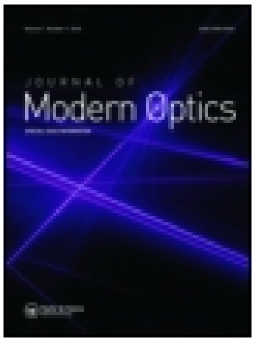


Journal of Modern Optics

ISSN: 0950-0340 (Print) 1362-3044 (Online) Journal homepage: http://www.tandfonline.com/loi/tmop20

Anomalous transmission in atom optics

Herman Batelaan , Ernst M. Rasel , Markus K. Oberthaler , Jörg Schmiedmayer & Anton Zeilinger

JOURNAL OF MODERN OPTICS, 1997, VOL. 44, NO. 11/12, 2629–2641

Anomalous transmission in atom optics

HERMAN BATELAAN†, ERNST M. RASEL‡,
MARKUS K. OBERTHALER†, JÖRG SCHMIEDMAYER†
and ANTON ZEILINGER†

† Institut für Experimentalphysik, Universität Innsbruck,
A-6020 Innsbruck, Austria
‡ Laboratoire Kastler-Brossel, Ecole Normale Supérieure,
24 rue Lhomond, F-75005 Paris, France

(*Received 20 November 1996; revision received 27 May 1997*)

Abstract. Anomalous transmission for X-rays in crystals as discovered by Borrmann can also be observed for atoms interacting with a standing light wave on resonance. It may be interpreted as a manifestation of a grey state. We describe phenomena based on anomalous transmission for various regimes of the atom–light interaction and elucidate these by numerical calculations for argon and calcium. The grey state in anomalous transmission is linked to the dark state in velocity selective coherent population trapping in atomic three-level Λ systems.

1. Introduction

Anomalous transmission of light and matter waves, called '*Übernormale Durchlässigkeit*' by Borrmann [1] in 1941, is the remarkable increase in the transmission of waves in an absorbing periodic medium at the Bragg angle. This effect was observed for X-rays, neutrons, electrons and recently for atoms [1–5]. It cannot be explained by a simple classical shadow effect of rows of atoms in the crystal, but it is caused by interference between two momentum states propagating in the crystal [6, 7].

Experimentally there are two characteristics of anomalous transmission. First, an increase in the transmitted intensity can be observed by tilting the crystal across the Bragg angle. Second, the Pendellösung is damped, leading for thick crystals to a far-field diffraction pattern consisting of two equally high peaks at the Bragg angles, which correspond to the two interfering momentum states inside the crystals.

In the field of atom optics, Bragg scattering of atoms from a refractive light crystal, created by a far detuned standing light wave, has been predicted and demonstrated [8–14]. Likewise, anomalous transmission of atoms through an absorptive light crystal can be observed [5]. The necessary absorption mechanism can be realized by a two-level atom where the excited state decays in a third non-interacting and non-detected state. The absorption process starts with excitation by resonant light and is followed by spontaneous decay to the non-observed atomic state. Therefore atoms emitting a spontaneous photon can be regarded as absorbed.

0950–0340/97 $12·00

In this paper we present an analysis based on quantized momentum states to show the anomalous transmission for atoms through a light crystal. With a simple numerical model we investigate two different regimes for anomalous transmission, namely the regime of very long lifetime and the regime of very short lifetime respectively of the excited atomic state. In both regimes we find parameters for which this effect should be observable in an experiment. We shall show that the excitation of the atoms inside a standing light field leads to extinction of the oscillating behaviour of the Pendellösung belonging to Bragg scattering and that the atom is left in a grey state. This connection between X-ray and atomic anomalous transmission is a very good example of the possibility to transfer elements from optics to atom optics.

We conclude this paper with a discussion of the connection between anomalous transmission and velocity selective coherent population trapping (VSCPT) [15]. Where VSCPT is observed in a three-level Λ system with orthogonal polarizations of light, anomalous transmission requires only an absorption mechanism combined with a two-level system. We shall demonstrate that a truncated set of equations for Bragg scattering is very similar to the equations describing VSCPT, which immediately suggests that, with the right conditions, one may apart from observing a grey state in Bragg scattering (which is the basis for anomalous transmission) also observe Bragg like scattering in the VSCPT case [10].

2. Building a 'light-field crystal'

Atoms can easily be manipulated via their interaction with light fields. We consider here a two-level atom with an additional decay channel of the excited state to a third non-interacting state. The interaction between the light field and the atom can then be described by a complex optical potential [16]

$$U(x,y) = \hbar \frac{d^2 E(x,y)^2}{\Delta + \mathrm{i}\gamma/2}. \tag{1}$$

Here $E(x,y)$ is the electric field connected to the light, d is the dipole matrix element of the transition, Δ represents the difference between the driving light frequency and the eigenfrequency of the transition, and γ is the loss rate from the excited level to the non-interacting state. Thus light fields can model a complex potential for atoms. A significant property is that, using diffractive optics and holography, one can in principle build any desirable light field pattern in the laboratory. This makes the atom–light system a model system for investigating matter waves in periodic potentials with adjustable ratio of refraction and absorption. Therefore we call our standing light waves a '*light crystal*'. Generalization to two- or three-dimensional light crystals is straightforward.

