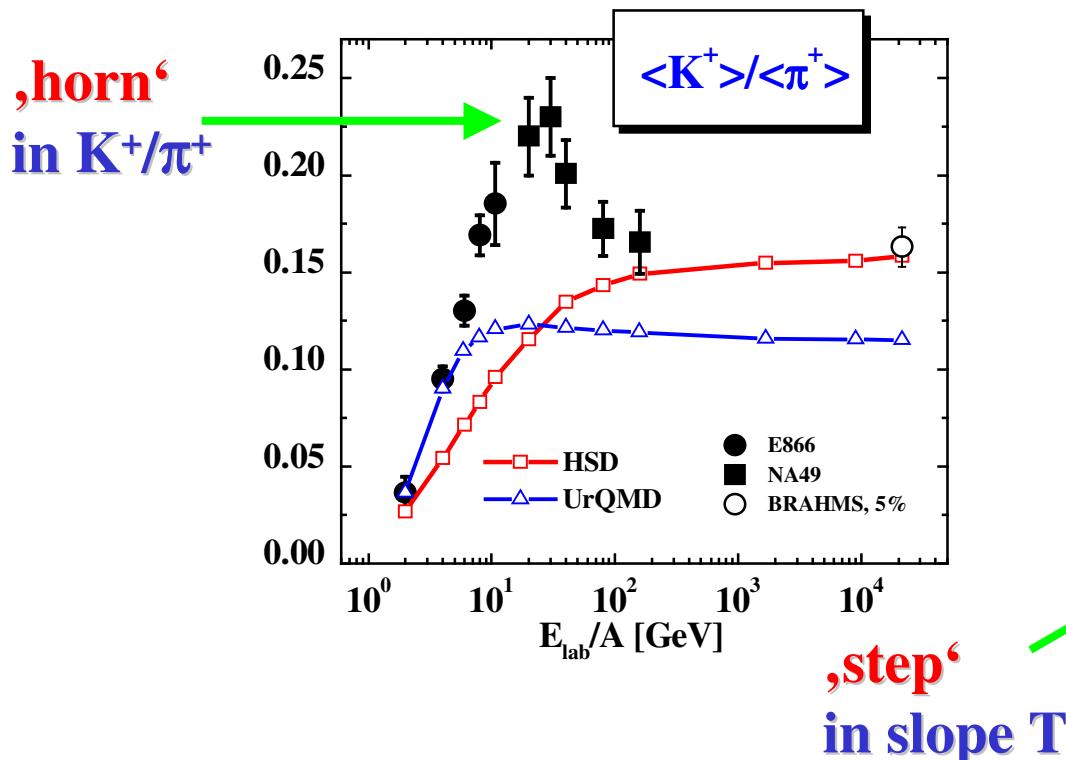
- very good description of particle production in pp, pA, AA reactions
- unique description of nuclear dynamics from low ( $\sim 100$  MeV) to ultrarelativistic ( $> 20$  TeV) energies



HSD predictions from 1999; data from the new millenium

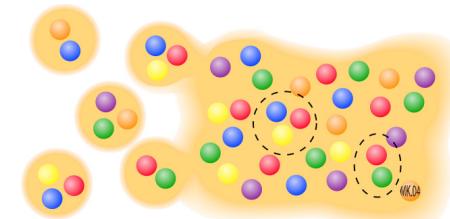
# Hadron-string transport models (HSD, UrQMD) versus observables

## ● Strangeness signals of QGP



Exp. data are not reproduced in terms of the hadron-string picture  
=> evidence for nonhadronic degrees of freedom

# Goal: microscopic transport description of the partonic and hadronic phase



## Problems:

- How to model a **QGP phase** in line with lQCD data?
- How to solve the **hadronization problem**?

## Ways to go:

### pQCD based models:

- QGP phase: pQCD cascade
  - hadronization: quark coalescence
- AMPT, HIJING, BAMPS

### ‘Hybrid’ models:

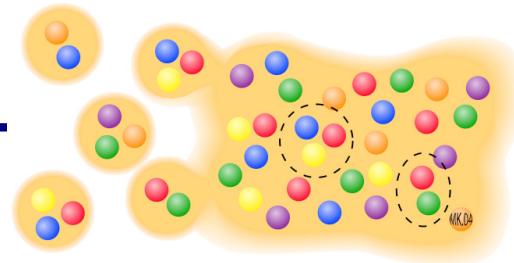
- QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner
- hadron-string transport model

→ Hybrid-UrQMD

- **microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons**

→ PHSD

# From hadrons to partons



In order to study the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma** – we need a consistent non-equilibrium (transport) model with

- explicit parton-parton interactions (i.e. between quarks and gluons) beyond strings!
- explicit phase transition from hadronic to partonic degrees of freedom
- lQCD EoS for partonic phase

**Transport theory:** off-shell Kadanoff-Baym equations for the Green-functions  $S_h^<(x,p)$  in phase-space representation for the partonic and hadronic phase



**Parton-Hadron-String-Dynamics (PHSD)**



QGP phase described by

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;  
NPA831 (2009) 215;  
W. Cassing, EPJ ST 168 (2009) 3

**Dynamical QuasiParticle Model (DQPM)**

A. Peshier, W. Cassing, PRL 94 (2005) 172301;  
Cassing, NPA 791 (2007) 365; NPA 793 (2007)

# The Dynamical QuasiParticle Model (DQPM)

**Basic idea:** Interacting quasiparticles

- massive quarks and gluons ( $g, q, \bar{q}$ ) with spectral functions :

$$\rho(\omega) = \frac{\gamma}{E} \left( \frac{1}{(\omega - E)^2 + \gamma^2} - \frac{1}{(\omega + E)^2 + \gamma^2} \right)$$

$$E^2 = p^2 + M^2 - \gamma^2$$

## ■ quarks

**mass:**  $m^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left( T^2 + \frac{\mu_q^2}{\pi^2} \right)$

**width:**  $\gamma_q(T) = \frac{N_c^2 - 1}{2N_c} \frac{g^2 T}{4\pi} \ln \frac{c}{g^2}$

**running coupling:**  $\alpha_s(T) = g^2(T)/(4\pi)$

$$g^2(T/T_c) = \frac{48\pi^2}{(11N_c - 2N_f) \ln(\lambda^2(T/T_c - T_s/T_c)^2)}$$

## ■ gluons:

$$M^2(T) = \frac{g^2}{6} \left( (N_c + \frac{1}{2}N_f) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

$$\gamma_g(T) = N_c \frac{g^2 T}{4\pi} \ln \frac{c}{g^2}$$

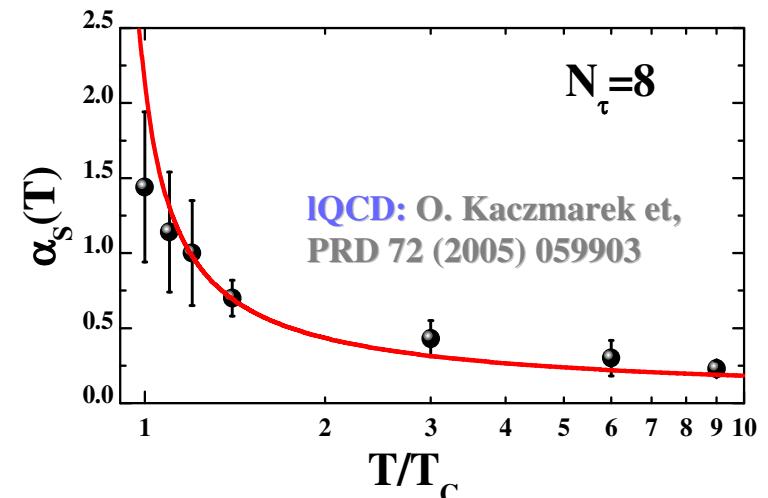
A. Peshier, PRD 70 (2004) 034016

$$N_c = 3, N_f = 3$$

➤ fit to lattice (lQCD) results (e.g. entropy density)

with 3 parameters:  $T_s/T_c = 0.46$ ;  $c = 28.8$ ;  $\lambda = 2.42$

→ quasiparticle properties (mass, width)



