Atom interferometry [1] is a powerful technique for the precise measurement of quantities such as acceleration, rotation, and frequency [2–5]. Since state-of-the-art atom interferometers already operate at the classical limit for phase precision [6,7], given by the projection noise $\Delta \theta_{\text{cl}} = 1/\sqrt{N}$ [8] for $N$ detected particles, quantum entangled input states are a viable route for further improving the sensitivity of these devices.

One class of such states is spin squeezed states, which outperform the classical limit at a level given by the metrological spin squeezing parameter $\xi$ with $\Delta \theta_{\text{sq}} = \xi R = \xi R$ [9,10]. In the photonic case, quantum-enhanced interferometry with squeezed states is routinely employed in optical gravitational wave detectors [11]. For atoms, proof-of-principle experiments have shown that spin squeezed states can be generated in systems ranging from high-temperature vapors to ultracold Bose-Einstein condensates (BECs) [12–21], surpassing the classical limit in atom interferometry [14,22], atomic magnetometers [19], and atomic clocks [23,24].

BECs are particularly well suited for applications that require long interrogation times, high spatial resolution, or control of motional degrees of freedom, such as in measurements of acceleration and rotation, due to their high phase-space density. However, in these systems scaling of the squeezed states to large particle numbers is intrinsically limited by density dependent losses, e.g., due to molecule formation. Keeping these processes negligible implies that, for larger numbers, the volume has to be increased, which in turn limits the generation of squeezed states due to uncontrolled nonlinear multimode dynamics. Here, we show how this limitation can be overcome by realizing an array of many individual condensates with an optical lattice, increasing the local trap frequencies but keeping the density small.

In our experiment, we simultaneously prepare up to 30 independent BECs by superimposing a deep 1D optical lattice (period 5.5 $\mu$m) on a harmonic trap with a large aspect ratio, which provides transverse confinement [see Fig. 1(a)]. Each lattice site contains a condensate with $N = 300$ to 600 atoms in a localized spatial mode and the internal state $|a\rangle = |F, m_F\rangle = |1, 1\rangle$ of the lowest hyperfine manifold. Using a two-photon radio frequency and microwave transition, we apply phase and amplitude controlled coupling of the states $|a\rangle$ and $|b\rangle = |2, -1\rangle$, forming an effective two-level system. A magnetic bias field of 9.12 G brings the system near a Feshbach resonance, changing the interspecies interaction and leading to the nonlinearity necessary for squeezed state preparation. The nonlinear evolution is governed by the one-axis twisting Hamiltonian $\mathcal{H} = \chi \hat{J}_z^2$ [25], where the interaction strength is parametrized by $\chi$, and $\hat{J}_z = (\hat{N}_b - \hat{N}_a)/2$ is the $z$ component of the Schwinger pseudospin. As indicated in Fig. 1(a), the evolution of an initial coherent spin state under this Hamiltonian leads to an elongated squeezed state with reduced quantum uncertainty along one direction. Details about the experimental sequence, which includes a spin-echo pulse to reduce the influence of technical fluctuations during state preparation, can be found in the Supplemental Material [26].

In order to get access to the axis of minimal fluctuations, we rotate the state by an angle $\sigma$ around its mean spin direction. A projective measurement of the population imbalance $z = 2J_z/N = (\hat{N}_b - \hat{N}_a)/N$ is implemented by state-selective absorption imaging [see Fig. 1(a)]. The lower quantum uncertainty of the state translates into reduced fluctuations of $z$ for repeated experiments, and is quantified using the number squeezing parameter $\xi_N^2 = N \text{Var}(z)$ [26].

High-resolution imaging of the individual lattice sites allows us to study $\xi_N^2$ for different system sizes by summing the populations of several sites [see Fig. 1(b)]. We develop a relative squeezing analysis that is insensitive to the...
magnetic field fluctuations present in our system (±45 μG for several days). For that, we divide the lattice in half and evaluate the difference of the population imbalances \( \delta N_{\text{left}} - \delta N_{\text{right}} \) of the two regions [see inset Fig. 1(b)], which rejects common mode fluctuations. The corresponding squeezing parameter is given by \( \xi_{\text{rel}}^2 = \frac{N_{\text{tot}} \text{Var}(\delta z)^2}{4} \) for equal particle numbers on both sides and \( \delta z_{\text{av}} \approx 0 \) [26], which is directly connected to the quantum enhancement of gradiometry as described below. We find \( \xi_{\text{rel}}^2 = -5.3(5) \) dB for the full ensemble of 12 300 atoms after 20 ms of nonlinear evolution [Fig. 1(b), blue circles]. The remaining decrease of squeezing for large \( N_{\text{tot}} \) is due to the atom number dependent parameters of the single-site Hamiltonian. This affects the squeezing as well as the optimal tomography angles for different ensemble sizes. From our observations, we infer that by extending our one-dimensional array (30 lattice sites) to three dimensions (30⁴ sites), ensembles as large as \( 10^7 \) atoms can be squeezed to the same level. Exploiting these resources does not require optically resolving the single lattice sites, and global population measurements are sufficient during the detection process.