3. Bragg scattering and grey states

Before discussing anomalous transmission in detail we give a short description of the standard view and calculation of Bragg scattering of an atomic beam from a standing light wave following Stenholm [17] and Gould *et al.* [18]. Additionally, we show how Bragg scattering is described in an intuitive picture involving grey states which will later be important for understanding the anomalous transmission effect.

For Bragg scattering, one may consider a two-level atom which is interacting with two extended counterpropagating laser beams. We shall assume that only the motion of the atom parallel to the laser beams in influenced and all subsequent discussion of energy and momentum conservation should be considered in this one-dimensional approximation. In particular, for an experimental set-up where a transversely, well collimated atomic beam with thermal velocity is crossed by extended laser beams, this assumption is valid because the direction of the photon recoil $\hbar k_{\text{photon}}$ is parallel to the laser beams, and nearly orthogonal to the atomic beam and additionally $k_{\text{atom}} \gg k_{\text{photon}}$.

Following Stenholm's [17] approach the evolution of the amplitudes of the ground states g_j and excited states e_j for the atomic motional states can be obtained from the Heisenberg equation. The subscript j indicates that the atomic momentum along the laser beam is quantized in multiples of the photon momentum $(j = -\infty, \infty)$:

$$
\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} g_{j+1} \\ e_j \\ g_{j-1} \end{bmatrix}
= \begin{bmatrix} (j+1)W_0 + (j+1)^2\epsilon & -\Omega/2 & 0 \\ -\Omega/2 & jW_0 + j^2\epsilon + \Delta - \mathrm{i}\frac{1}{2}\gamma & -\Omega/2 \\ 0 & -\Omega/2 & (j-1)W_0 + (j-1)^2\epsilon \end{bmatrix},
\times \begin{bmatrix} g_{j+1} \\ e_j \\ g_{j-1} \end{bmatrix}, \tag{2}
$$

where $\epsilon = \hbar k^2/2M$ is the recoil energy, k is the wave-vector of the light field, γ is the loss rate to the non-observed state, and Δ and Ω are the detuning and the standing-wave Rabi frequency. We have added the spontaneous emission to allow for an on-resonance calculation and used a quantum Monte Carlo method [19] for the numerical calculations. The spontaneous emission occurs to a state that is not explicitly included in the calculation. This set of equations is intuitively clear. Each ground state g_j is coupled through the Rabi frequency with its neighbouring excited states e_{j+1} and e_{j-1}, meaning that an atom can absorb or emit only one photon at a time. The diagonal terms give the energy shift belonging to each state, which consists of the detuning Δ, the recoil shift ϵ and the Doppler shift W_0.

The above set of equations is extremely rich and makes it necessary to point out the cases that we want to consider. Throughout this paper we will limit ourselves to long interaction times Δt corresponding to Bragg scattering. This means that the energy transfer to the atom (in our one-dimensional approximation) has been determined so sharply, $\Delta E = \hbar/\Delta t$, that higher-transverse-momentum states cannot be reached owing to the recoil shift ($\Delta E \ll \epsilon$) (figure 1). Typically, interaction times much longer than the recoil frequency should be chosen to

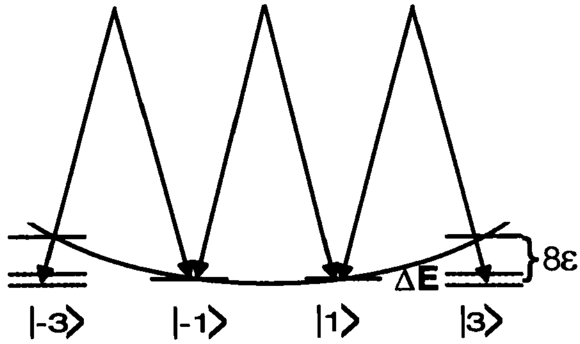


Figure 1. The energy levels of the different atomic momentum ground states $|j\hbar k\rangle$ are shown including the kinetic shifts in the one-dimensional approximation. In the standing light wave the atom can only change its momentum through absorption and stimulated emission by two photon recoils ($2\hbar k$). The comparison between the energy uncertainty $\Delta E(=\hbar/\Delta t)$ in the scattering process and the kinetic energy shifts is critical. For Bragg scattering the uncertainty is much smaller, $\Delta E \ll 8\epsilon$, and an atom starting in the $|-1\hbar k\rangle$ or $|+1\hbar k\rangle$ state cannot reach higher-momentum states. If the atom does not start in one of these states (equal to the Bragg angles), then no scattering will take place because of these kinetic shifts.

satisfy this condition. Furthermore the laser intensity (Rabi frequency) will be chosen just strong enough for some coupling between different momentum states to occur, that is the atom will scatter some photons (weak-coupling limit). Under these conditions we may further discriminate between the following possibilities depending on the chosen conditions of the atom–laser interaction.

(i) The laser is tuned far off resonance, $\Delta \gg \gamma$, in which case the spontaneous emission term can be omitted.
(ii) Alternatively, on resonance $\Delta < \gamma$.