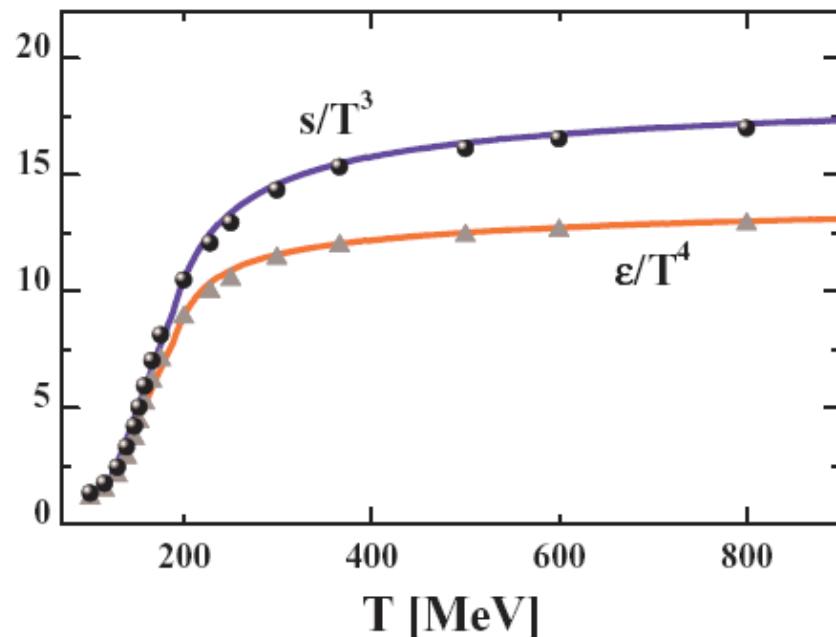
DQPM: Peshier, Cassing, PRL 94 (2005) 172301;  
Cassing, NPA 791 (2007) 365; NPA 793 (2007)

# DQPM thermodynamics ( $N_f=3$ ) and lQCD

**entropy**     $s = \frac{\partial P}{dT}$     **→ pressure P**

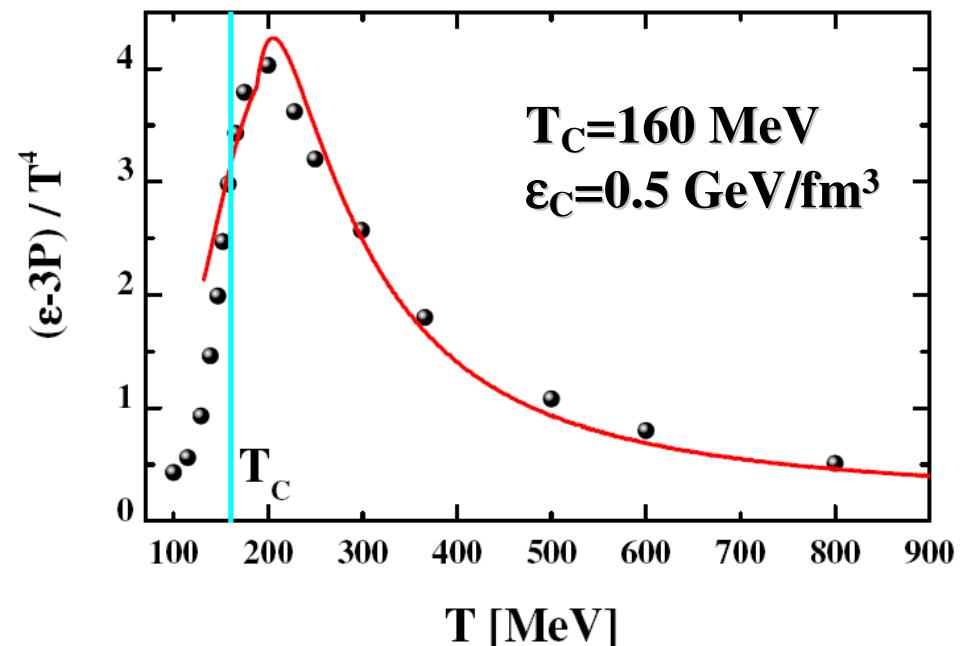
**energy density:**     $\epsilon = Ts - P$

**lQCD:** Wuppertal-Budapest group  
Y. Aoki et al., JHEP 0906 (2009) 088.



**interaction measure:**

$$W(T) := \epsilon(T) - 3P(T) = Ts - 4P$$

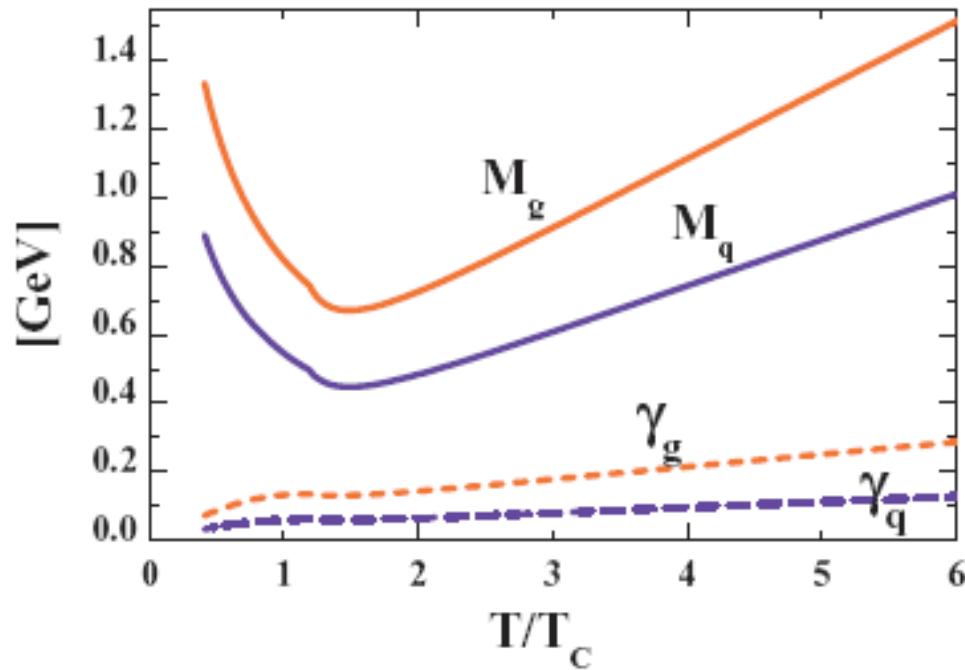


**DQPM gives a good description of lQCD results !**

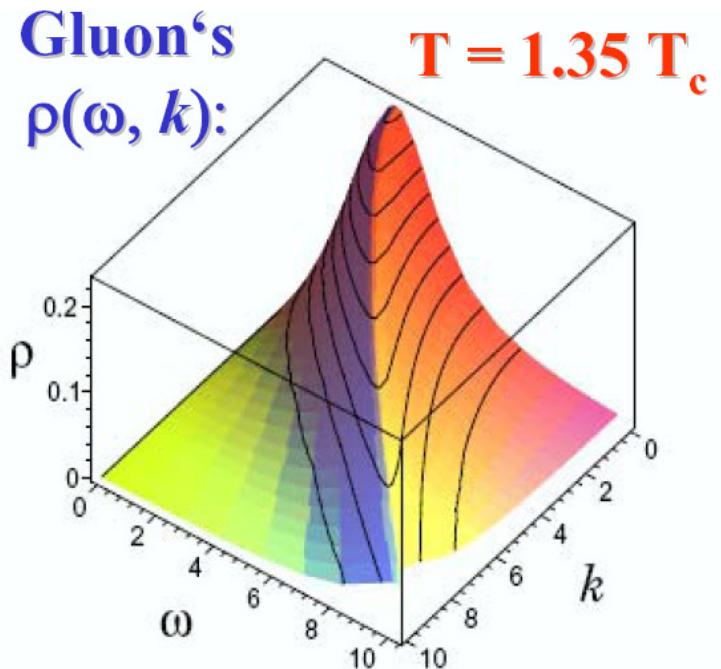
# The Dynamical QuasiParticle Model (DQPM)

→ Quasiparticle properties:

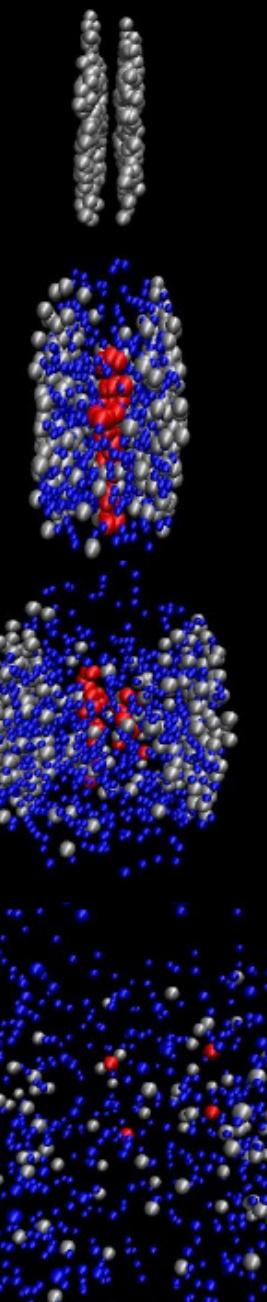
■ large width and mass for gluons and quarks



