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Anti-matter wave interferometry with positronium

M.K. Oberthaler

Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany

Abstract

Atomic matter wave optics is a well-developed field in physics. Many different coherent manipulation techniques for atoms have been developed. In this paper the application of one of these techniques, Bragg scattering at near-resonant standing light waves, for ortho-positronium is discussed. Utilizing Bragg diffraction as a coherent splitting mechanism the realization of a Positronium interferometer is discussed. Estimates for the measurement of gravitational acceleration for Positronium are given. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Positronium; Interferometry; Light interaction; Matter wave; Anti-matter

1. Introduction

Interferometry with neutral matter waves is a well-developed field in physics [1]. Many different demonstrations and precision experiments have been performed. In this paper I would like to discuss a possible experimental realization of a Mach–Zehnder type interferometer for positronium (Ps) utilizing far off-resonant standing light waves. Such an interferometer allows for the investigation of relativistic forces like the Anandan force [2] which leads to a quantum phase accumulated by a magnetic dipole propagating in a homogeneous electric field. A more fundamental question like the measurement of the gravitational force on a purely leptonic and anti-matter system will be discussed in detail.

Studying anti-matter gravity with an interferometer was first put forward theoretically by Phillips [3]. He proposed a positronium interferometer with three massive absorption gratings as it was already used for neutral atom interferometry [4]. The main drawback of that interferometer is the low throughput since at least half of the atoms are absorbed per grating due to the open fraction of about 0.5. Furthermore the technology for producing open absorptive diffraction gratings only allows gratings with interferometric quality on the order of square millimeters, which also limits the throughput. The usage of phase gratings realized with standing light waves resolves both drawbacks. On the one hand all atoms are transmitted since a standing light wave acts like a pure phase object. On the other hand the size of the standing light waves can be easily scaled up to some square centimeters.

Why positronium interferometry? Positronium (Ps) is a pure leptonic system consisting of an electron and its anti-particle, the positron. One important feature of Ps is that it is a neutral anti-matter particle. There is no Coulomb interaction present which can mask phase shifts of interest, e.g. inertial

E-mail address: markus.oberthaler@uni-konstanz.de (M.K. Oberthaler).

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forces. The Coulomb interaction was the main problem in the proposed anti-proton gravity experiment as described by Goldman and Nieto [5]. This anti-proton experiment has been closed down with no gravity results. The lightness of Ps is interesting for general investigations of relativistic quantum mechanics. There the Compton wavelength appears as a characteristic length, which is inversely proportional to the rest mass of the particle. For Ps this characteristic length is half the electron Compton wavelength. The big advantage of Ps over electrons lies in the fact that Ps interacts resonantly with light and thus allows the application of techniques developed in the field of atom optics. This makes Positronium a unique particle for investigations of relativistic quantum motion. Another important point is that Ps can be produced with a fairly cheap tabletop source for demonstration purposes. High intensity sources for precision measurements are available at the ISA slow positron facility in Aarhus, Denmark or at the Lawrence Livermore National Laboratory, in the USA.

2. Matter wave interferometer

The basic setup of the proposed matter wave interferometer is similar to the Mach–Zehnder type matter wave interferometers already realized for neutrons and atoms [6]. The schematic is shown in Fig. 1. It consists of three main parts: the formation of a collimated Ps beam, the actual interferometer consisting of three diffraction gratings and the detection.

The details of the experimental realization of the main three parts will be discussed in the following sections. Here I will discuss briefly the basic features of a Mach–Zehnder type interferometer, how gravity leads to a phase shift and what the intrinsic limit of phase shift resolution is.

The beam separation and re-combination in the proposed interferometer occurs by diffraction at three diffraction gratings. Incident waves are divided at the first grating which creates a spatially separated coherent superposition of two matter waves. The achieved separation of the two beams is given by the diffraction angle θ and the length L of the interferometer. The second grating acts like a mirror such that the two coherent beams overlap at the third grating. There the beams are coherently re-combined and the two output beams are formed. The interference signal is detected by translating the third grating and observing the modulation of the number particles in either of the two outgoing beams in the far-field.

The interference pattern is given by

$$N_{\text{out}}(\Delta x) = N_0\left[1 + C\cos\left(2\pi\frac{\Delta x}{d} + \phi\right)\right], \tag{1}$$

where N_0 is the mean number of particles detected in one output, C represents the contrast of the fringes sometimes called visibility, Δx is the distance the last grating is moved and d describes the grating period. An additional phase shift such as gravity or Anandan phase can be described by ϕ.