In the last case ($\Delta < \gamma$) we may further discriminate between the specific properties of the involved atomic transition.

(*a*) An atomic transition is chosen with a natural linewidth much larger than the recoil shift ($\gamma \gg \epsilon$).
(*b*) Alternatively, a natural linewidth is chosen much smaller than the recoil shift ($\gamma \ll \epsilon$).

For a large detuning, $\Delta \gg \gamma$, the spontaneous emission term $i\gamma/2$ can be neglected. Treating first-order Bragg scattering, we can now proceed to truncate the set of equations at e_2 and e_{-2} since close to the Bragg resonance the higher-momentum states are weakly populated. Here, we are considering that the atoms enter the laser light with the momentum $|+\hbar k\rangle$ ($g_1 = 1$), which corresponds to incidence at the exact Bragg angle. The Heisenberg equation is chosen such that for this propagation direction the Doppler shift vanishes ($W_0 = 0$).

After truncating this set of equations at e_2 and e_{-2} one obtains

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} e_2 \\ g_1 \\ e_0 \\ g_{-1} \\ e_{-2} \end{bmatrix} = \begin{bmatrix} \Delta + 4\epsilon & -\Omega/2 & 0 & 0 & 0 \\ -\Omega/2 & \epsilon & -\Omega/2 & 0 & 0 \\ 0 & -\Omega/2 & \Delta & -\Omega/2 & 0 \\ 0 & 0 & -\Omega/2 & \epsilon & -\Omega/2 \\ 0 & 0 & 0 & -\Omega/2 & \Delta + 4\epsilon \end{bmatrix} \begin{bmatrix} e_2 \\ g_1 \\ e_0 \\ g_{-1} \\ e_{-2} \end{bmatrix}. \tag{3}$$

We now discuss case (i). For *large detunings* ($\Delta \gg \gamma$) and small Rabi frequencies ($\Delta \gg \Omega$) we can adiabatically eliminate the excited states at $2\hbar k$, $0\hbar k$ and $-2\hbar k$ momentum and shift the energy with the transformation $g_{-1,1} \to g_{-1,1}\exp(\mathrm{i}U_0 t)$, to find that

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} g_1 \\ g_{-1} \end{bmatrix} = \begin{bmatrix} \epsilon - U_0 & -U_0 \\ -U_0 & \epsilon - U_0 \end{bmatrix}\begin{bmatrix} g_1 \\ g_{-1} \end{bmatrix}, \tag{4}$$

where $U_0 = \Omega^2/4\Delta$ is the light shift potential of the atom in the laser light. In the basis $|-\hbar k\rangle$, $|+\hbar k\rangle$ this equation is analytically solved by

$$g_1 = \exp(-\mathrm{i}\epsilon t)\,[A - B\exp(\mathrm{i}2U_0 t)] \tag{5}$$

$$g_{-1} = \exp(-\mathrm{i}\epsilon t)\,[A + B\exp(\mathrm{i}2U_0 t)], \tag{6}$$

where A and B are determined by initial conditions. This equation could also have been directly obtained from a description more familiar to the two-beam approximation in dynamical diffraction theory. The wavefunction of the atoms inside the standing light wave is described by the $|+\rangle$ and $|-\rangle$ eigenstates of the atom in the periodic potential (also called the Bloch states) which are superpositions of the incoming and the Bragg deflected beam. A transformation to this basis

$$\begin{bmatrix} |+\rangle \\ |-\rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{2^{1/2}}(|-\hbar k\rangle + |+\hbar k\rangle) \\ \frac{1}{2^{1/2}}(|-\hbar k\rangle - |+\hbar k\rangle) \end{bmatrix}, \tag{7}$$

which corresponds to two standing matter waves with a π phase shift, decouples the above two equations and yields, in terms of their amplitudes b_+ and b_-,

$$\begin{aligned} \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_+ &= (\epsilon - 2U_0)b_+, \\ \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}b_- &= \epsilon b_-. \end{aligned} \tag{8}$$

Although this basis transformation is mathematically trivial, the first basis is physically more appropriate for a well collimated atomic beam outside the light crystal, while the second basis is more appropriate inside the light crystal. The two equations mirror the fact that the two states couple differently to the light field; therefore we call the $|+\rangle$ state a bright (or strong-coupling) state and the $|-\rangle$ state a grey (or weak-coupling) state.

For initial condition of Bragg incidence $g_1 = 1$, $g_{-1} = 0$ and $W_0 = 0$, we find that $|g_1|^2 = \cos^2(U_0 t)$ (figure 2). This means that, when an atomic beam is directed at the Bragg angle with the standing light waves, the forward scattered intensity shows a periodic dependence on the potential height $2U_0$ and interaction time t. This is known as Pendellösung. This could be regarded as an interference between the $|-\rangle$ state and the $|+\rangle$ state which accumulate because of their different