→ Broad spectral function :



- DQPM matches well lattice QCD
- DQPM provides mean-fields (1PI) for gluons and quarks as well as effective 2-body interactions (2PI)
- DQPM gives transition rates for the formation of hadrons → PHSD



# PHSD - basic concept

**Initial A+A collisions – HSD: string formation and decay to pre-hadrons**

**Fragmentation of pre-hadrons into quarks:** using the quark spectral functions from the **Dynamical QuasiParticle Model (DQPM)** - approximation to QCD

**Partonic phase:** quarks and gluons (= ,dynamical quasiparticles') with off-shell spectral functions (width, mass) defined by the DQPM

**elastic and inelastic parton-parton interactions:**

using the effective cross sections from the DQPM

✓ q + qbar (flavor neutral)  $\leftrightarrow$  gluon (colored)

✓ gluon + gluon  $\leftrightarrow$  gluon (possible due to large spectral width)

✓ q + qbar (color neutral)  $\leftrightarrow$  hadron resonances

**self-generated mean-field potential for quarks and gluons**

**QGP phase:**

$$\varepsilon > \varepsilon_{\text{critical}}$$

**Hadronization:** based on DQPM - massive, off-shell quarks and gluons with broad spectral functions hadronize to off-shell mesons and baryons:

gluons  $\rightarrow$  q + qbar; q + qbar  $\rightarrow$  meson (or string);

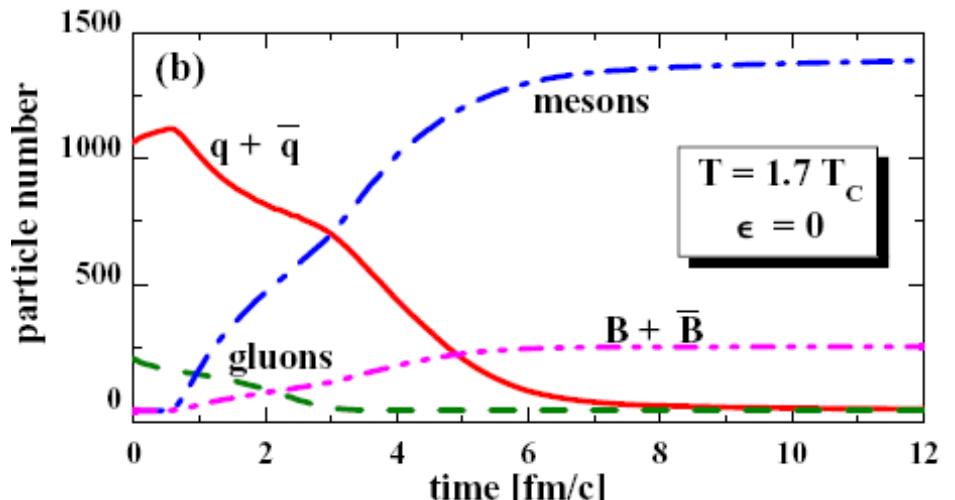
q + q + q  $\rightarrow$  baryon (or string) (strings act as ,doorway states' for hadrons)

**Hadronic phase:** hadron-string interactions – off-shell HSD

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;  
NPA831 (2009) 215; EPJ ST 168 (2009) 3; NPA856 (2011) 162.

# PHSD: hadronization of a partonic fireball

E.g. time evolution of the partonic fireball at initial temperature  $1.7 T_c$  at  $\mu_q=0$



## Consequences:

- Hadronization:  $q+q_{\bar{q}}$  or  $3q$  or  $3q_{\bar{q}}$  fuse to color neutral hadrons (or strings) which subsequently decay into hadrons in a microcanonical fashion, i.e. obeying all conservation laws (i.e. 4-momentum conservation, flavor current conservation) in each event!
- Hadronization yields an increase in total entropy  $S$  (i.e. more hadrons in the final state than initial partons ) and not a decrease as in the simple recombination models!
- Off-shell parton transport roughly leads a hydrodynamic evolution of the partonic system

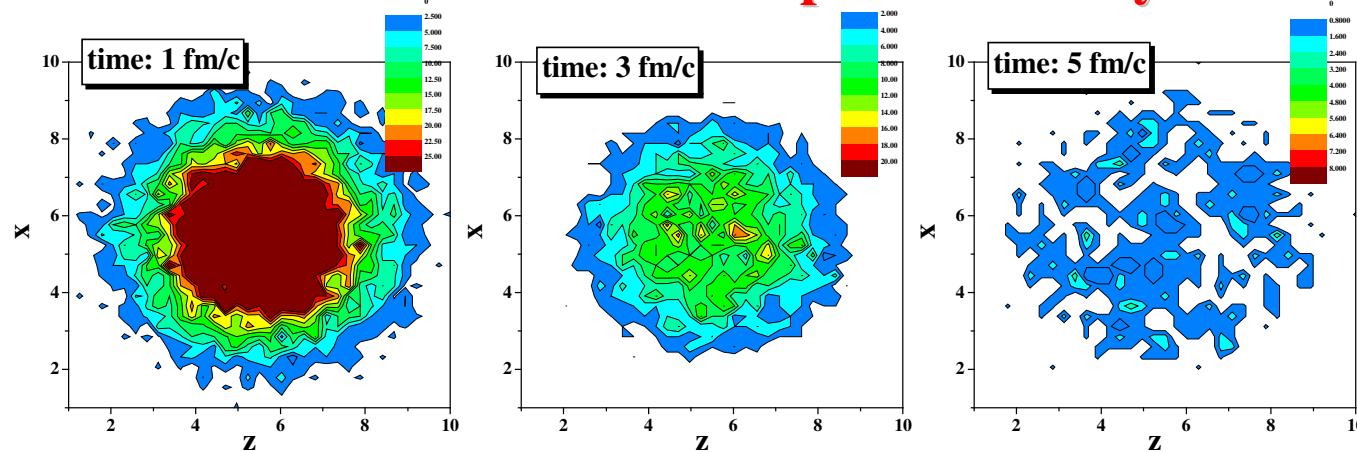
W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;

NPA831 (2009) 215;

