The LIF circuit

In the leaky-integrate-and-fire (LIF) model the neuron’s soma integrates synaptic input and constantly leaks charge. It fires when its voltage crosses the threshold value $v_{\text{th}}$. Whenever a neuron emits a spike the exponentially decaying synaptic conductance $g_{\text{syn}}$ is increased by the synaptic strength $w$. 

$$
\frac{C}{\text{d}t} = g_{\text{syn}} (E - v) + \sum g_{\text{syn}} (E_{\text{syn}} - v) \\
\frac{dv}{dt} = \frac{g_{\text{syn}}}{\tau_{\text{syn}}} + w \sum_{j \neq i} \delta ( t - t_{\text{ref}} )
$$

After a spike the membrane is clamped to a reset potential $v_{\text{ref}}$ for a fixed period of time $\tau_{\text{ref}}$. In the limit $\tau_{\text{ref}} \to 0$ the transfer function of the neuron is close to linear. Finite $\tau_{\text{ref}}$ limit the output. Adding Poisson noise softens the onset of the activation, furthermore this renders the neuron stochastic, enabling an ensemble to sample from a Boltzmann distribution [1].


**Relation to Spin Glasses**

Boltzmann machines are mathematically equivalent to spin glasses. In networks of LIF neurons we can therefore observe known physical phenomena such as the Curie law and hysteresis.

$$W = 4J \\
b = 2h - 2W$$

For $J = \frac{1}{4} W$ and $b = \frac{1}{2} b + W$, the Curie law

$$S \sim \frac{T}{T_c - T}$$

To translate between different interaction shapes we match the mean interaction strength within the refractory time (shaded area).

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Network and future hardware implementation

Implementation on the BrainScaleS 1 system uses the 2D topology of the Ising network. We segment the “magnet” into smallish slices which can be handled by a single chip. Each slice consists of it’s “heat bath”, an “external field” neuron and the network. This implementation restricts chip-to-chip communication to nearest neighbors.

Each BSS1 wafer should be able to support more than 10 000 network neurons. The heat bath is implemented as a network of randomly inhibitory connected neurons with leak-over-threshold, in order to reduce the IO requirement. External fields will be emulated by providing two (inhibitory and excitatory) external spike sources that project on to all network neurons.