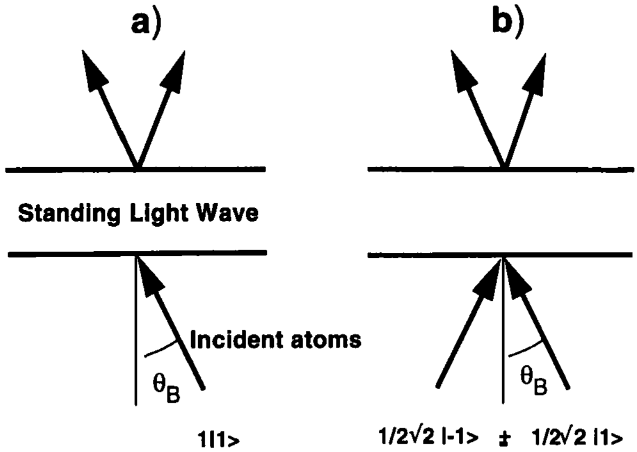


Figure 2. (*a*) A well collimated atomic beam crossing a thick standing light wave at the Bragg angle θ_B results in an oscillating behaviour in the intensity of the two outgoing beams (Pendellösung). (*b*) Initial conditions where two atomic beams with a well defined phase difference enter the standing light wave lead to a time independence for both indicated phases. This is a feature of grey and dark states.

couplings to the light field different phases. Preparing only one eigenstate by combining two coherent atomic beams incident symmetrically at the '+' and '−' Bragg angle corresponding to $g_1 = 1/2^{1/2}$, $g_{-1} = -1/2^{1/2}$ ($|g_1|^2 = |g_{-1}|^2 = 0{\cdot}5$) the Pendellösung vanishes.

A numerical solution of the original set of equations truncated not at $j = 1$ but at $j = 5$ at very weak coupling for the case (i) (*a*) ($\Delta \gg \gamma \gg \epsilon$) shows that this grey-state behaviour does not disappear when allowing coupling to higher-momentum states.

4. Anomalous transmission

When the laser is chosen *on resonance* (ii), coupling to the excited state becomes important. Some of the consequences of this case for a closed two-level system have been carefully studied by Schumacher *et al.* [20]. In contrast, we shall consider an open two-level system where the emission of a photon leads to an unobserved state and therefore can be viewed as absorption of the atom. We must now consider two cases: the recoil energy is much smaller than the atomic linewidth (case (ii) (*a*)) and the recoil energy is much larger than the atomic linewidth (case (ii) (*b*)).

4.1. *Recoil shift smaller than natural line width* ($\epsilon \ll \gamma$)

For small recoil (ii) (*a*) ($\epsilon \ll \gamma$), we are not allowed to reduce the set of equations (2) to only g_1 and g_{-1}, but have to include e_2, e_0 and e_{-2}. Still, the presence of the grey state is felt. This is best seen in a description using the Bloch state basis (equation (7)).

As before, we consider the motion in the light crystal in a truncated set of quantized momentum states $\{|g, +\hbar k\rangle, |g, -\hbar k\rangle\}$; here the ground state g is explicitly indicated. We chose the basis (equation (7)) $\{|+\rangle, |-\rangle\}$ for a simple and intuitive picture of the atomic motion in the standing light wave by calculating the coupling of an atom to the resonant light field that forms the light crystal. On resonance we have to take into account the excited momentum states $\{|e, +2\hbar k\rangle, |e, 0\hbar k, |e, -2\hbar k\rangle\}$. The coupling strength to the excited state is given by $\langle e, n\hbar k|\mathbf{d}\cdot\mathbf{E}|g, m\hbar k\rangle = M\delta_{n,m=n\pm 1}$, where $M = -\Omega/2$. We neglect the effect of the recoil shift in this equation because the natural line width is large. The coupling of the $|-\rangle$ state and $|+\rangle$ state to the various excited states follows directly:

$$\begin{aligned} \langle e, +2\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle &= \frac{1}{2^{1/2}} M, \\ \langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle &= 0, \\ \langle e, -2\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle &= -\frac{1}{2^{1/2}} M, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \langle e, +2\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle &= \frac{1}{2^{1/2}} M, \\ \langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle &= 2^{1/2} M, \\ \langle e, -2\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle &= \frac{1}{2^{1/2}} M. \end{aligned} \tag{10}$$

To get the total transition amplitude for each state ($|-\rangle$ and $|+\rangle$), we have to sum over the three momentum states:

$$\begin{aligned} |\langle e|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 &= |\langle e, -2\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle + \langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle + \langle e, +2\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2, \\ |\langle e|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 &= |\langle e, -2\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle + \langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle + \langle e, +2\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2, \end{aligned} \tag{11}$$

where the question of interference arises. Note that the different excited states are distinguishable in the far field because of their momentum states and therefore no interferences between the different amplitudes arise and one finds the following coupling to the excited states:

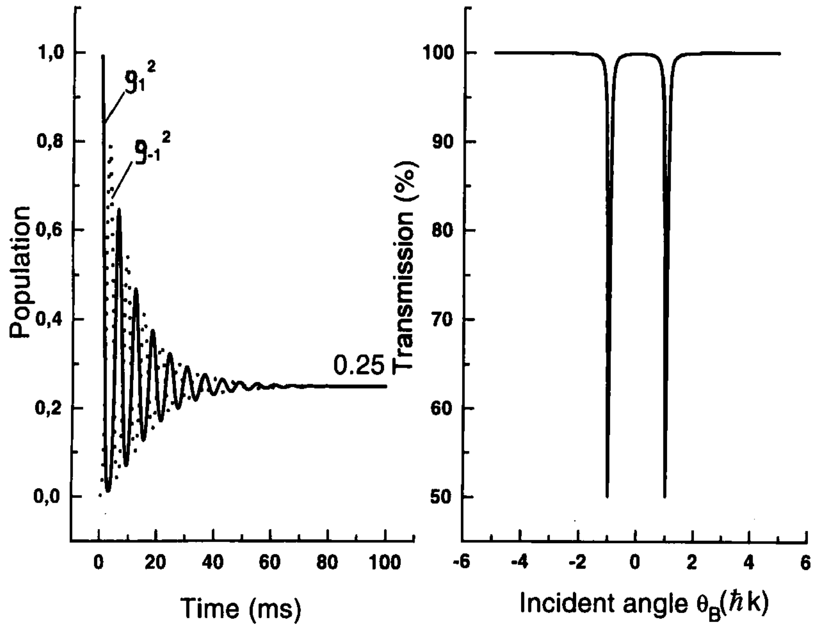
$$\begin{aligned} |\langle e|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 &= |\langle e, -2\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 + |\langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 + |\langle e, +2\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 \\ &= \tfrac{1}{2}M^2 + 2M^2 + \tfrac{1}{2}M^2 \\ &= 3M^2, \end{aligned} \tag{12}$$

$$\begin{aligned} |\langle e|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 &= |\langle e, -2\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 + |\langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 + |\langle e, +2\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 \\ &= \tfrac{1}{2}M^2 + 0M^2 + \tfrac{1}{2}M^2 \\ &= M^2. \end{aligned} \tag{13}$$

On the other hand for a simple incoming beam $|g, p\rangle$ ($p \neq \hbar k$) we find that

$$\begin{aligned} |\langle e|\mathbf{d}\cdot\mathbf{E}|g, p\rangle|^2 &= |\langle e, p - \hbar k|\mathbf{d}\cdot\mathbf{E}|g, p\rangle|^2 + |\langle e, p + \hbar k|\mathbf{d}\cdot\mathbf{E}|g, p\rangle|^2 \\ &= M^2 + M^2 \\ &= 2M^2. \end{aligned} \tag{14}$$

Therefore the $|-\rangle$ state couples more weakly to the radiation field than an atom incident in an arbitrary direction does, and the $|+\rangle$ state couples stronger. Once in the excited state the atom will spontaneously decay, and therefore the $|-\rangle$ state will



(*a*) (*b*)

Figure 3. (*a*) The damping of the Pendellösung for the case of Ar* is shown. The departure of the minima from zero indicates the presence of the grey state. The inset shows that the atomic beam is transmitted through the light crystal longer than would be expected from considering absorption without taking coherences into account. (*b*) The transmission of atoms is increased at the Bragg angles just as for anomalous transmission of X-rays.

scatter less light and the $|+\rangle$ state will scatter more light than an atom incident at arbitrary direction. A well collimated beam incident at the Bragg angle is described by a 50 : 50 superposition of the $|+\rangle$ and $|-\rangle$ states. In this case the overall scattering is less than for arbitrary incidence, because the reduced scattering of the grey state is not compensated by the enhanced scattering of the bright state.

We performed a numerical integration of the full set of equations (2) truncated at $j = \pm 20$ including the spontaneous emission term, which confirms the physical picture above. We chose as an example the Ar* line $1s_5$–$2p_4$ ($4\,^1S$–$4\,^3P_0$). For this line the excited state decays preferably to the states $1s_3$ and $1s_2$ (branching ratio $1s_5 : 1s_3 : 1s_2 = 2 : 56 : 42$). For our purposes this can be viewed as an open two-level system, because atoms in the states $1s_3$ and $1s_2$ can be filtered out. The numerical results for exact Bragg incidence and for a detuning of half a linewidth (as shown in figure 3 (*a*)) demonstrate that the Pendellösung is damped exponentially owing to the absorption. The total transmission of the state $1s_5$ as shown in the insert decays half as fast as a beam which pass the light field at arbitrary direction for long interaction times. The characteristic time scale is given by the absorption time $1/\gamma_p = (\gamma^2 + 2\Omega^2 + 4\Delta^2)/2\Omega^2\gamma$ (figure 3 (*b*)) [5, 21]. For an interaction, which lasts longer than the characteristic time, the atomic population survives essentially in the grey state.