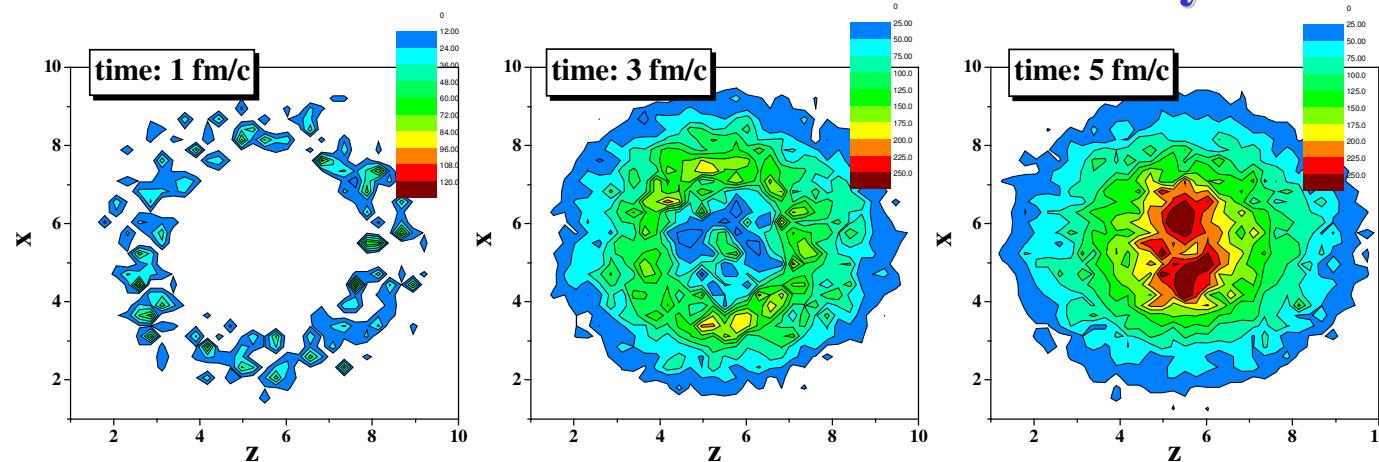
W. Cassing, EPJ ST 168 (2009) 3

# PHSD: Expanding fireball

Time-evolution of parton density



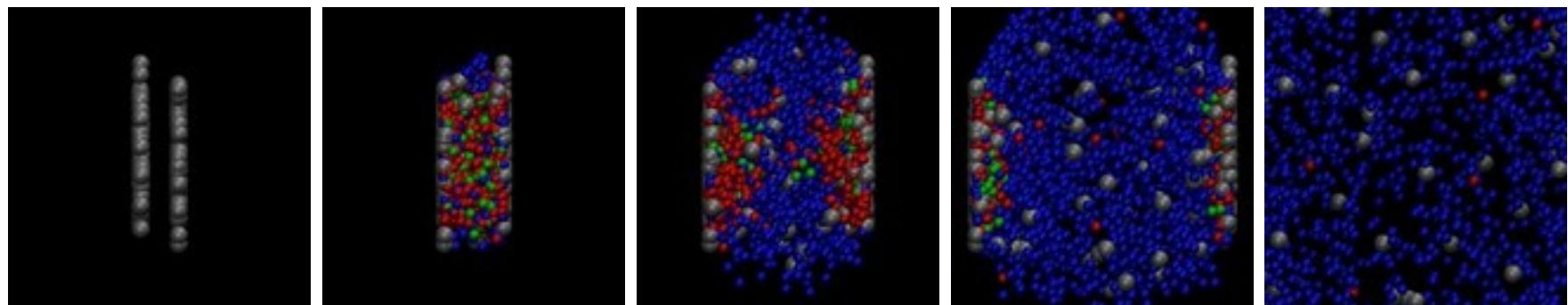
Time-evolution of hadron density



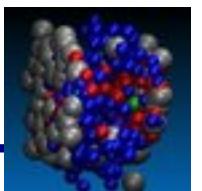
Expanding grid:  $\Delta z(t) = \Delta z_0(1+a t)$  !

PHSD: spacial phase ,co-existence‘ of partons and hadrons, but NO interactions between hadrons and partons (since it is a cross-over)

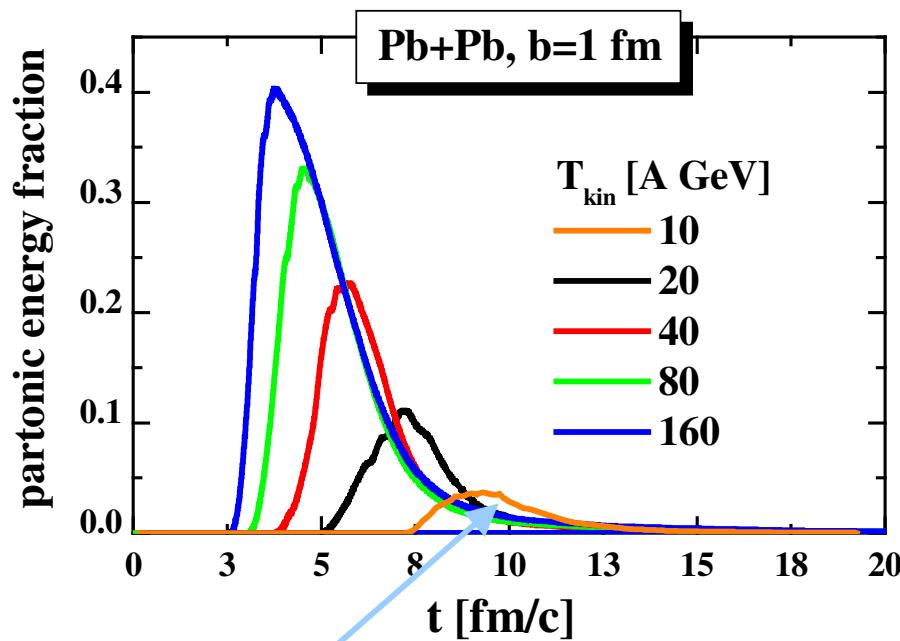
# **Bulk properties: rapidity, $m_T$ -distributions, multi-strange particle enhancement in Au+Au**



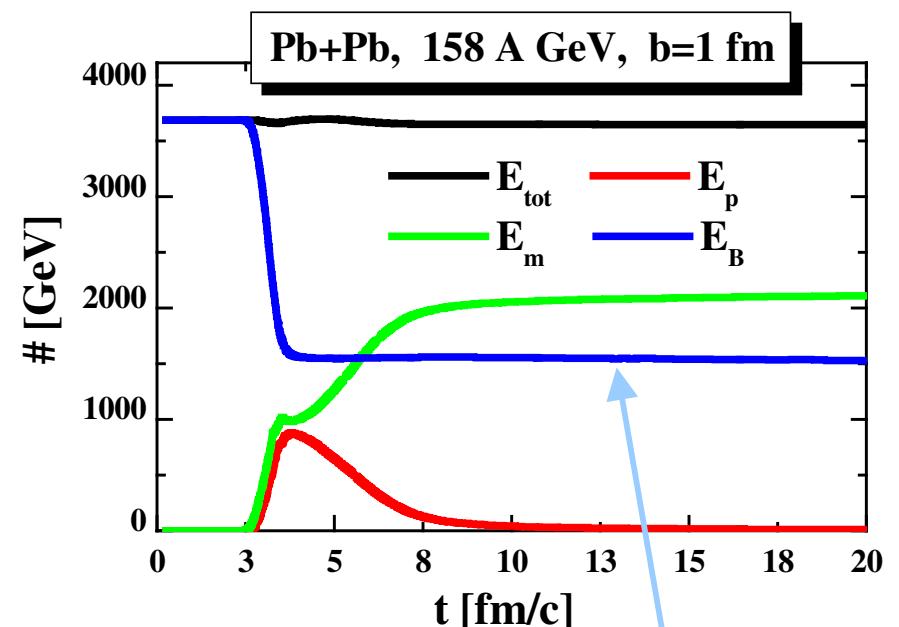
# Application to nucleus-nucleus collisions



