Experimental observation of oscillating and interacting matter wave dark solitons

A. Weller, J.P. Ronzheimer, C. Gross, J. Esteve, and M.K. Oberthaler
Kirchhoff Institute for Physics, University of Heidelberg, INF 227, 69120 Heidelberg, Germany

D.J. Frantzeskakis
Department of Physics, University of Athens, Panepistimiopolis, Zografos, Athens 157 84, Greece

G. Theocharis and P.G. Kevrekidis
Department of Mathematics and Statistics, University of Massachusetts, Amherst MA 01003-4515, USA

(Dated: June 10, 2008)

We report on the generation, subsequent oscillation and interaction of a pair of matter wave dark solitons. These are created by releasing a Bose-Einstein condensate from a double well potential into a harmonic trap in the crossover regime between one dimension (1D) and three dimensions (3D). Multiple oscillations and collisions of the solitons are observed, in quantitative agreement with simulations of the Gross-Pitaevskii equation. An effective particle picture is developed and confirms that the deviation of the observed oscillation frequencies from the asymptotic prediction $\nu_z/\sqrt{2}$, where ν_z is the longitudinal trapping frequency, results from the dimensionality of the system and the soliton interactions.

Solitons are one of the most prominent features of nonlinear dynamics emerging in diverse fields extending from hydrodynamics to solid state physics and from nonlinear optics to biophysics. Dark solitons are the fundamental excitations of the defocusing nonlinear Schrödinger equation [1], and have the form of a localized "dip" on a background wave, accompanied by a phase jump [2]. These localized waveforms have been demonstrated experimentally in different contexts, including liquids [3], discrete mechanical systems [4], thin magnetic films [5, 6], optical media [7, 8, 9, 10, 11], and, more recently, Bose-Einstein condensates (BECs) [12, 13, 14, 15, 16, 17, 18]. The possibility of creating pairs of dark solitons [8, 10] has stimulated considerable interest in the repulsive [19] collisional interactions between them [20, 21, 22]. The fundamental features of soliton collisions have a universal character and thus, e.g., optical solitons interact essentially the same way as matter-wave solitons.

In this letter we report on the systematic generation of a pair of matter wave dark solitons which is subsequently oscillating and colliding in a harmonic trap. Our experiment is performed in the crossover regime between 1D and 3D [23], where dark solitons exist and are robust [24]. This allows us to monitor, to our knowledge for the first time in any field, multiple oscillations and collisions of dark solitons, permitting the precise measurement of their oscillation frequency and their mutual repulsive interactions. Previous experiments have been performed in a genuine 3D regime where dark solitons are unstable due to the so-called snaking instability and eventually decay into vortex rings [14, 24]. In these experiments solely their translation in the trap has been shown [12, 13, 14]. Only very recently dark solitons have been reported to undergo a single oscillation period in a harmonic trap [18].

Different methods have been explored to create dark solitons in Bose-Einstein condensates [12, 13, 14, 15, 16, 17, 18]. In our experiment, the solitons are generated by merging two coherent condensates initially prepared in a double well potential. The observed evolution in the trap, after the preparation process, is shown in Fig. 1a revealing that an even number of solitons is generated. This formation process of the dark solitons can be regarded as a consequence of matter wave interference of the two condensates [25, 26, 27, 28]. Our procedure is very similar to the recently reported generation of vortices out of a triple well potential [29].

Since the two dominant solitons are created with a distance of a few healing lengths, the repulsive interaction between them leads to a significant modification of the oscillation frequency. The measured frequencies deviate up to 16% from the single soliton asymptotic Thomas-Fermi 1D (TF1D) prediction of $\nu_z/\sqrt{2}$ [30] where ν_z is the longitudinal trapping frequency. Our experimental results are in quantitative agreement with numerical simulations of the Gross-Pitaevskii equation (GPE). They reveal that dark solitons can behave very similar to particles. This is confirmed by explaining the essential features of the dynamics within a simple physical picture regarding the dark solitons as particles in an effective potential due to the external trap and their mutually repulsive interactions. Being in the crossover regime, the role of the transverse degrees of freedom has to be included in the effective potential in order to get quantitative agreement between theory and experiment [31].