One important feature of a Mach–Zehnder interferometer is that the visibility of the interference pattern is independent of the initial wavelength, i.e. of the velocity distribution. It is a white light

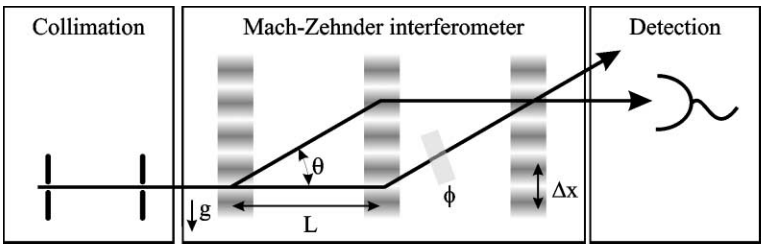

Fig. 1. Schematic for realization of a matter wave interferometer: Two slits are used to collimate the beam. The interferometer is set up with three diffraction gratings equally spaced. A spatially resolving particle detection allows distinction between the two complementary outputs of the Mach–Zehnder interferometer.

interferometer. This is a consequence of the high symmetry which guarantees that the path length difference between the two interferometer arms is zero, independent of the initial wavelength (velocity).

The expected phase shift due to gravity can be calculated in a straightforward way by realizing that the energy difference between the two arms of the interferometer is given by $\Delta U = mgz$ (m is the mass of the particle, g is the gravitational acceleration, z is the spatial splitting between the two paths). Since the splitting results from diffraction, the angle between the two beams is given by $\Theta = \lambda_{\rm dB}/d$ with de Broglie wavelength $\lambda_{\rm dB}$. The phase shift due to gravity is given by

$$\phi_{\rm g} = \frac{\Delta U}{\hbar}\tau = \frac{2\pi}{d} g\left(\frac{L}{v}\right)^2 = \frac{2\pi}{d} g\tau^2, \qquad (2)$$

where $\tau = L/v$ is the interaction time. This result shows that the phase shift within a factor of two can be understood in a classical way. It is given by the distance the particle falls during the interaction measured with the ruler given by the diffraction gratings. Since the divisions are very closely spaced, high resolution can be obtained.

An important figure of merit of an interferometer is the sensitivity, which describes the minimal acceleration that can be detected during a given measurement time. It is given for a beam with a mean velocity v and N_0 detected particles by [7]

$$S = \frac{1}{C\sqrt{N_0}} \frac{d}{2\pi} \frac{1}{\tau^2}. \qquad (3)$$

It is important to note that decreasing the grating period d allows one to measure smaller accelerations. An increase of the interaction time and the throughput of the interferometer have the same effect. Since the lifetime of Ps is comparable with the transit time through the interferometer, an optimal size of the interferometer for best performance can be found (Section 5).

3. Diffraction of positronium by standing light waves

In atom optics diffraction of atoms by standing light waves is a well-established phenomenon [8]. In the following I will discuss the important experimental parameters and what they imply for diffraction of positronium.

One of the important feature of atoms is their resonant interaction with light. As a consequence light can induce a large oscillating dipole moment in the atom which interacts with the light itself. This is quantum mechanically described in the simplest way by a two-level system. The atom is described by one ground and one excited state which has a finite lifetime and thus a line width of Γ. Positronium has a level scheme which comes very close to the two level system, as can be seen in Fig. 2. In this paper only the ortho-Ps energy levels are discussed since the gravitational phase shift is proportional to the square of the interaction time and thus only the 'long' living ortho-Ps can be used.

The interaction energy U for an induced electric dipole moment with the inducing electric field is proportional to the light intensity and is inverse proportional to the detuning of the laser frequency $\nu_{\rm l}$ to the atomic transition ν_0 as described by $\delta = \nu_{\rm l} - \nu_0$. In the limit of large detuning, i.e. $\delta \gg \Gamma$ and $\delta \gg \Gamma\sqrt{I/(2I_{\rm s})}$, the interaction energy and thus the potential for the ground state is given by [9]

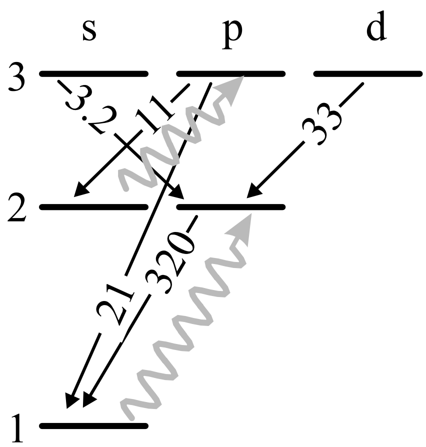

Fig. 2. Grotrian diagram of the lowest energy levels of *ortho*-Ps: The two optical transitions which are discussed are indicated: 1s–2p at a wavelength of 243 nm and 2s–3p at a wavelength of 1.3 μm. The Einstein coefficients are given in units of 10^6 s^{-1}.

$$U(r) = \frac{\hbar\Gamma^2}{8I_s}\frac{I(r)}{\delta}, \qquad (4)$$

where I_s is the saturation intensity and $I(r)$ is the spatial intensity distribution of the light. In the literature the potential is often called the light shift potential.