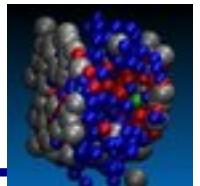
partonic energy fraction vs energy



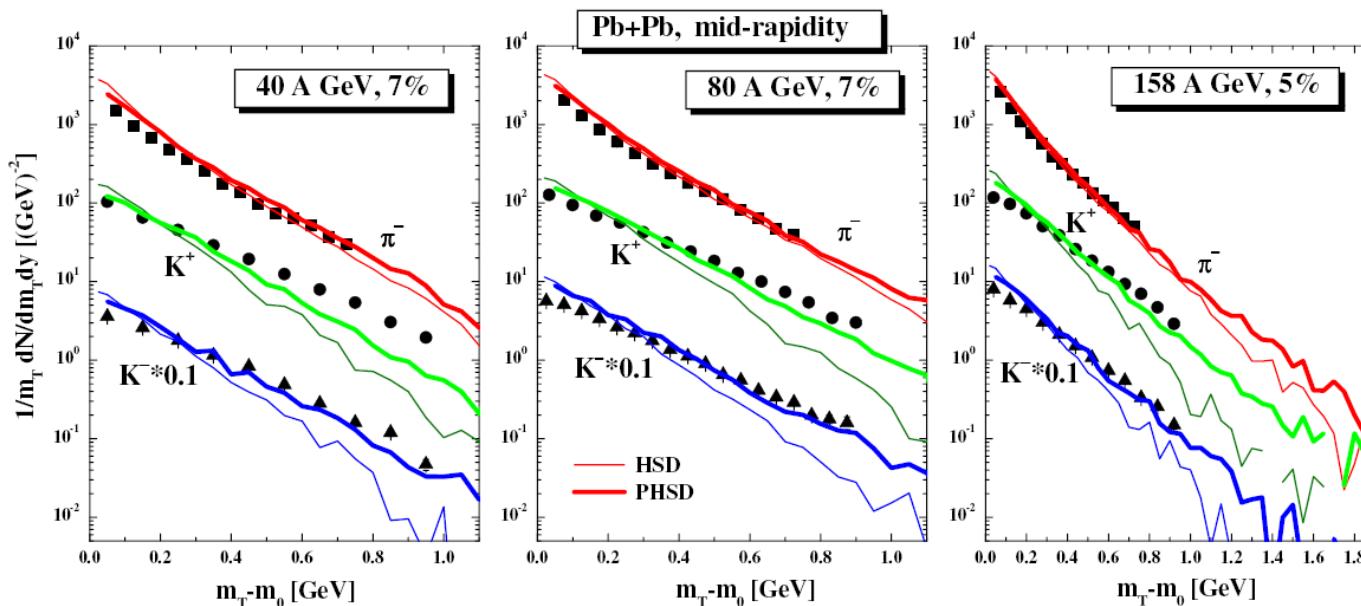
energy balance



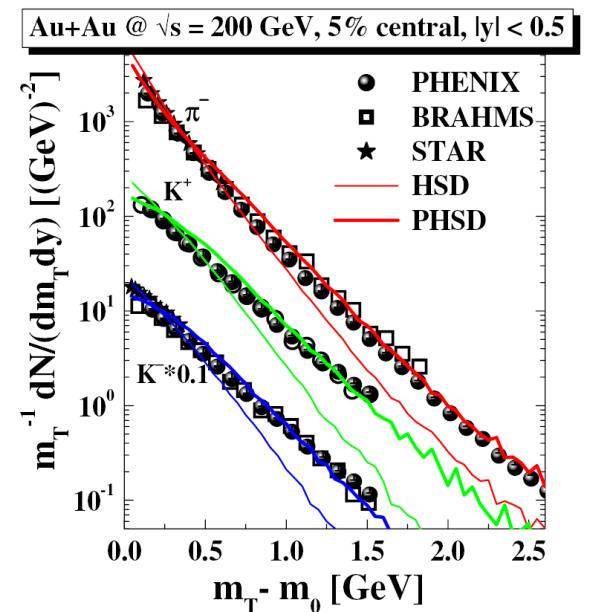
- Dramatic decrease of partonic phase with decreasing energy
- Pb+Pb, 160 A GeV: only about 40% of the converted energy goes to partons; the rest is contained in the large hadronic corona and leading partons!



## Central Pb + Pb at SPS energies

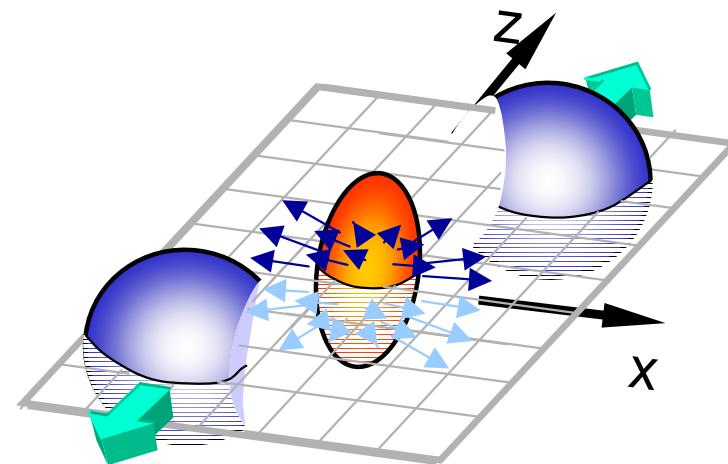


## Central Au+Au at RHIC



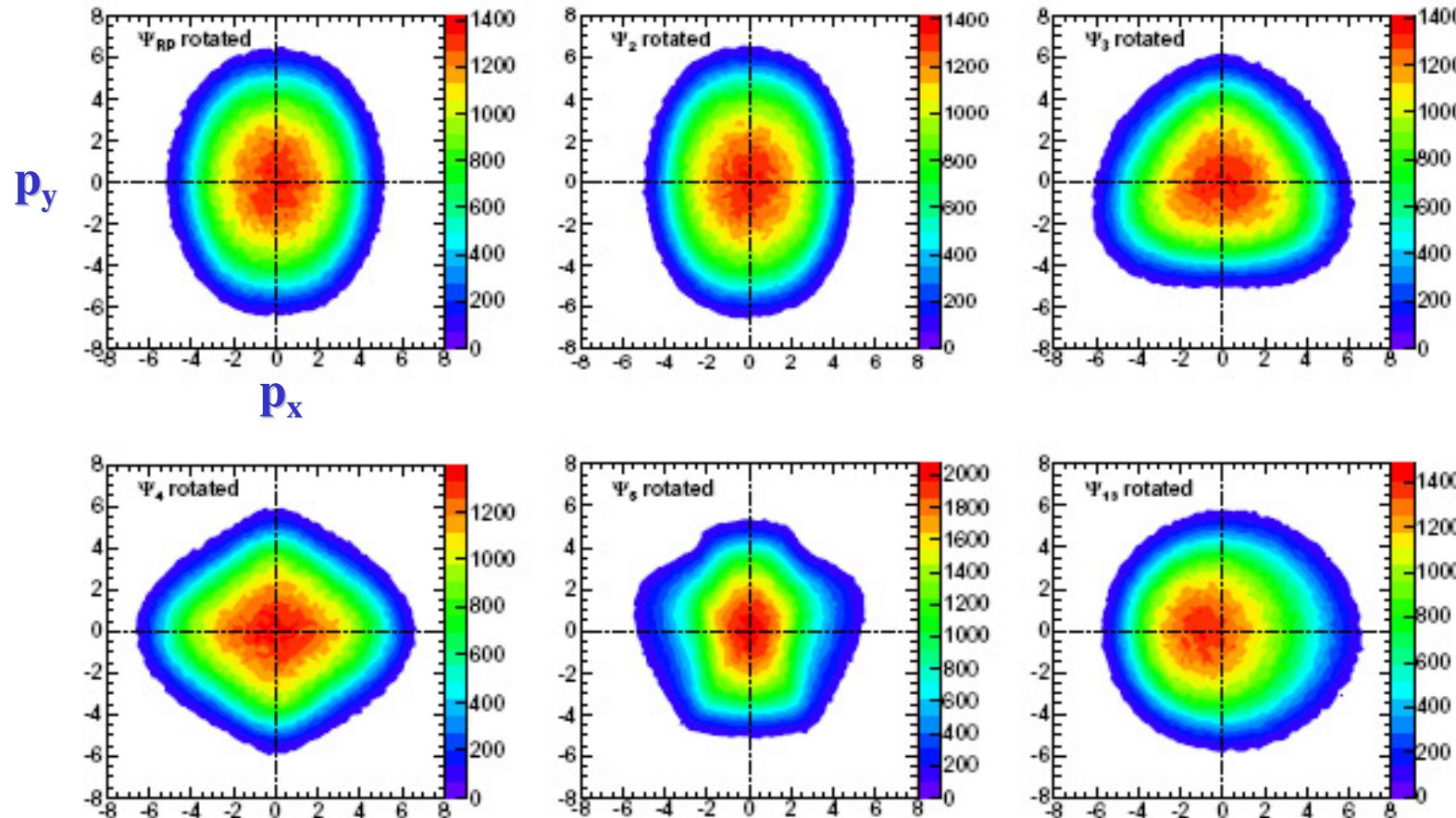
- PHSD gives **harder  $m_T$  spectra** and works better than HSD **at high energies**  
– RHIC, SPS (and top FAIR, NICA)
- however, at low SPS (and low FAIR, NICA) energies the effect of the partonic phase decreases due to the decrease of the partonic fraction