Before elaborating on the theoretical models and systematic studies we will briefly describe the details of the experimental setup. We prepare a BEC of ^{87}Rb in the $|F=2, m_F=2\rangle$ state containing about $N = 1500$ atoms in a double well potential [32]. This potential is realized by superimposing a far detuned crossed optical dipole trap ($\lambda = 1064$ nm) and a one dimensional optical lattice

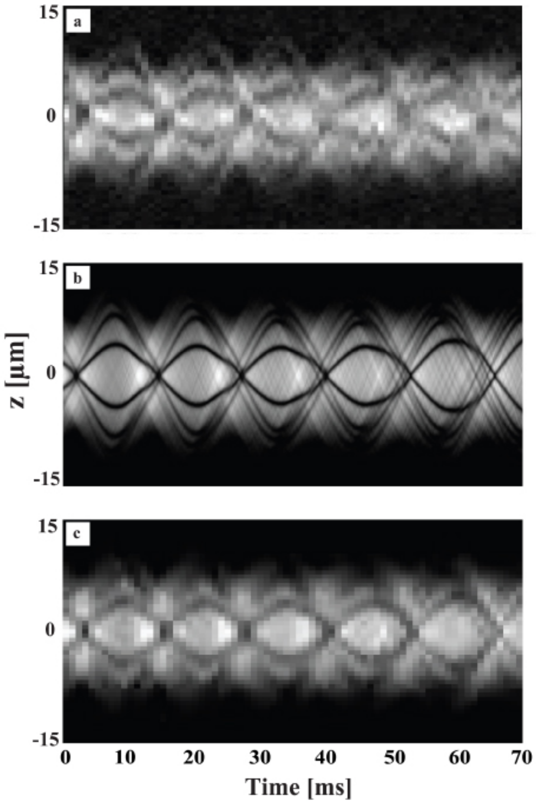

FIG. 1: Observation of the time evolution of dark solitons in a harmonic trap. a) Experimental observation of the dynamics of the longitudinal atomic density for the case of $N = 1700$ atoms and trapping frequencies $(\nu_z, \nu_\perp)$=(53 Hz,890 Hz) after a short time of flight. The shown images are averaged over 10 realizations of the experiment. b) Result of the numerical integration of the 3D GPE taking into account the full preparation process. c) The theoretical prediction taking into account the finite spatial as well as temporal resolution of the experiment.

($\lambda = 843$ nm). The first beam of the dipole trap has a gaussian waist of 5 μm and results in a strong transversal and weak longitudinal confinement. The second beam orthogonally crosses the first one and has an elliptic shape (60 μm $\times$ 230 μm waist) leading to an extra adjustable confinement only in the longitudinal direction of the trap. We start our experiments with a transversal frequency of the total harmonic trap of $\nu_\perp = 408$ Hz and a longitudinal one of $\nu_z = 63$ Hz. This corresponds to a power of about 1 mW in the first beam and 400 mW in the second beam. The barrier height of the optical lattice is chosen to be approximately 1 kHz and the lattice spacing is 5.7 μm. This results in a double well potential with a well distance of 5.4 μm.

In order to start with a well defined phase between the two condensates the barrier height is chosen to be low enough such that thermal phase fluctuations are negligible for the measured temperature of $T \approx 10$ nK [33] (the critical temperature for condensation is $T_c \approx 110$ nK) and high enough so that high contrast solitons are formed. Switching off the optical lattice transforms the double well potential into a harmonic trapping potential. Right after the switching off, we ramp to the trap parameters of interest $(\nu_z, \nu_\perp)$ and use an optimized ramping time to minimize the excitation of quadrupole oscillations (e.g. from $(\nu_z, \nu_\perp)$=(63 Hz, 408 Hz) to (53 Hz, 890 Hz) within 10 ms for $N = 1700$ atoms, or to (58 Hz, 408 Hz) within 3 ms for $N = 950$). The distance between the formed solitons can be adjusted by choosing different sets of final trap frequencies. Imaging the atomic density after a certain time of evolution in the harmonic trap is done using standard absorption imaging. Although our optical resolution of ≈ 1 μm allows the direct observation of the solitons in the trap, we use a short time of flight on the order of 1 ms to enhance the contrast.

