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sub. to Advances in Solid State Physics

HD-KIP-00-15

KIRCHHOFF-INSTITUT FÜR PHYSIK

Beyond the tunneling model — Elastic properties of vitreous silica at low temperatures*

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Summary: The internal friction Q^{-1} and the sound velocity $\delta v/v$ of vitreous silica were measured at temperatures between 6 mK and 40 K using mechanical double paddle resonators. This experimental technique allows measurements to be performed at different frequencies (0.33 to 14 kHz) and with very small background loss. Above ~ 50 mK the elastic properties can be described by the tunneling model with overall good agreement, provided that at temperatures above 5 K allowance is made for thermally activated relaxation processes. In contrast, at very low temperatures both internal friction and sound velocity deviate substantially from the predictions of the tunneling model. The results clearly indicate the relevance of mutual interaction between tunneling states in the Millikelvin temperature range but a satisfactory quantitative understanding of the observed phenomena is still lacking.

1 Introduction

When a perfect crystal is cooled to low temperatures its elastic constants will not change noticeably below 1 K as the number of elementary excitations is very small. In contrast, glasses or, more generally, disordered solids exhibit significant changes of their elastic properties even at temperatures in the Millikelvin range. The remarkable temperature dependence is caused by tunneling states — localized low-energy excitations present in virtually all disordered solids [1, 2]. Their microscopic nature is still only vaguely known; in a simplifying picture they can be thought of as particles — atoms or small clusters of atoms — being able to move between two neighboring equilibrium positions in the disordered environment. At sufficiently high temperatures the barrier between the potential minima can be overcome by thermally activated jumps; at low temperatures this is no longer possible but the particle can still tunnel through the barrier of the double well potential.

The theoretical baseline for the description of the elastic and also of the thermal and dielectric properties of glasses was marked by the phenomenological tunneling model [3]. In this model the existence of double well potentials with a broad distribution of their characteristic parameters — resulting from the large variety

* This paper is dedicated to Prof. F. Wegner on occasion of his 60$^{\text{th}}$ birthday.

of local configurations in a glass — is assumed. Moreover, it is supposed that tunneling states can interact with phonons via resonant and relaxation processes whereas a mutual interaction between tunneling states is neglected.

While early measurements of the low frequency elastic properties of vitreous silica indicated good overall agreement with the predictions of the tunneling model [4] more recent vibrating reed studies showed significant deviations, e.g., below 100 mK an unexpected temperature dependence [5], the occurrence of nonlinear behavior of the sound velocity [5, 6], and at temperatures below $\sim$ 2 mK a leveling-off or "saturation" of $\delta v/v$ [6]. Moreover, the internal friction of glasses at very low temperatures did not obey the predicted [4] T^3-behavior but exhibited a weaker temperature dependence [5, 6]. However, vibrating reeds usually have fairly large background losses of the order of 10^{-5} or even higher [6]. Since at low temperatures the internal friction of glasses becomes very small, the measured value of Q^{-1} may be strongly influenced or even dominated by the clamping losses, and its temperature dependence may be significantly distorted.

In this paper we present new data of $\delta v/v$ and Q^{-1} of vitreous silica which were obtained using mechanical double paddle oscillators [7]. Major advantages of this experimental technique are the significantly lower background of Q^{-1} and the possibility to perform measurements at different eigenmodes, i.e., different frequencies. Results of the measurements below 1 K have been published recently [8, 9]. The measurements presented here cover the extended temperature range 6 mK–40 K and the frequency range 0.33–14.0 kHz. The data taken above helium temperature clearly confirm the assumption that the dynamics of double well defect states changes from tunneling to thermally activated relaxation [10, 11]. Another change of the dynamics of the tunneling states can be observed at temperatures below $\sim$ 30 mK: The internal friction exhibits a surprising temperature and frequency dependence and clearly exceeds the values expected from the tunneling model. The most likely origin of this behavior appears to be the mutual interaction between tunneling states which is omitted in the standard tunneling model but whose relevance has been emerged with increasing clarity during the past years [12, 13, 14, 15, 16, 17, 18].