4.2. *Recoil shift larger than natural linewidth* ($\epsilon \gg \gamma$)

Now we shall consider the case (ii) (*b*) where the recoil energy is much larger than the linewidth of the atomic transition ($\epsilon \gg \gamma$). For the case (ii) (*a*) ($\epsilon \ll \gamma$)

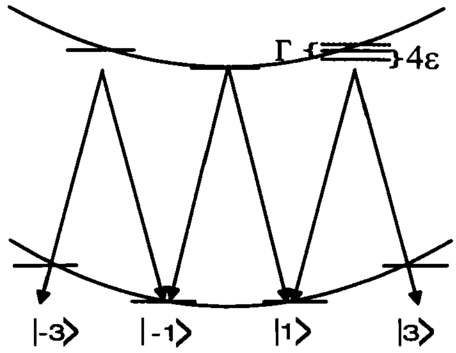


Figure 4. The coupling between the momentum ground and excited states is shown for the on-resonance case where in contrast with figure 1 the excited state cannot be neglected. Here the comparison between the natural linewidth γ of the atom and the recoil shift in energy is essential. For a large recoil shift, $4\epsilon \gg \gamma$, the system is reduced to two momentum ground states and one excited state.

discussed above, we could not truncate the set of equations at g_1 and g_{-1}. Here we are allowed to do so because the excited states are recoil shifted with respect to each other and are therefore shifted out of resonance (figure 4). Note that because of the narrow atomic linewidth the resonance condition is not $\Delta = 0$ but $\Delta = -\epsilon$.

The analysis of our problem is again simplified by using the basis (7). The atom inside the resonant light field couples only via the excited state with zero momentum $|e, 0\hbar k\rangle$, because of the recoil shift (the amplitude of the Rabi flopping to the $|\pm 2\hbar k\rangle$ states is reduced to $\Omega^2/[\Omega^2 + (4\epsilon)^2]$. The result can now be copied from the previous section:

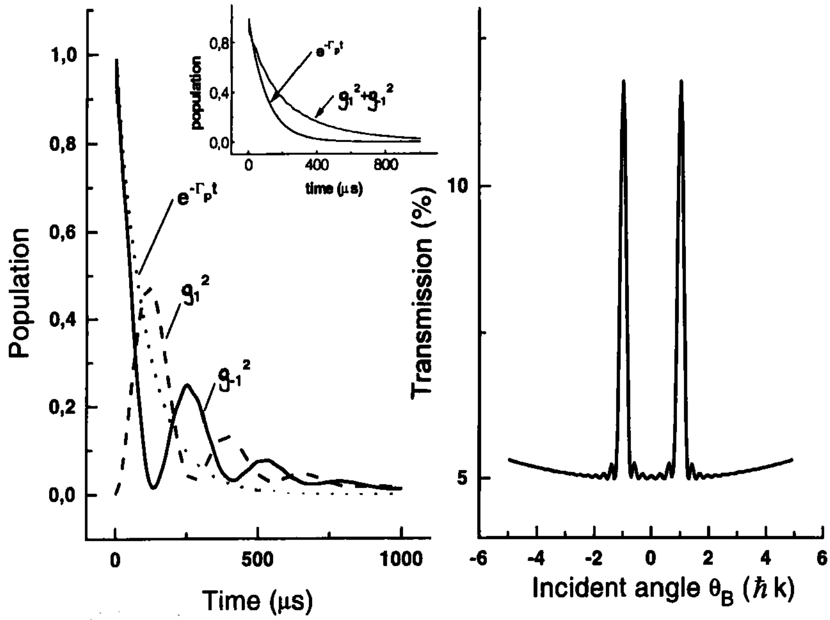
$$\begin{aligned} |\langle e|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 &= |\langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|+\rangle|^2 = 2M^2, \\ |\langle e|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 &= |\langle e, 0\hbar k|\mathbf{d}\cdot\mathbf{E}|-\rangle|^2 = 0. \end{aligned} \tag{15}$$

Contrary to the previous case ($\gamma \gg \epsilon$) the atom–light interaction depends strongly on the angle of incidence. For a detuning $\Delta = -\epsilon$ the light field is only resonant with the atomic transition at the two Bragg angles; hence absorption occurs only at these angles of incidence. Decomposing the incident atomic beam in the eigenstates $|-\rangle$ and $|+\rangle$, one can easily deduce that half the atoms are in a dark state and pass the crystal also for times much longer than the absorption time. The momentum distribution in the far field is then given by two equally high peaks.

On the contrary, the atomic interaction with two counterpropagating incoherent light waves tuned on the atomic resonance would result in a complete absorption of the atoms for the same angle of incidence (figure 5).

We solved our system numerically by integrating the set of equations in time. The interaction time must be chosen to be at least as long as the spontaneous decay time. For a shorter interaction time the Pendellösung will not have been damped out yet and also the absorption will not be efficient. We checked using a quantum Monte Carlo method [19] that recoil due to spontaneous emission back to the initial state is not important at our indicated parameters.

Only a few atomic lines have such small linewidths. The intercombination lines of alkaline earths such as Ca (and Mg) are examples (table), which would



(*a*) (*b*)

Figure 5. (*a*) The damping of the Pendellösung into a grey state for the case of Ca is shown as a function of interaction time. As for anomalous transmission for X-rays the atoms take much longer to pass through the crystal than would be expected from a simple absorption picture with no coherences. (*b*) The transmission of atoms as a function of incident angles does not have maxima at the Bragg angles but shows that only half of the atoms are absorbed at the Bragg incidence owing to interference.

The atomic constants and calculation parameters used for our two examples shown in figures 3 and 5.

Case	Atom	λ (nm)	γ (kHz)	τ_{int} (ms)	$2\Omega^2/\gamma^2$	Δ	ϵ (kHz)
$\epsilon \gg \gamma$	Ca	657	0·408	50	10	$-\epsilon$	23
$\epsilon \ll \gamma$	Ar* $1s_5$–$2p_4$	715	650	0·3	0·5	4γ	23

experimentally be very close to a closed two-level system. An experiment for this long interaction time could be performed in a trap or with a laser-slowed atomic beam.