It is straightforward to see from Eq. (4) that a standing light wave with the wavelength λ leads to a periodic potential for the atoms with a period of $d = \lambda/2$ and thus to a diffraction angle of $\theta = 2\lambda_{dB}/\lambda$.

In order to get only one diffracted beam, as indicated in Fig. 1, one has to realize a standing light wave in the Bragg regime [10], i.e. the interaction length has to be bigger than the Talbot length given by

$$L_T = \frac{2d^2}{\lambda_{dB}}. \qquad (5)$$

In the following I always assume that the extension of the standing light wave is given by the Talbot length L_T.

The Bragg diffraction efficiency for such a light crystal is given by [10]

$$I_B = \sin^2\left(2\pi\frac{h^2}{8m\lambda^2}U_{max}\right), \qquad (6)$$

where m is the mass of the particle, λ the wavelength of the resonantly interacting light wave and U_{max} is the modulation depth of the light shift potential. The formula shows that Bragg scattering can be used to create arbitrary splitting ratios from perfect beam splitters $I_B = 0.5$ to mirrors $I_B = 1$. The light intensity requires to diffract all incoming atoms (Bragg mirror) by a standing light wave with detuning δ can be calculated as

$$I_{in} = \frac{2\pi^2}{\lambda^2}\frac{\hbar}{m\Gamma^2}\delta I_s. \qquad (7)$$

This implies that by decreasing the laser detuning, the necessary light intensity can be reduced. However there is a limit for the detuning δ. This arises from the incoherent process of spontaneous emission.

The probability for spontaneous emission is proportional to the probability of exciting an atom during the interaction with the Bragg standing light wave, which is given by [9]

$$p_s = \frac{2\Gamma^3\lambda^2 m}{h}\frac{I_{in}}{I_s}\frac{1}{\delta^2}. \qquad (8)$$

The important result is that spontaneous emission scales with $1/\delta^2$ in comparison to the optical potential, which is proportional to $1/\delta$. This makes it always possible to reduce spontaneous emission for a given potential height by increasing the laser intensity and the detuning. Combining this result with the Eq. (6) one finds

$$p_s = 2\pi\frac{\Gamma}{\delta}. \qquad (9)$$

The experimental parameters for Bragg diffraction of *ortho*-Ps in the 1s and 2s state are given in Table 1. The estimates show that the necessary light power for Bragg diffraction of the 1s *o*-Ps state currently cannot be produced with continu-

Table 1
A summary of the standing light wave diffraction characteristics of a 10 eV 1s and 2s *ortho*-Ps state

Positronium state	1s *o*-Ps	2s *o*-Ps
Lifetime	142 ns	1.1 μs
Optical transition	1s–2p	2s–3p
Optical wavelength, λ	243.1 nm	1312.5 nm
Diffraction angle, θ	2.3 mrad	0.42 mrad
Talbot length, L_T	0.1 mm	3.1 mm
Einstein coefficient [11], A	3.17×10^8	1.13×10^7
Saturation intensity [9], I_s	460 mW/cm^2	0.1 mW/cm^2
Necessary laser power	42 W	260 mW

The standing light wave is assumed to have an interaction length equal to the corresponding Talbot length and a height of 5 mm. For the estimate of the necessary laser power the detuning of the laser light was chosen to correspond to a spontaneous emission probability of 1% and a diffraction efficiency of 100%.

ous-wave lasers. However pulsed excimer lasers can be used to deliver the required high powers. A commercial KrF excimer laser at a wavelength of 248 nm has enough power even for a detuning as large as of 5 nm. Although the spontaneous emission is absolutely negligible for such a large detuning, the ionization might become a problem due to the high light intensity. For the 2s–3p transition the laser power can be realized readily with commercially available laser diodes and fiber amplifiers. This wavelength is used in fiber communications and thus narrow line width and high power laser diodes for tabletop experiments are available.

4. Positronium interferometer

One crucial issue for the realization of a Ps-interferometer is a collimated metastable Ps beam. Using diffraction at a standing light wave of wavelength λ the splitting angle is given by the diffraction angle $\Theta = 2\lambda_{dB}/\lambda$. Since the proposed interferometer is realized with phase crystals (no absorption), one has to distinguish between the two complementary output ports of the interferometer in order to be able to observe interference fringes. Thus the impinging particle beam has to be collimated better than the diffraction angle, which is 2.4 mrad for 243 nm and 0.4 mrad for 1.3 μm standing light wave.