# **Collective flow: anisotropy coefficients ( $v_1, v_2, v_3, v_4$ ) in A+A**



# Final angular distributions of hadrons

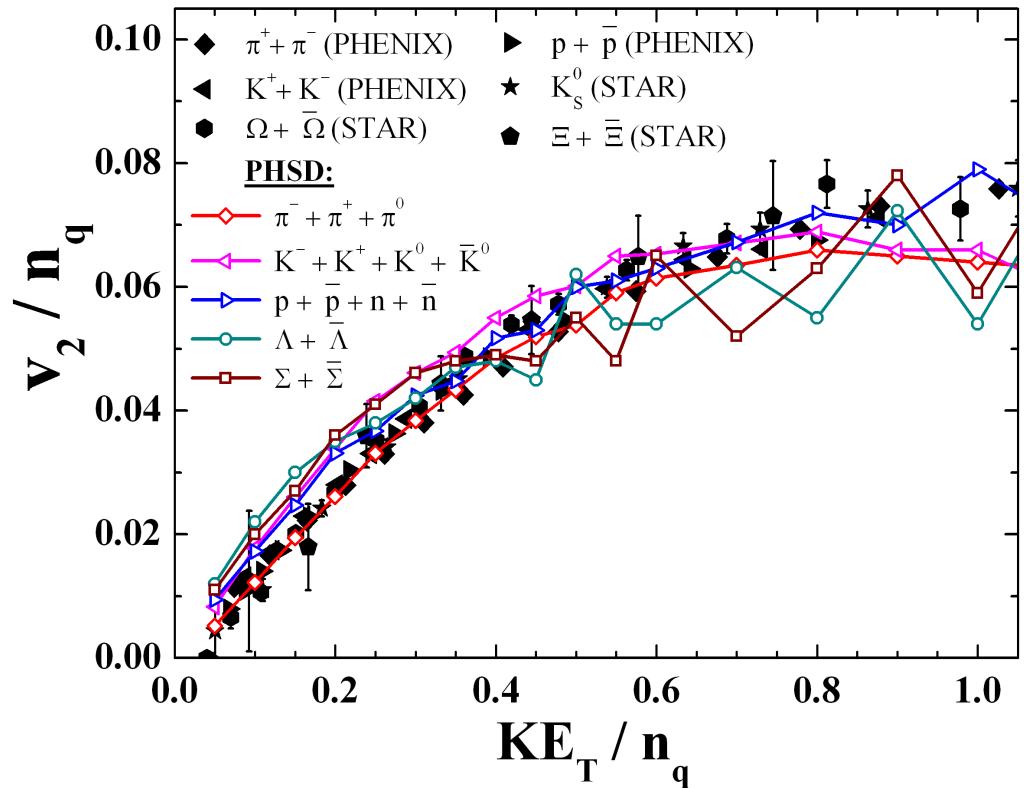
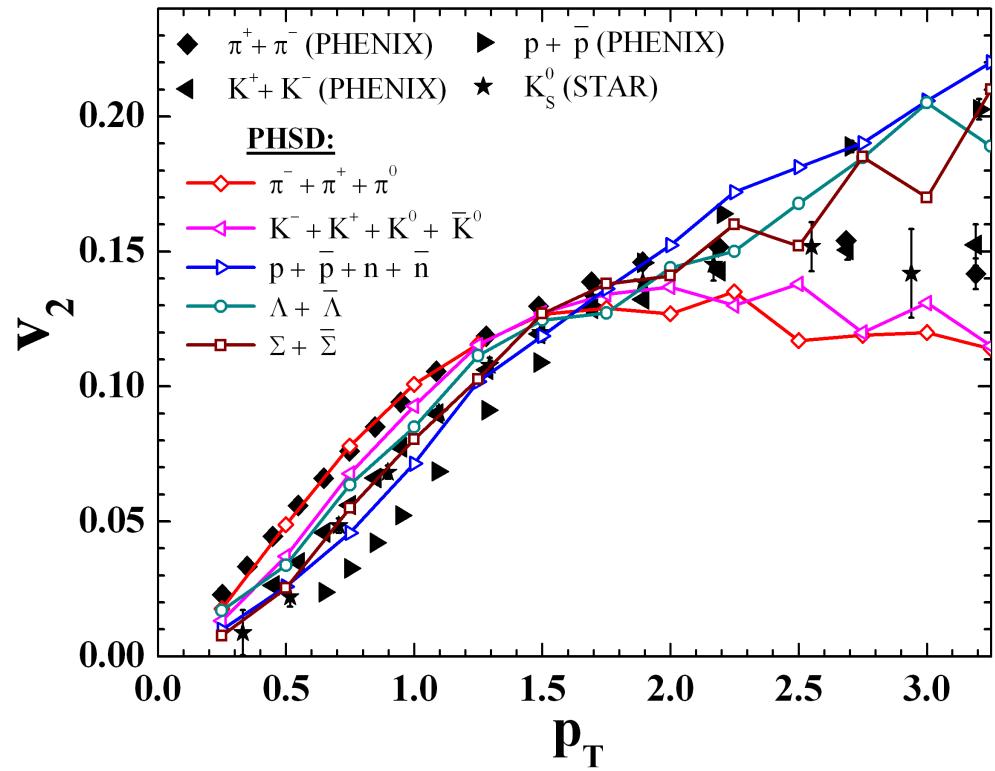
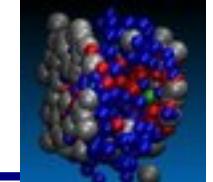
10k Au+Au collision events at  $b = 8 \text{ fm}$  at 21 TeV rotated to different event planes:



$$E \frac{d^3 N}{d^3 p} = \frac{d^2 N}{2\pi p_T dp_T dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n(p_T) \cos(n(\psi - \Psi_n)) \right)$$

show higher order harmonics  $v_n$

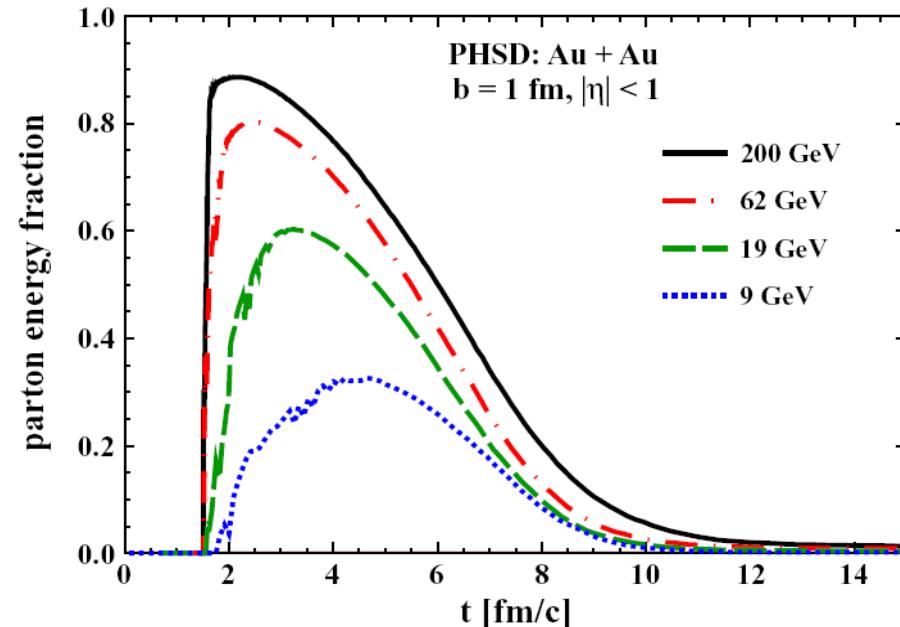
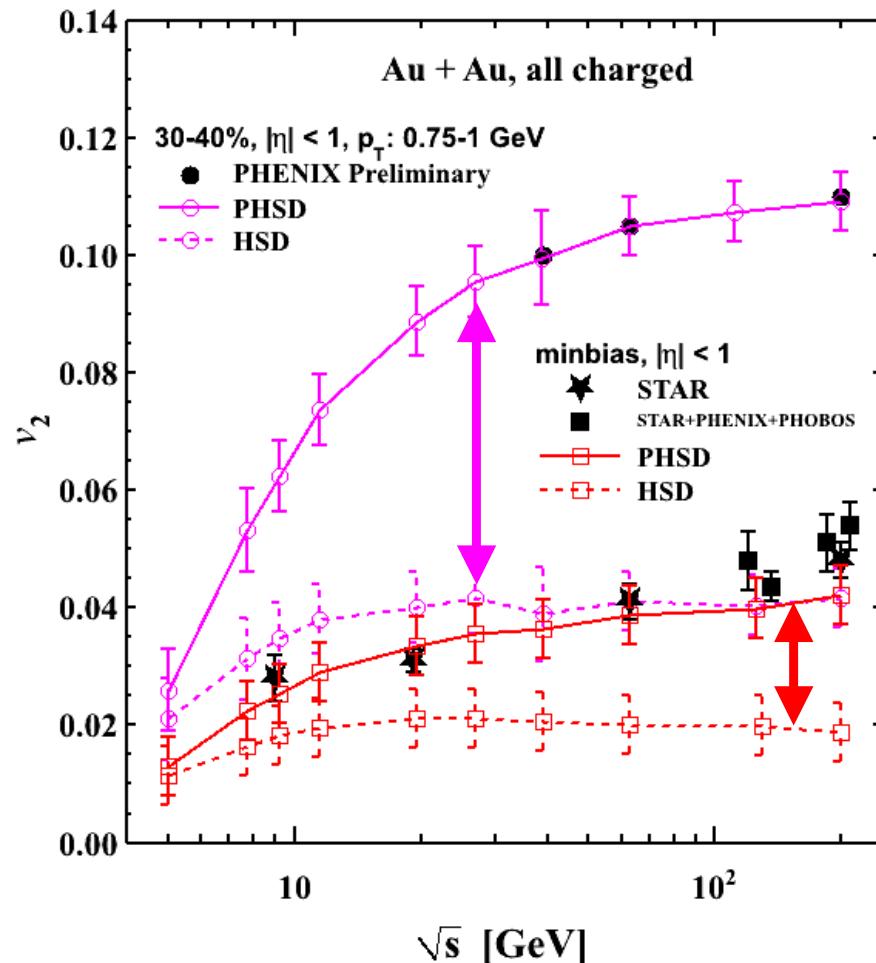
# Elliptic flow scaling at RHIC



- The mass splitting at low  $p_T$  is approximately reproduced as well as the meson-baryon splitting for  $p_T > 2 \text{ GeV}/c$  !
- The scaling of  $v_2$  with the number of constituent quarks  $n_q$  is roughly in line with the data .