To compare the scaling of our squeezed state with the best attainable classical state, we show the values of \( \xi_{\text{rel}}^2 \) obtained for the initial coherent spin state (black diamonds), yielding the expected classical shot noise limit. In the case of squeezed states, even the direct analysis of summing all ensembles, which does not reject technical fluctuations, yields squeezing of \( \xi_{\text{N}}^2 = -1.3(6) \) dB for 12 300 particles [Fig. 1(b), open blue squares]. For all given variances, the independently characterized photon shot noise of the detection process was subtracted.

Figure 1(c) shows the tomographic characterization of fluctuations for different readout rotation angles \( \alpha \) for the ensemble containing \( 10^4 \) particles, revealing the expected sinusoidal behavior. The difference between the relative and the direct analysis is consistent with an independent characterization of the technical noise [26] and can be further reduced with an optimized spin-echo pulse.

A natural application of atomic squeezed states is the measurement of magnetic fields, where BECs are an ideal system to achieve both high sensitivity and spatial resolution [22,46,47]. We implement a quantum-enhanced magnetometer using a modified Ramsey sequence that coherently transfers the population of one level to a different hyperfine state for the interrogation time. The advantage of this state swapping is twofold: the nonlinear interaction becomes negligible on our interferometric time scales, and the magnetic sensitivity is significantly increased from \( \approx 1 \) Hz/μT (second order Zeeman shift at the operating field of \( B_0 = 9.12 \) G) to \( \approx 140 \) Hz/μT (first order Zeeman shift). Figure 2(a) depicts the implemented experimental sequence. After generating the squeezed state in the levels \( |a\rangle \) and \( |b\rangle \) (left panel), we rotate it for maximum phase sensitivity (middle panel). The interrogation time \( t_{\text{int}} \) of the Ramsey sequence starts with a microwave \( \pi \) pulse \( (\pi) \) which swaps the level \( |b\rangle \) of the phase squeezed state to the level \( |c\rangle = |1, -1\rangle \), yielding increased magnetic sensitivity (right panel). After a hold time \( t_{\text{hold}} \), we swap the state back to the original level. During this sequence, the state acquires a phase \( q = 2\pi|S(B - B_0) + \delta(t_{\text{hold}} + 2\tau_G)| \), where \( \delta \) describes the relative detuning of the \( \pi \) pulse. A Ramsey fringe is obtained by a final \( \pi/2 \) rotation with varied pulse phase \( \phi \).

First, we confirm that the level of squeezing is maintained during state swapping by performing an interferometric sequence with \( t_{\text{hold}} = 1 \) μs followed by a tomographic analysis [Fig. 2(b)]. We find \( \xi_{\text{rel}}^2 = -5.1_{-0.7}^{+0.6} \) dB at the optimum tomography angle and a Ramsey fringe visibility of \( V = 0.950(5) \) [Fig. 2(c)], revealing no significant reduction of the squeezing initially present. Without subtraction of detection noise, we find squeezing of \( -3.8(5) \) dB, which corresponds to metrologically relevant spin squeezing of \( -3.4(5) \) dB for the visibility \( V = 0.95 \).