An established method for Ps formation is scattering of positrons from atoms in gas phase [12] for which both states of interest can be formed [13]. Theoretical estimates of the angular dependence of the Ps formation cross-section predict that a substantial fraction of the Ps atoms are emitted in a cone of a few degrees around the incident positron direction. Experimental results confirm these theoretical predictions. The directionality of the formation allows the realization of a collimated Ps beam. By applying static field 'electron optics' [14] and a special design of the radioactive source, one can adapt the positron beam shape to match the one-dimensional collimation of the Ps beam. It has been shown that a positron beam can be focused down to 20 μm. By sending such a focused positron beam into a gas cell and mounting a massive 20 μm wide slit 10 cm down stream one should be able to realize a Ps beam with the requisite collimation.

The expected flux can be estimated by assuming the cross-section for $n = 2$ Ps formation for positron–potassium scattering, a positron source with 10^6 positrons per second and a detection efficiency of $\sim$1%. One can expect to detect a few counts/second in the collimated Ps beam [15]. Such low count rates allow the demonstration of the interferometer and the measurement of phase shifts on the order of a few mrad.

In order to distinguish between the two different output ports, a spatially sensitive detector can be used. The proof of principle has been shown in [16], but for applying this technique to Ps interferometry the reported angular resolution of 8 mrad has to be improved. For the 2s *o*-Ps interferometer another atom-optical method can be used. By tuning the last standing light wave exactly on resonance an absorptive periodic grating can be realized [17], and thus the integral transmission exhibits the fringe pattern.

5. Sensitivity for gravity measurement

In the following Section 1 will give an estimate for the measurement of gravitational acceleration of anti-matter using a positronium interferometer. The problem with the measurement of gravity for anti-matter to date is that a charged anti-matter particle, the anti-proton, was used [5]. The Coulomb interaction makes these experiments very difficult. Thus the direct demonstration of the action of gravity on anti-matter has still to be performed. From these other experiments one cannot rule out a difference between the gravitational acceleration of matter and anti-matter on the order of few percent.

The interferometer proposed here is intrinsically very weakly dependent on electric and magnetic fields, since Ps is neutral. In order to estimate the expected sensitivity for gravity one has to take into account the visibility, mean count rate including the finite lifetime and the expected resolution for accelerations.

The minimum detectable acceleration per square root of the measurement time is shown in

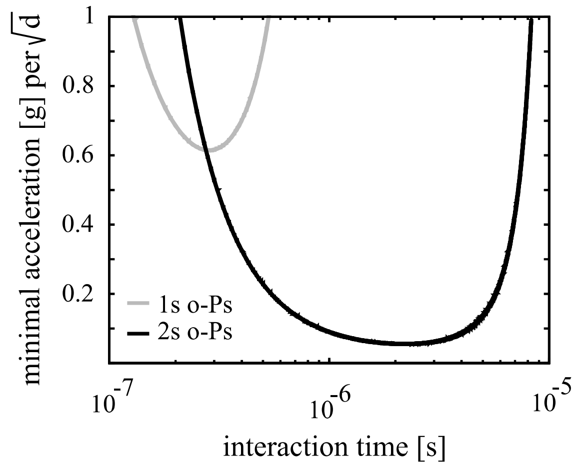

Fig. 3. Sensitivity estimates for 1s and 2s *o*-Ps interferometer. The graph shows the minimal gravitational acceleration (within 1σ) in units of the earth's gravitational acceleration for an incident flux of 10^6 Ps/s for a measurement lasting for one day.

Fig. 3 using Eq. (3). One can see that for interaction times longer than 0.25 μs the 1321 nm interferometer is preferable. With an optimized Ps source of 10^6 Ps/s and a measurement time of 1 day one can measure 0.07 g within one standard deviation. Note that such an optimized interferometer would be $2L = 5$ m long. It is obvious that such an experiment is very difficult since the thermal drifts of the interferometer have to be kept smaller than the phase shift which corresponds to 0.04 nm. An improvement can be achieved by utilizing higher-order Bragg scattering which has been observed up to the sixth order [18] leading to a sixfold improvement of the sensitivity. By applying more elaborate manipulation techniques an improvement by a factor of 20 seems possible.

6. Conclusions

In summary, it has been shown that a 2s ortho-Ps interferometer realized with standing light waves is experimentally feasible. The necessary experimental parameters are discussed. The estimate for using the interferometer to measure gravitational acceleration of purely leptonic system shows that with current techniques it is a very challenging experiment. It seems to be a good time to start working on the realization of a collimated Ps beam. This would immediately allow for very interesting investigations due to the lightness of Ps such as relativistic forces like the Anandan force or atom optical experiments utilizing the large photon recoil.

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