In our experiment the distance $D = 5.4$ μm between the two colliding condensates is well within the regime where the formation of dark solitons is expected due to nonlinear interference. Dark solitons are generated if the distance D is smaller than the critical distance $D_c = \pi(6\frac{N\hbar a_s}{\nu_z m})^{1/3} = 25.8$ μm with N being the number of atoms, a_s the s-wave scattering length, ν_z the longitudinal trap frequency and m the atomic mass; if $D < D_c$, the interaction-energy exceeds the kinetic energy and hence nonlinear dynamical phenomena are expected, such as the formation of dark soliton pairs [26]. This is confirmed by 3D GPE simulations of the soliton formation and their subsequent evolution in the trap as shown in Fig. 1. Including the optical and time resolution, the experimentally observed density profile evolution is well reproduced. A dominant pair of solitons oscillates close to the center of the cloud and we can also distinguish additional pairs of solitons with much lower contrast. In the following, we focus on the dynamics of the dominant central pair and show that its oscillation frequency is well described within a two soliton approximation.

We experimentally investigate the oscillation frequency of the dominant soliton pair for different trap parameters and different inter-soliton distances. A typical data set consists of 50 time steps and 10 pictures per time step. Our data does not allow to distinguish if the two solitons do or do not cross at the collision point. Hence, for each time step we measure the soliton distance which is well defined and reconstruct its time evolution from which we extract an oscillation frequency as shown in the inset of Fig. 2. The obtained frequency is divided by two in order to compare it to the oscillation frequency expected for a single trapped soliton. The shot to shot reproducibility of the soliton dynamics up to 100 ms allows the observation of up to 7 oscillation periods and hence the deduction of the frequency with high accuracy. The typical statistical experimental error in the frequency measurement is $\pm 1.5\%$. Fig. 2 shows the results of our frequency measurements and their comparison with numerical simulations for the motion of two trapped solitons using the Nonpolynomial Schrödinger equation (NPSE) [34]. Note that this equation has been shown to be an excellent approximation to the 3D GPE in the dimensionality crossover regime where our experiments are performed, especially for studying dark soliton dynamics [31].

In order to capture the essentials of the dynamics of the experimentally realized soliton pairs, we initialize the condensate with two solitons in the simulations such that the rms amplitude of their oscillating motion matches the one observed experimentally. The good agreement between numerics and experiments shows that the dynamics produced by our method is well described within a two soliton approximation even though extra solitons are produced. From our experiment and the NPSE simulations, we observe an upshift up to 16% from the $\nu_z/\sqrt{2}$ prediction which was the first value theoretically derived for the oscillation frequency of a single trapped soliton [30]. It is expected to be valid in a 1D trap in the asymptotic Thomas-Fermi limit ($N\Omega a_s/a_\perp \ll 1$ and $((N/\sqrt{\Omega})a_s/a_\perp)^{1/3} \gg 1$) [23] where $\Omega = \nu_z/\nu_\perp \ll 1$ is the aspect ratio of the trap and $a_\perp$ the transverse harmonic oscillator length.