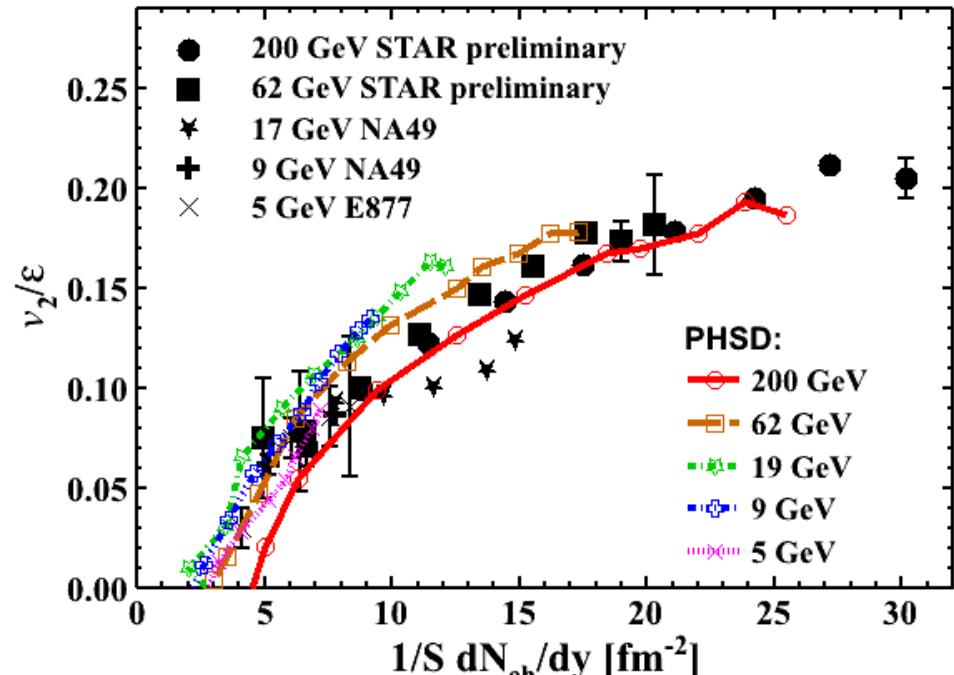
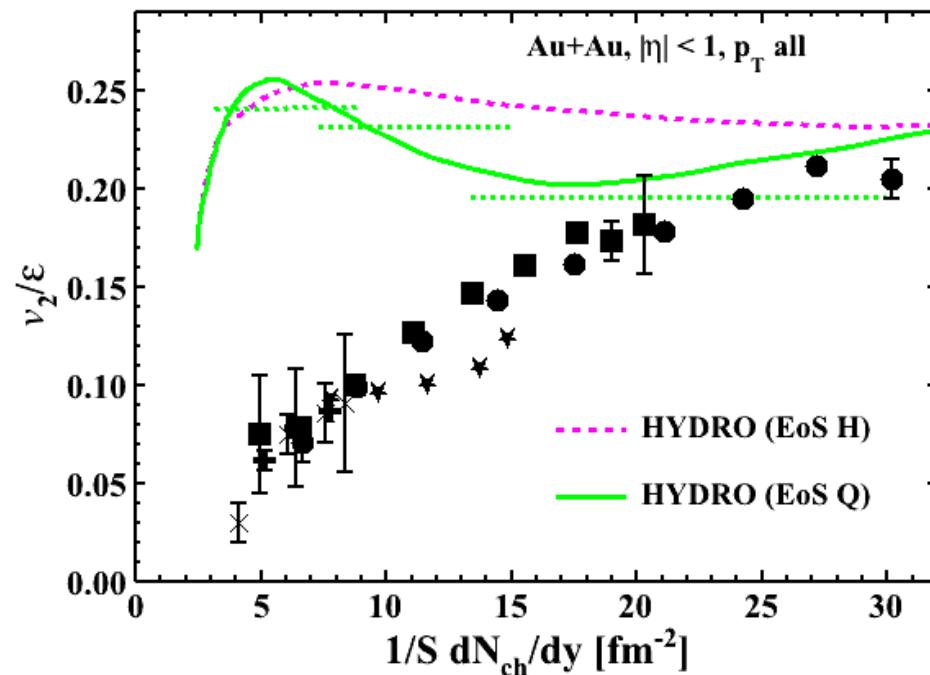
E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk,  
NPA856 (2011) 162

# Elliptic flow $v_2$ vs. collision energy for Au+Au



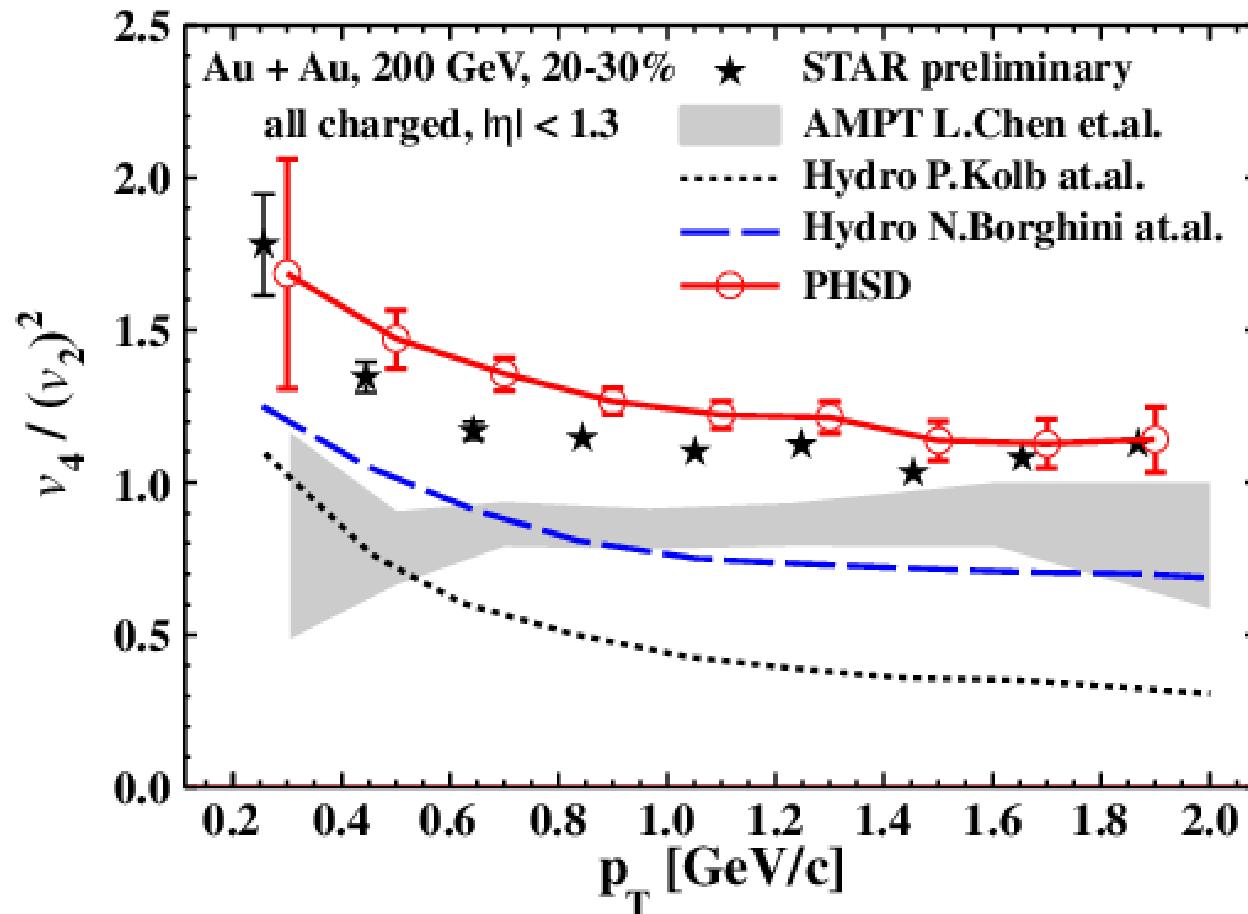
- $v_2$  in PHSD is larger than in HSD due to the repulsive scalar mean-field potential  $U_s(p)$  for partons
- $v_2$  grows with bombarding energy due to the increase of the parton fraction

