

# Direct Observation of Tunneling and Nonlinear Self-Trapping in a single Bosonic Josephson Junction

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(Dated: April 4, 2006)

We report on the first realization of a single bosonic Josephson junction, implemented by two weakly linked Bose-Einstein condensates in a double-well potential. In order to fully investigate the nonlinear tunneling dynamics we measure the density distribution in situ and deduce the evolution of the relative phase between the two condensates from interference fringes. Our results verify the predicted nonlinear generalization of tunneling oscillations in superconducting and superfluid Josephson junctions. Additionally we confirm a novel nonlinear effect known as macroscopic quantum self-trapping, which leads to the inhibition of large amplitude tunneling oscillations.

PACS numbers: 03.75.Lm, 05.45.-a

Tunneling through a barrier is a paradigm of quantum mechanics and usually takes place on a nanoscopic scale. A well known phenomenon based on tunneling is the Josephson effect [1] between two macroscopic phase coherent wave functions. This effect has been observed in different systems such as two superconductors separated by a thin insulator [2] and two reservoirs of superfluid Helium connected by nanoscopic apertures [3, 4]. In this letter we report on the first successful implementation of a bosonic Josephson junction consisting of two weakly coupled Bose-Einstein condensates in a macroscopic double-well potential.

In contrast to all hitherto realized Josephson junctions in superconductors and superfluids, in this new system the interaction between the tunneling particles plays a crucial role. This nonlinearity gives rise to new dynamical regimes. Anharmonic Josephson oscillations are predicted [5, 6, 7], if the initial population imbalance of the two wells is below a critical value. The dynamics changes drastically for initial population differences above the threshold of macroscopic quantum self-trapping [8, 9, 10] where large amplitude Josephson oscillations are inhibited. The two different dynamical regimes have been experimentally investigated in the context of Josephson junction arrays [11, 12, 13]. However, the small periodicity of the optical lattice does not allow to resolve individual wells and thus the dynamics between neighboring sites. Our experimental implementation of a single weak link makes it possible for the first time to directly observe the density distribution of the tunneling particles in situ. Furthermore we measure the evolution of the relative quantum mechanical phase between both condensates by means of interference [14].

The experimentally observed time evolution of the atomic density distribution in a symmetric bosonic Josephson junction is shown in Fig. 1 for two different initial population imbalances (depicted in the top graphs). In Fig. 1(a) the initial population difference between the

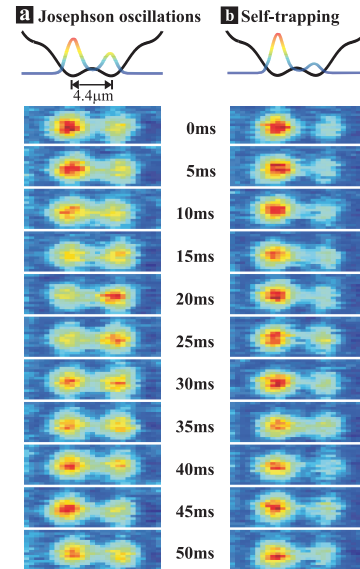


FIG. 1: Observation of the tunneling dynamics of two weakly linked Bose-Einstein condensates in a symmetric double-well potential as indicated in the schematics. The time evolution of the population of the left and right potential well is directly visible in the absorption images ( $19.4 \mu\text{m} \times 10.2 \mu\text{m}$ ). The distance between the two wavepackets is increased to  $6.7 \mu\text{m}$  for imaging (see text). (a) Josephson oscillations are observed when the initial population difference is chosen to be below the critical value  $z_C$ . (b) In the case of an initial population difference greater than the critical value the population in the potential minima is nearly stationary. This phenomenon is known as macroscopic quantum self-trapping.

two wells is chosen to be well below the self-trapping threshold. Clearly nonlinear Josephson oscillations are observed i.e. the atoms tunnel right and left over time. The period of the observed oscillation is  $40(2)\text{ms}$  which is much shorter than the tunneling period of approximately  $500\text{ms}$  expected for non-interacting atoms in the realized potential. This reveals the important role of

the atom-atom interaction in Josephson junction experiments with Bose-Einstein condensates. A different manifestation of the nonlinearity is shown in Fig. 1(b) exhibiting macroscopic quantum self-trapping, which implies that the population imbalance does not change over time within the experimental errors. The only difference to the experiment shown in Fig. 1(a) is that the initial population imbalance is above the critical value.