We now give a theoretical description of the different effects leading to the observed upshift for our situation of two oscillating and interacting dark solitons including the dimensionality of the trap. We consider the two solitons as particles moving in an effective potential which arises from the combination of a harmonic potential due to the trap [30] (see Fig. 3a) and a repulsive potential due to the interaction between the solitons [35]. Because of the spatially symmetric preparation, the effective potential is a symmetric double well potential which is depicted in Fig. 3b. This potential can be expressed as a function of the distance z of each of the solitons from the trap center and its time derivative $\dot{z}$:

$$V(z,\dot{z}) = (2\pi\nu_{1s})^2\,\frac{z^2}{2} + \frac{\mu B^2}{2m\cosh^2(2Bz/\xi)} \qquad (1)$$

where $B = \sqrt{1-(\dot{z}/\xi)^2(\hbar/\mu)^2}$ denotes the darkness of the solitons, μ is a typical interaction energy on the order of the chemical potential, $\xi = \sqrt{\hbar/(m\mu)}$ the associated healing length and ν_{1s} the oscillation frequency of a single trapped soliton. The frequency of the motion is obtained by solving the Euler-Lagrange equation of motion associated with the Lagrangian $\mathcal{L}(z,\dot{z}) = \dot{z}^2/2 - V(z,\dot{z})$. To obtain quantitative agreement, the model has to take into account correctly both the free propagation of the solitons in the trap when they are far away from each other ($z \gg \xi$) and the repulsive interaction when they approach each other.

Good estimates for the single soliton parameter ν_{1s} and the soliton interaction strength μ can be calculated as follows. The single soliton frequency ν_{1s} is obtained by numerical integration of the NPSE describing a single soliton. Because our experimental parameters

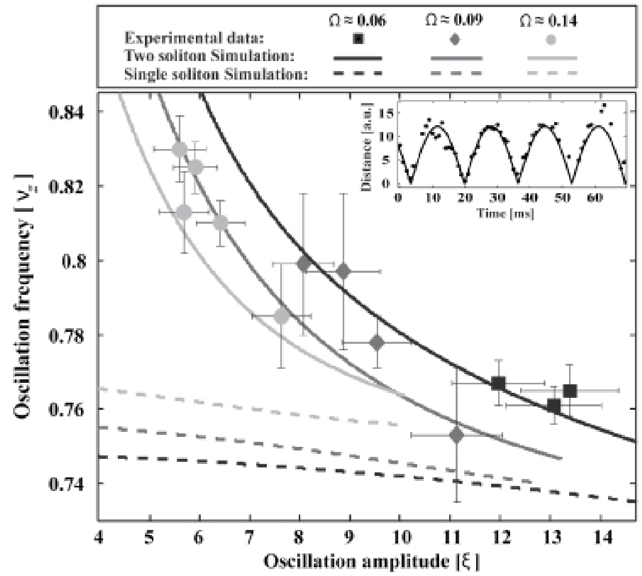

FIG. 2: Comparison between experimentally obtained soliton oscillation frequencies and NPSE simulation for one and two solitons. Each frequency point is deduced from the temporal evolution of the soliton distance as shown in the inset. Different symbols correspond to different aspect ratios Ω of the trap. NPSE simulations are represented by solid lines for the two soliton case, and by dashed lines for the respective single soliton oscillations. The error bars on the measured frequencies account for statistical errors on the measured soliton and trap frequencies and systematic errors on the atom number used to calculate the healing length.

$\Omega \approx 0.06-0.14$ and $N\Omega a_s/a_\perp \approx 1.2-1.8$ are both in the crossover regime and far from the Thomas-Fermi limit, substantial corrections to the asymptotic value $\nu_z/\sqrt{2}$ are expected. Therefore the oscillation frequency of a single dark soliton is upshifted by a few percent from the asymptotic value as discussed in detail using the Bogoliubov-de Gennes analysis of the NPSE in [31] (see Fig. 3a). Our simulations reveal that for the example of our parameter sets with $\Omega \approx 0.06$ this upshift is 5% (see Fig. 3c). Note that the frequencies obtained by 1D GPE simulations are approximately 2% higher than the asymptotic limit because the Thomas-Fermi limit is not reached for our experimental parameters as discussed in [36]. The effect of dimensionality of the system, i.e. the role of the transverse degrees of freedom which is captured only by the NPSE or the 3D GPE, accounts for the remaining 3%. Fig. 3c shows the comparison between the asymptotic limit $\nu_z/\sqrt{2}$ and the single soliton NPSE simulation for one specific trap. The simulation results for the three different parameter sets used in the experiment are shown in Fig. 2. As expected, the single soliton frequency increases with the aspect ratio.