Up to now the boundary conditions for a single incident beam led to the population of both the bright and the dark grey states.

We would like to mention another possibility of how to load one of these states. One can populate one particular state by entering the interaction zone with two coherent atomic beams, which could be realized by a two-grating Mach–Zehnder interferometer. Here an incoming atomic beam may be split into two parts by Bragg scattering; then the resulting beams are redirected in an additional interaction zone and finally superposed in order to form one of the eigenstates. By adjusting the relative phase between the two atomic beams one can choose between the bright and the grey state. A similar method has been used very recently for photon scattering of a photorefractive crystal [22]. Recently case (ii) (*b*) has been studied theoretically [23, 24]. The difference is that for this particular case we are considering scattering of an atom at one incident angle to the laser beam, while in [23] a wide distribution of incident angles with transverse Doppler cooling is considered. The very interesting result reached in [23] is a 'quasi-VSCPT' process that fills the grey state belonging to the two-level system. We think that this is caused by the grey state in the anomalous transmission effect. The work in [24] for a more complicated system is directed towards the application of the case (ii) (*b*) with respect to atom interferometers.

5. Anomalous transmission and velocity-selective coherent population trapping

Bragg scattering [9] and VSCPT [15] have been demonstrated to split an atomic beam into two parts by interaction with laser light. For Bragg scattering, one part is the beam giving straight through, while the other part is scattered with a change in transverse momentum of two photon recoils, $2\hbar k$. One photon recoil comes from absorption directly from the laser beam, while the other recoil is due to stimulated emission into the retroreflected laser beam or vice versa.

Also in the field of laser cooling the effect of VSCPT results in two emerging beams. Following a random walk in momentum space due to spontaneous emission, atoms are trapped in a dark state which is a superposition of two momentum states again separated by $2\hbar k$. The question of whether both effects are due to the same physical cause or not seems natural.

The grey-state-like behaviour discussed in the Bragg case and in anomalous transmission are both related to the dark state of VSCPT. This relation becomes clear when comparing equation (2) with the equations that describe VSCPT. For VSCPT, one may consider a Λ system where the counterpropagating laser beams have a $\sigma_+ - \sigma_-$ configuration. This can be realized in an $F = 1 \rightarrow 1$ transition. Following Marte *et al.* [10] the evolution of the amplitudes is given by

$$\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} g_1 \\ e_0 \\ g_{-1} \end{bmatrix} = \begin{bmatrix} W_0 + 2\epsilon & -\frac{1}{2}\Omega & 0 \\ -\frac{1}{2}\Omega & \epsilon - \Delta - \mathrm{i}\frac{1}{2}\gamma & -\frac{1}{2}\Omega \\ 0 & -\frac{1}{2}\Omega & -W_0 + 2\epsilon \end{bmatrix}\begin{bmatrix} g_1 \\ e_0 \\ g_{-1} \end{bmatrix}. \tag{16}$$

A transformation of the basis to

$$\begin{aligned} |NC\rangle &= \tfrac{1}{2}\sqrt{2}(|-1\rangle + |1\rangle), \\ |C\rangle &= \tfrac{1}{2}\sqrt{2}(|-1\rangle - |1\rangle), \\ |e\rangle &= |0\rangle, \end{aligned} \tag{17}$$

shows upon substitution into the truncated set of equations that the dark state $|NC\rangle$ is decoupled from the light. A trivial shift in energy of ϵ and realizing that the detuning has been defined with opposite sign shows that the equations for VSCPT are embedded in the truncated set of equations (2) for Bragg scattering. So both have for the appropriate initial conditions as a solution the dark state that is the basis for VSCPT.

The selection criterion for VSCPT is the internal atomic state and not the external state as for the two-level system in a standing light wave. If the atom is in the $m = -1$ ground state, it can absorb only a σ_+ photon to excite the $m = 0$ state. Because this photon can come from only one side, higher-momentum states cannot be reached and thus results for all regimes in a perfect dark state.

The initial conditions $g_1 = \frac{1}{2}2^{1/2}$, $g_{-1} = -\frac{1}{2}2^{1/2}$ corresponds to the coherent dark state of VSCPT just as for the two-level case; note that for VSCPT there is a π phase shift between the two momentum states. It is very interesting to reverse the argument and to ask what the initial condition $g_1 = 1$ and $g_{-1} = 0$ amounts to. It is straightforward to show that this condition gives a Pendellösung for the $\sigma_+ - \sigma_-$ configuration. One might argue that the Pendellösung in the three-level Λ system cannot be stable because of spontaneous emission, which causes momentum diffusion. However, with a suitable choice of the Rabi frequency Ω and the detuning Δ it is possible to keep the potential U_0 constant, while reducing the optical pumping time $\gamma_p = \Omega^2\gamma/2\Delta^2$. It is even possible to chose a short interaction time, because, as discussed above, the selection criterion is not given by the interaction time.†

6. Conclusion

We feel that anomalous transmission is a very good example showing the similar behaviours of the external states of X-rays, neutrons, electrons and atoms in periodic potentials.