The experimental setup and procedure to create the  $^{87}\text{Rb}$  Bose-Einstein condensates is similar to that used in our previous work [15]. A sufficiently precooled thermal cloud is loaded into an optical dipole trap consisting of two crossed, focussed laser beams and is subsequently evaporatively cooled by lowering the light intensities. We produce pure condensates consisting of  $1150 \pm 150$  atoms and final trap frequencies of  $\omega_x = 2\pi \times 78(1)\text{Hz}$ ,  $\omega_y = 2\pi \times 66(1)\text{Hz}$  and  $\omega_z = 2\pi \times 90(1)\text{Hz}$ , with gravity acting in the  $y$ -direction. Subsequently we adiabatically ramp up a periodic one-dimensional light shift potential in  $x$ -direction to a depth of  $2\pi \times 412(20)\text{Hz}$  with periodicity  $5.2(2)\mu\text{m}$  realized by a pair of laser beams at a wavelength of  $811\text{nm}$  crossing at a relative angle of  $9^\circ$ . The superposition of this periodic potential with the strong harmonic confinement creates an effective double-well potential in  $x$ -direction with a barrier height of  $2\pi \times 263(20)\text{Hz}$ , which splits the initial condensate into two parts separated by  $4.4(2)\mu\text{m}$  realizing a single weak link (see schematics in Fig. 1).

The initial population difference between the left and right component is obtained by loading the Bose-Einstein condensate into an asymmetric double-well potential, which is created by a displacement of the harmonic confinement with respect to the periodic potential. The asymmetry can be adjusted by shifting the focussed laser beam which realizes the harmonic confinement in  $x$ -direction. This is done by means of a piezo actuated mirror mount. A relative shift of only  $350\text{nm}$  leads to a relative population difference corresponding to the self-trapping threshold. This demands high passive stability of the mechanical setup and makes it necessary to actively stabilize the phase of the periodic potential. With our setup we can adjust any initial population imbalance with a standard deviation of  $\Delta z = 0.06$ . The Josephson dynamics is initiated at  $t = 0$  by non-adiabatically (with respect to the tunneling dynamics) changing the potential to a symmetric double-well (see schematics in Fig. 2). After a variable evolution time the potential barrier is suddenly ramped up and the harmonic potential in  $x$ -direction is switched off. This results in dipole oscillations of the atomic clouds around two neighboring minima of the periodic potential. Thus by imaging at the time of maximum separation ( $1.5\text{ms}$ ) we can observe clearly distinct wave packets with a distance of  $6.7(5)\mu\text{m}$ . The atomic density is deduced from absorption images with a spatial resolution of  $2.2(2)\mu\text{m}$ . In previously reported realizations of Bose-Einstein conden-

ates in double-well potentials [16, 17] the time scale of tunneling dynamics was in the range of thousands of seconds. In contrast, our small interwell distance combined with a negligible thermal atomic fraction allows the realization of tunneling times on the order of  $50\text{ms}$ , which makes the direct observation of the nonlinear dynamics in a single bosonic Josephson junction possible for the first time.

The physics of Josephson junctions is based on the presence of two weakly coupled macroscopic wave functions separated by a thin potential barrier. Insight into the dynamics of the system can be gained by employing a two mode approximation which characterizes the wave function by only two parameters, the fractional relative population  $z = (N_l - N_r)/(N_l + N_r)$  and the quantum phase difference  $\phi = \phi_r - \phi_l$  between the left ( $l$ ) and right ( $r$ ) component. In this framework the resulting quantum dynamics in a symmetric double-well potential is described by two coupled differential equations

$$\begin{aligned} \dot{z} &= -\sqrt{1-z^2} \sin \phi \\ \dot{\phi} &= \Lambda z + \frac{z}{\sqrt{1-z^2}} \cos \phi \end{aligned} \quad (1)$$

where  $\Lambda$  is proportional to the ratio of the on-site interaction energy and the coupling matrix element given in [9]. These equations represent the nonlinear generalization of the sinusoidal Josephson oscillations occurring in superconducting junctions. An intuitive understanding of the rich dynamics of this system can be gained by considering a descriptive mechanical analog. The equations given above describe a classical non-rigid pendulum of tilt angle  $\phi$ , angular momentum  $z$ , and a length proportional to  $\sqrt{1-z^2}$ . In the following discussion we will only consider the case of vanishing initial phase difference  $\phi(0) = 0$ . If the initial population imbalance is below the critical value [10]  $|z(0)| < z_C$  (from our experimental results we deduce  $z_C \approx 0.5$  corresponding to  $\Lambda \approx 15$ ), equ. 1 describes oscillations in  $z$  and  $\phi$ . In the limit of  $|z(0)| \ll z_C$  this reduces to a harmonically oscillating mathematical pendulum. In the context of Josephson junctions this behavior is known as plasma oscillations. A different dynamical phenomenon arises if the initial population imbalance is above the critical value. This implies that the difference between the two on-site interaction energies becomes larger than the tunneling energy splitting [18]. In this case the relative phase rapidly increases in time leading to a rapidly alternating tunneling current according to equ. 1. This results in a population imbalance which performs small oscillations around the initial value (self-trapping, running phase modes [10]).