As shown in Fig. 3c, the repulsive interaction between the solitons results in an upshift of the oscillation frequency, compared to the single soliton case, that strongly depends on the oscillation amplitude. Our model accurately reproduces the upshift if the interaction parameter μ is set to be the chemical potential of the condensate obtained from the 3D GPE equation. In our experimentally accessible parameter range, the agreement of the model with NPSE simulations is better than 5%. This allows us to clearly identify the significant role of the repulsive interactions and shows that the effective repulsive potential in Eqn. (1) obtained in the 1D homogeneous case is a good approximation to our complex situation.

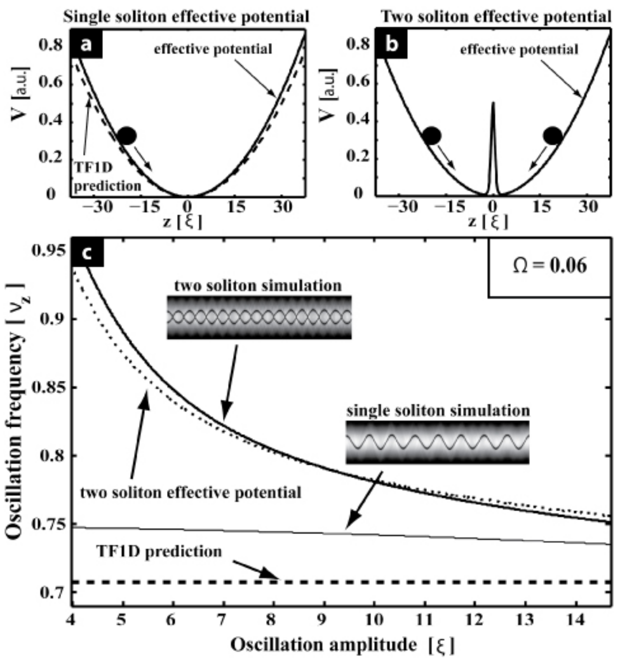

FIG. 3: The oscillation dynamics of dark solitons in a trapped BEC is well captured in an effective particle picture. For one soliton the particle moves in a harmonic trap (a), while for two solitons an additional barrier due to the repulsive interaction appears (b). The dependence of the soliton oscillation frequency on the oscillation amplitude from the trap center is shown in (c). The dashed line shows the TF1D GPE result, the thin solid line indicates the upshift mainly due to the dimensionality, while the thick solid line includes the upshift due to the inter-soliton interaction obtained by solving the NPSE. The dotted line represents the result obtained by the simple effective particle model from Eqn. (1).

In conclusion we controllably create pairs of dark solitons by colliding two atomic clouds released from a double well potential in a harmonic trap. The full dynamics of multiple dark soliton oscillations and collisions can be observed for the first time, allowing for precise frequency measurements and showing that dark solitons are still stable after several collisions. The experimentally observed total upshifts from the TF1D frequency prediction are up to 16%. A simple effective particle picture confirms that the final oscillation frequency of two solitons is affected by two effects namely the single soliton frequency upshift and the inter-soliton interaction. The presented robust method for preparing solitonic excitations will be a starting point for further studies of dark soliton dynamics in the presence of designed potentials as well as for a possible route towards multi-soliton interaction and perhaps even dark soliton gases.

We would like to thank Peter Schmelcher for discussions at the early stages of this project, and Börge Hemmerling, Rudolf Gati and Timo Ottenstein for their help in the experimental setup. We gratefully acknowledge support from NSF, DFG and from the Alexander von Humboldt Foundation. J.E. aknowledges support from the EC Marie-Curie program (EIF-040721).