Atoms moving in light crystals offer an advantage over other particles for studying such effects because the light fields can be very precisely engineered using laser technology. Using diffractive optics and holography, one can, in principle, build any desirable structure in the laboratory [25]. In addition, one can change the interaction between the object and the atom at will, so that one may consider firstly very weak elastic interactions such as those in dynamical diffraction, secondly very strong interactions as in channelling and thirdly those interactions which are dominated by dissipative processes using on-resonance light. One therefore can build complex potentials with almost arbitrary spatial symmetry which may lead to a new tool for creating, manipulating and investigating matter wave fields. In addition this offers a nice example of the wave–particle duality through the use of the laser light to form a crystal for atomic matter waves.

Acknowledgments

We thank Helmut Ritsch for helpful discussions. This work is supported by grant No. S065-04 from the Austrian Fond zur Förderung der wissenschaftlichen Forschung and a Human Capital and Mobility network grant of the European Community (contract No. ERBCHBGCT 940664). H.B. is supported by a Lise Meitner Fellowship, E.M.R. by an European Training Project and J.S.

† This gives the interesting possibility of splitting or reflecting an atomic beam with a short interaction time and with $\Omega \gg \epsilon$ in contrast with the inverse condition for the usual Bragg scattering [10].

acknowledges an Austrian program for Advanced Research and Technology Fellowship from the Austrian Academy of Sciences.

References

[1] Borrmann, G., 1941, *Phys. Z.*, **42,** 157.

[2] Knowles, J. W., 1961, *Acta crystallogr.*, **9,** 61.

[3] Hashimoto, H., Howie, A., and Whelan, M. J., 1962, *Proc. R. Soc.* A, **269,** 80.

[4] Rauch, H., and Petrascheck, D., 1978, *Neutron Diffraction*, edited by H. Dachs (Berlin: Springer), p. 303 and therein.

[5] Oberthaler, M. K., Abfalterer, R., Bernet, S., Schmiedmayer, J., and Zeilinger, A., 1996, *Phys. Rev. Lett.*, **77,** 4980.

[6] Cowley, J. M., 1990, *Diffraction Physics* (Amsterdam: North-Holland).

[7] Batterman, B. W., and Cole, H., 1964, *Rev. mod. Phys.*, **36,** 681; Rauch, H., and Petrascheck, D., 1978, *Neutron Diffraction*, edited by H. Dachs (Berlin: Springer), p. 303.

[8] Bernhardt, A. F., and Shore, B. W., 1981, *Phys. Rev.* A, **23,** 1290.

[9] Martin, P. J., Oldaker, B. G., Miklich, A. H., and Pritchard, D. E., 1988, *Phys. Rev. Lett.*, **60,** 515.

[10] Marte, M. A. M., Cirac, J. I., and Zoller, P., 1991, *J. mod. Optics*, **38,** 2265.

[11] Marte, M., and Stenholm, S., 1992, *Appl. Phys.* B, **54,** 443.

[12] Giltner, D. M., McGowen, R. W., and Lee, S. A., 1995, *Phys. Rev. Lett.*, **75,** 2638; 1995, *Phys. Rev.* A, **52,** 3966.

[13] Kunze, S., Dürr, S., and Rempe, G., 1996, *Europhys. Lett.*, **34,** 343.

[14] Adams, C., Sigel, M., and Mlynek, J., 1994, *Phys. Rep.*, **240,** 143.

[15] Aspect, A., Arimondo, E., Kaiser, R., Vansteenkiste, N., and Cohen-Tannoudji, C., 1988, *Phys. Rev. Lett.*, **61,** 826.

[16] Chudesnikov, D. O., and Yakovlev, V. P., 1991, *Laser Phys.*, **1,** 110.

[17] Stenholm, S., 1992, *Laser Manipulation of Atoms and Ions*, Proceedings of the International School of Physics 'Enrico Fermi', edited by E. Arimondo, W. D. Phillips and F. Strumia (Amsterdam: North-Holland), pp. 29–70.

[18] Gould, P. L., Ruff, G. A., and Pritchard, D. E., 1986, *Phys. Rev. Lett.*, **56,** 827.

[19] Dum, R., Zoller, P., and Ritsch, H., 1992, *Phys. Rev.* A, **45,** 4879.

[20] Schumacher, E., Wilkens, M., Meystre, P., and Glasgow, S., 1992, *Appl. Phys.* B, **54,** 451.

[21] Batelaan, H., Bernet, S., Oberthaler, M. K., Rasel, E. M., Schmiedmayer, J., and Zeilinger, A., 1997, Atom Interferometry, edited by P.R. Berman (New York: Academic Press), p. 85.

[22] Mair, A., and Rupp, R., 1996, private communication.

[23] Doery, M. R., Vredenbregt, E. J. D., and Bergeman, T., 1994, private communication.

[24] Horak, P., Gheri, K. M., and Ritsch, H., 1996, private communication.

[25] Collier, R. J., Burckhardt, C. B., and Lin, L. H., 1971, *Optical Holography* (New York: Academic Press).