In this case the population difference is self-locked to the initial value and the relative phase is increasing monotonically (running phase modes [10]). In the mechanical analogue this critical imbalance corresponds to an initial angular momentum sufficiently large that the

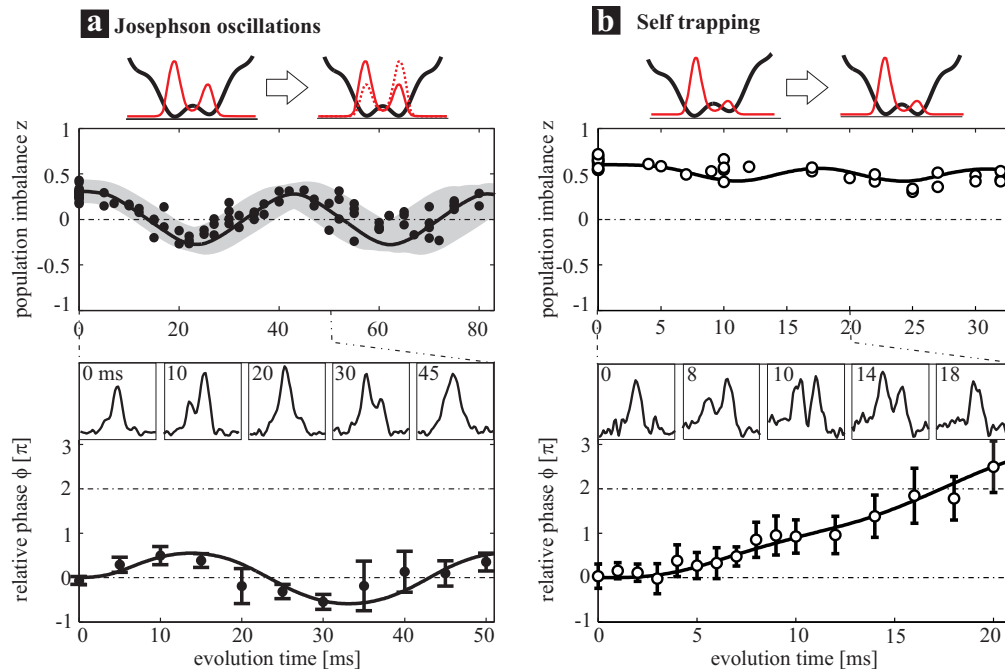


FIG. 2: Detailed analysis of the time dependence of the two dynamical variables  $z$  and  $\phi$  describing the system. The top graphs depict the experimental preparation scheme implemented to realize different initial atomic distributions. The dynamics is initiated at  $t = 0$  by switching non-adiabatically to the symmetric double-well potential. Graph (a) shows the familiar oscillating behavior of both the population imbalance and the relative phase in the Josephson regime. The solid lines represent the results obtained by numerically integrating the non-polynomial Schrödinger equation, and are in excellent agreement with our experimental findings. The shaded region shows the theoretically expected scattering of the data due to the uncertainties of the initial parameters and broadens for large evolution times due to different oscillation frequencies for different initial population imbalances. The insets depict representative atomic interference patterns obtained by integrating the absorption images along the  $y$ - and  $z$ -direction after the indicated evolution times. In graph (b) the totally different dynamics in the regime of macroscopic quantum self-trapping becomes obvious. The population imbalance exhibits no dynamics within the experimental errors and reveals the expected nonzero average  $\langle z \rangle \neq 0$ . Clearly the phase is unbound and winds up over time. The error bars in the phase measurements denote statistical errors arising from the uncertainty of the initial population imbalance.

pendulum reaches the top position and continues to rotate with a non vanishing angular momentum.

In order to fully characterize the evolution of the system we measure not only the population imbalance but also the relative phase of the macroscopic wave functions. This is achieved by releasing the Bose-Einstein condensates from the double-well potential after different evolution times. After a time of flight of 5ms in the Josephson and 8ms in the self-trapping regime the wave packets interfere unveiling the relative phase in a direct way since the resulting atomic fringes are similar to a double slit diffraction pattern.