[1] V. E. Zakharov and A. B. Shabat, Sov. Phys. JETP **37**, 823 (1973).
[2] Y. S. Kivshar and B. Luther-Davies, Phys. Rep. **298**, 81 (1998).
[3] B. Denardo and S. Wright, W. andPutterman, Phys. Rev. Lett. **64**, 1518 (1990).
[4] B. Denardo et al., Phys. Rev. Lett. **68**, 1730 (1992).
[5] M. Chen et al., Phys. Rev. Lett. **70**, 1707 (1993).
[6] B. A. Kalinikos, M. Scott, and C. E. Patton, Phys. Rev. Lett. **84**, 4697 (2000).
[7] P. Emplit et al., Opt. Comm. **62**, 374 (1987).
[8] D. Krökel et al., Phys. Rev. Lett. **60**, 29 (1988).
[9] A. M. Weiner et al., Phys. Rev. Lett. **61**, 2445 (1988).
[10] D. R. Andersen et al., Opt. Lett. **15**, 783 (1990).
[11] G. A. Swartzlander Jr. et al., Phys. Rev. Lett. **66**, 1583 (1991).
[12] S. Burger et al., Phys. Rev. Lett. **83**, 5198 (1999).
[13] J. Denschlag et al., Science **287**, 97 (2000).
[14] B. P. Anderson et al., Phys. Rev. Lett. **86**, 2926 (2001).
[15] Z. Dutton et al., Science **293**, 663 (2001).
[16] P. Engels and C. Atherton, Phys. Rev. Lett. **99**, 160405 (2007).
[17] G.-B. Jo et al., Phys. Rev. Lett. **98**, 180401 (2007).
[18] C. Becker, Nature Physics, nphys962 (2008).
[19] K. J. Blow and N. J. Doran, Phys. Lett. A **107**, 55 (1985).
[20] D. Foursa and P. Emplitt, Phys. Rev. Lett. **77**, 4011 (1996).
[21] A. Dreischuh et al., Phys. Rev. Lett. **96**, 043901 (2006).
[22] K. Bongs et al., C.R. Acad. Sci. Paris, t.2, **4**, 671 (2001).
[23] C. Menotti and S. Stringari, Phys. Rev. A **66**, 043610 (2002).
[24] A. E. Muryshev, H. B. van Linden van den Heuvell, and G. V. Shlyapnikov, Phys. Rev. A **60**, R2665 (1999).
[25] W. P. Reinhardt and C. W. Clark, J. Phys. B At. Mol. Opt. **30**, L785 (1997).
[26] T. F. Scott, R. J. Ballagh, and K. Burnett, J. Phys. B At. Mol. Opt. **31**, L329 (1998).
[27] C. Lee, E. A. Ostrovskaya, and Y. S. Kivshar, J. Phys. B At. Mol. Opt. **40**, 4235 (2007).
[28] Since the submission of the manuscript we learned about a related experiment: I. Shomroni et al., arXiv:0805.3263v1 (2008).
[29] D. R. Scherer et al., Phys. Rev. Lett. **98**, 110402 (2007).
[30] T. Busch and J. R. Anglin, Phys. Rev. Lett. **84**, 2298 (2000).

[31] G. Theocharis et al., Phys. Rev. A **76**, 045601 (2007).
[32] M. Albiez et al., Phys. Rev. Lett. **95**, 010402 (2005).
[33] R. Gati et al., Phys. Rev. Lett. **96**, 130404 (2006).
[34] L. Salasnich, A. Parola, and L. Reatto, Phys. Rev. A **65**, 043614 (2002).
[35] Y. S. Kivshar and W. Królikowski, Opt. Comm. **114**, 353 (1995). This potential is valid provided that the solitons do not overlap.
[36] V. A. Brazhnyi and V. V. Konotop, Phys. Rev. A **68**, 043613 (2003).