# $v_2/\varepsilon$ vs. centrality at different collision energies



- PHSD:  $v_2/\varepsilon$  vs. centrality follows an approximate scaling with energy in line with experimental data

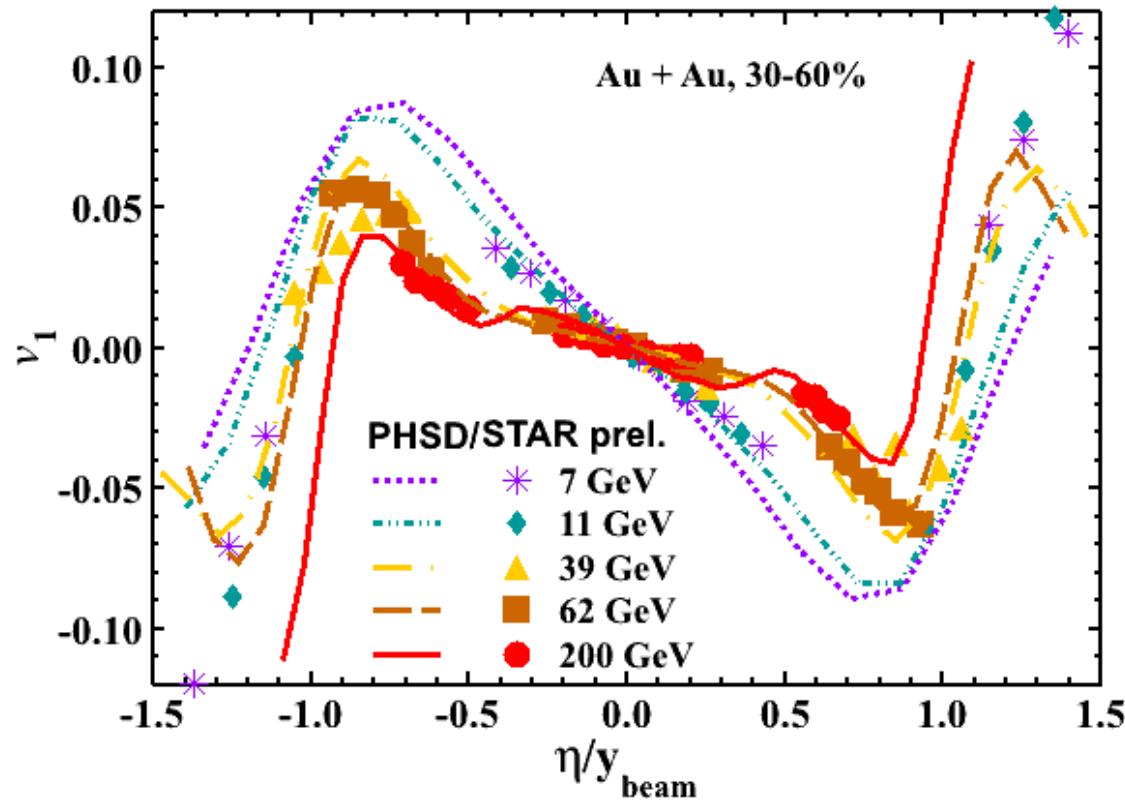
# Ratio $v_4/(v_2)^2$ vs. $p_T$



- The ratio  $v_4/(v_2)^2$  from PHSD grows at low  $p_T$  - in line with exp. data

V. Konchakovski, E. Bratkovskaya, W. Cassing, V. Toneev, V. Voronyuk,  
 Phys.Rev. C85 (2012) 044922

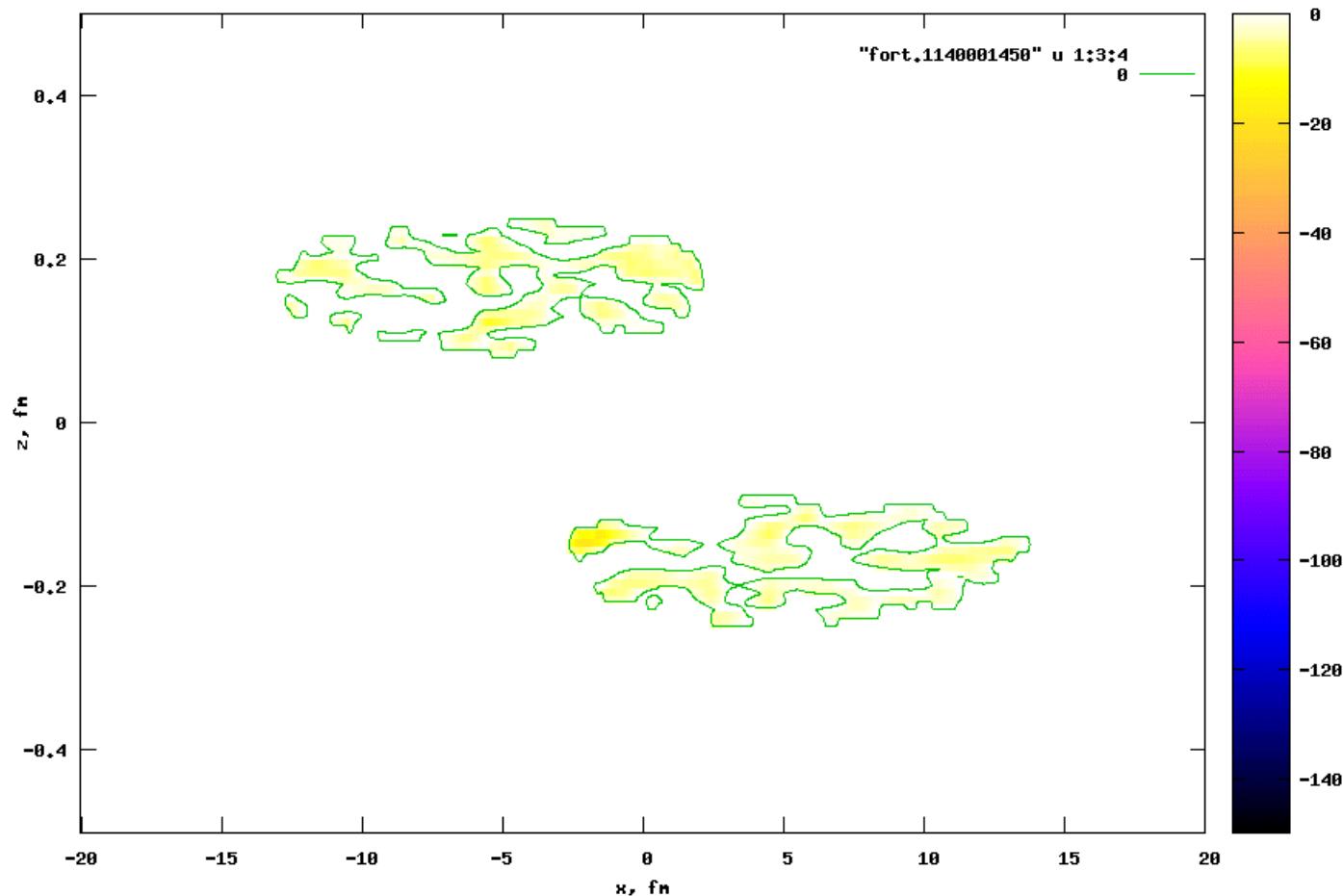
# $v_1$ vs. pseudo-rapidity at different collision energies



- PHSD:  $v_1$  vs. pseudo-rapidity follows an approximate scaling for high invariant energies  $s^{1/2}=39, 62, 200$  GeV - in line with experimental data – whereas at low energies the scaling is violated!

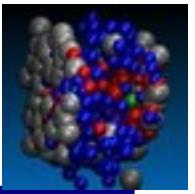
# ,Turbulence‘ in heavy-ion collisions

- Au+Au @  $s^{1/2}=200$  GeV : rotating charge density





# Summary



- PHSD provides a consistent description of off-shell parton dynamics in line with the lattice QCD equation of state (from the BMW collaboration)
  - PHSD versus experimental observables:
    - enhancement of meson  $m_T$  slopes (at top SPS and RHIC)
    - strange antibaryon enhancement (at SPS)
    - partonic emission of high mass dileptons at SPS and RHIC
    - enhancement of collective flow  $v_2$  with increasing energy
    - quark number scaling of  $v_2$  (at RHIC)
    - jet suppression
- ...
- ⇒ evidence for strong nonhadronic interactions in the early phase of relativistic heavy-ion reactions  
⇒ formation of the sQGP established!



# PHSD group



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