Since the single lattice sites are statistically independent, the single shot magnetic field sensitivity using this resource can be extracted from a differential measurement. This rejects common mode fluctuations of homogeneous fields.
which would, otherwise, mask the actual sensitivity of the device. A magnetic field difference \( \delta B \) between the left and right part of the ensemble translates into a differential phase \( \delta \phi \) of the Ramsey fringes [Fig. 3(a)]. For a fixed pulse phase, this shows up as \( \delta \zeta \), the difference of the corresponding population imbalances. The optimal working point for estimating magnetic fields is close to the zero-crossings of the Ramsey fringes, where \( \delta \zeta \) is maximal. At this point, the difference in the magnetic field can be deduced as \( \delta B \approx (\delta \zeta)_{\text{max}}/2 \pi n_{\text{int}} SV \) with the interrogation time \( t_{\text{int}} = t_{\text{hold}} + 2t_r \). The single shot magnetic field sensitivity around the optimal working point follows from error propagation in the expression for \( \delta B \) using the measured values for \( \text{Var}(\delta \zeta) \) [26]. We find quantum-enhanced sensitivity up to interrogation times of 342 \( \mu s \) [see Fig. 3(b)]. For longer times, quantum enhancement is lost due to fluctuations of the magnetic field which translate into a significant reduction of the mean Ramsey contrast. We do not observe a decrease in single-shot visibility, indicating that no coherence is lost on these time scales.

For \( t_{\text{int}} = 342 \mu s \) and the full ensemble, we find a quantum-enhanced single shot sensitivity for static magnetic fields of 310(47) pT compared to the shot noise limit of 382 pT for a perfect classical device (same atom number, no detection noise, and \( \nu = 1 \)). With our current experimental duty cycle (36 s production, 342 \( \mu s \) interrogation), we realized a sub-shot-noise sensitivity of 1.86(28) nT/\( \sqrt{\text{Hz}} \) for static magnetic fields. The performance of our magnetometer is competitive with state-of-the-art devices with comparably small probe volume [48], such as micro-superconducting quantum interference devices or nitrogen vacancy centers (see [26] for an overview). The ultimate physical limitation is the residual nonlinearity of the employed states, which implies that further improvement of the sensitivity by at least 2 orders of magnitude can be achieved by increasing the interrogation time. Thus, for an interrogation time of 250 ms [46] and assuming a realistic cycle time of 5 s for an all-optical BEC apparatus, a sensitivity of \( \sim 1 \) pT/\( \sqrt{\text{Hz}} \) in a probe volume of just 90 \( \mu m^3 \) is feasible.
Our array of BECs is ideally suited for gradiometric measurements. The sensitivity for magnetic field gradients depends on interrogation time, the distance between the detectors (baseline length), and the noise of the magnetic field detection. We find the expected linear dependence of the signal amplitude \((\delta z)_{\text{max}}\) on interrogation time, as shown in Fig. 4(a). The gradiometric sensitivity for the specific interrogation time of 342 \(\mu s\) as a function of baseline length is shown in Fig. 4(b). Using single wells with varying distances, we find the expected linear gain in gradient sensitivity with baseline length (triangles) and observe quantum enhancement beyond the respective classical limits (dashed line).

The single shot sensitivity of the gradiometer can be further improved to 12(2) \(pT/\mu m\) by summing up to 20 adjacent lattice sites, exploiting the scalability of the squeezed state, and leading to an effective baseline length of up to 50 \(\mu m\) [Fig 4(b), squares]. This corresponds to a quantum-enhancement of 24\%, as expected from the independently determined 2.4 dB of squeezing [see Fig. 2(b) at \(\alpha = 90^\circ\), here, without subtraction of detection noise]. Specifically for our experiment, we determine the magnetic field gradient to be \(\partial B_z/\partial x = 19.6(6)\) \(pT/\mu m\).

The high spatial resolution, combined with the excellent magnetic field sensitivity, makes the array of spin squeezed BECs an ideal system for magnetic field microscopy, where squeezing allows for enhanced sensitivity at small length scales. In contrast to atoms magnetically trapped on atom chips, optical confinement also allows vector field reconstruction with high spatial resolution.

In conclusion, we demonstrated the scalability of squeezed ensembles in an optical lattice, directly applicable to high-precision atom interferometry with ultracold clouds. We show that swapping of squeezed states is experimentally feasible and allows both for control of nonlinear interaction and the field sensitivity. Both advantages are explicitly demonstrated with quantum-enhanced Ramsey magnetometry, achieving high sensitivity and spatial resolution. The flexibility of state swapping with large squeezed ensembles offers prospects for improved tests of general relativity or the detection of gravitational waves with atom interferometers employing motional degrees of freedom [49,50], controlled by Raman beam splitters. From a more fundamental perspective, the system of several independent ensembles entangled in the internal degrees of freedom combined with adjustable tunnel coupling is a perfect starting point to study the spread of quantum correlations [51,52] in the continuous variable limit and the role of entanglement in quantum phase transitions [53,54].

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