In Fig. 2 we present the quantitative analysis of our experimental results. The measured fractional population imbalance and the relative phase in the regime of Josephson oscillations ( $z(0) = 0.28(6) < z_C$ ) are shown in Fig. 2(a). As expected for a symmetric double-well potential the relative population oscillates around its mean value  $\langle z \rangle = 0$ . The relative phase of the two Bose-Einstein condensates oscillates with a finite amplitude of  $\phi = 0.5(2)\pi$  around  $\langle \phi \rangle = 0$ . The self-

trapping regime can be reached by simply increasing the initial asymmetry of the double-well potential as indicated in the schematic diagram in Fig. 2(b) realizing  $z(0) = 0.62(6) > z_C$ . In this case theory predicts that  $z$  exhibits only small amplitude oscillations which never cross  $z = 0$  i.e.  $\langle z \rangle \neq 0$ . Additionally the relative phase  $\phi$  is unbound and is supposed to wind up in time. In Fig. 2(b) these characteristics of macroscopic quantum self-trapping are evident. The population difference does not change over time within the experimental errors and the phase increases monotonically. The initial deviation from the linear time dependence of the phase is due to the finite response time of the piezo actuated mirror.

The experimentally obtained results can be understood quantitatively by going beyond the two mode model which assumes stationary wave functions in the individual wells which is only justified for  $N_l + N_r \ll 1000$  atoms [8]. Therefore we numerically integrate the non-polynomial Schrödinger equation [19] using the independently measured trap parameters and atom numbers. The calculations also include the fact that the piezo ac-

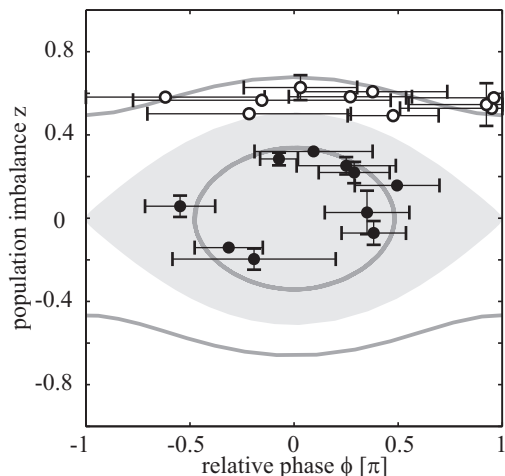


FIG. 3: Quantum phase-plane portrait for the bosonic Josephson junction. In the regime of Josephson oscillations the experimental data are represented with filled circles and in the self-trapping regime with open circles. The shaded region, which indicates the Josephson regime, and the solid lines are obtained by solving the coupled differential equ. 1 with our specific experimental parameters. The two mode model explains the observed  $z(\phi)$  dependence reasonably in both dynamical regimes. The error bars represent the statistical error and mainly result from the high sensitivity of the relative phase on the initial population imbalance especially for long evolution times.

tuated mirror initiating the Josephson dynamics reaches its final position only after 7ms. It is remarkable that all experimental findings are in excellent quantitative agreement with our numerical simulation without free parameters.

The distinction between the two dynamical regimes - Josephson tunneling and macroscopic self-trapping - becomes very apparent in the phase-plane portrait of the dynamical variables  $z$  and  $\phi$ . For our experimental situation this is shown in Fig. 3 where we compare our results with the prediction of the simple two mode model. From our experimental observations the critical population imbalance can be estimated to  $z_C = 0.50(5)$ . In the framework of the two mode model [10] this yields  $\Lambda = 15(3)$ . The corresponding solutions of equ. 1 are depicted with solid lines. Clearly the basic features of the dynamics are well captured by this approach. In the nonlinear Josephson tunneling regime ( $z < z_C$ ) the dynamical variables follow a closed phase plane trajectory as predicted by the simple model. Very recently a variable tunneling two mode model has been discussed by Ananikian and Bergeman [20] which is quantitative agreement with our experimental observations.

The successful experimental realization of weakly coupled Bose-Einstein condensates adds a new tool to quantum optics with interacting matter waves. It opens up new avenues ranging from the generation of squeezed atomic states [21] and entangled number states

(Schrödinger cat states) [22] to applications such as atom interferometry [23]. Moreover the detailed investigation of the self-trapping phenomenon could provide a test of the validity of the mean field description in atomic gases in the strong nonlinear regime [24].

We wish to thank Andrea Trombettoni, Augusto Smerzi, Tom Bergeman, and Luis Santos for very stimulating discussions. We would also like to thank Thomas Anker and Bernd Eiermann for their contributions to the experimental setup. This work was funded by Deutsche Forschungsgemeinschaft 'Emmy Noether Programm' and by the European Union, RTN-Cold Quantum Gases, Contract No. HPRN-CT-2000-00125. R. G. thanks the Landesgraduiertenförderung Baden-Württemberg for the financial support.

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