2 Theory

2.1 Standard tunneling model

In this section we briefly summarize the assumptions and predictions of the tunneling model that are relevant to low-frequency acoustic experiments. For more detailed discussions see, e.g., Refs. [4, 5]. The basic assumptions of the tunneling model are:

i) In amorphous solids some atoms or small groups of atoms can move between two almost degenerate configurations in double well potentials with asymmetry Δ and tunnel splitting $\Delta_0 \simeq E_0 \exp(-\lambda)$. Here E_0 is the ground state energy in a single well and $\lambda \simeq d\sqrt{2mV}/2\hbar$ the tunneling parameter which is determined by the distance d in configurational space between the two potential minima, the mass m of the "particle", and the barrier height V between the wells. The total energy splitting between the two lowest levels is given by $E=\sqrt{\Delta^2+\Delta_0^2}$.

ii) The parameters Δ and λ are independent of each other and they are widely distributed because of the randomness of the local environment. Usually, a distribution function

$$P(\Delta,\lambda)\,\mathrm{d}\Delta\,\mathrm{d}\lambda=\overline{P}\,\mathrm{d}\Delta\,\mathrm{d}\lambda \tag{1}$$

is assumed where $\overline{P}$ is a constant. Equation 1 may be rewritten as

$$P(\Delta,\Delta_0)\,\mathrm{d}\Delta\,\mathrm{d}\Delta_0=(\overline{P}/\Delta_0)\,\mathrm{d}\Delta\,\mathrm{d}\Delta_0\ . \tag{2}$$

To avoid a divergence of the total number of tunneling states, a low-energy cutoff $\Delta_{0,\mathrm{min}}$ is introduced which is usually assumed to be much smaller than 1 mK.

iii) At temperatures below 1 K the dominant relaxation mechanism for tunneling systems in insulators is the so-called one-phonon or direct process. The rate of this process is given by [19]

$$\tau_\mathrm{d}^{-1}=\frac{1}{2\pi\varrho\hbar^4}\left(\frac{\gamma_\mathrm{l}^2}{v_\mathrm{l}^5}+\frac{2\gamma_\mathrm{t}^2}{v_\mathrm{t}^5}\right)\Delta_0^2E\coth\frac{E}{2k_\mathrm{B}T}=A\,\Delta_0^2E\coth\frac{E}{2k_\mathrm{B}T} \tag{3}$$

where ϱ is the mass density, v the sound velocity, and $\gamma=(1/2)\,\mathrm{d}\Delta/\mathrm{d}e$ the deformation potential, i.e., the derivative of the asymmetry energy with respect to strain e. Indices l and t in Eq. 3 denote longitudinal and transversal polarization, respectively. For a given energy splitting, there is a wide distribution of relaxation times, and amongst this distribution symmetric tunneling systems ($E=\Delta_0$) have the shortest relaxation times, denoted by τ_min.

Using these assumptions the following predictions for the temperature dependence of the internal friction Q^{-1} and for the relative change of sound velocity $\delta v/v$ of an insulating glass below 1 K are obtained:

At low temperatures, when $\omega\tau_\mathrm{min}\gg 1$, the internal friction increases as the third power of temperature, and the sound velocity is expected to vary logarithmically with temperature as

$$\frac{\delta v}{v}=C\ln\left(\frac{T}{T_0}\right) \tag{4}$$

where T_0 is an arbitrary reference temperature, and the parameter C is given by $C=\overline{P}\gamma_i^2/\varrho v_i^2$; the index i stands for longitudinal or transversal polarization.

At higher temperatures, when $\omega\tau_\mathrm{min}\ll 1$, the internal friction approaches the value

$$Q^{-1}=\frac{\pi}{2}C \tag{5}$$

independent of temperature and frequency. The sound velocity passes a maximum and then decreases logarithmically with increasing temperature as

$$\frac{\delta v}{v}=-\frac{C}{2}\ln\left(\frac{T}{T_0}\right). \tag{6}$$

2.2 Extensions of the standard tunneling model

At temperatures above a few Kelvin the number of phonons becomes so large that relaxation does no longer occur predominantly via the one-phonon process; the probability of more complicated relaxation processes increases. Here we refrain from the discussion of Raman processes [20] and incoherent tunneling [21, 22] and focus on thermally activated processes [10, 11] which may be described as classical Arrhenius type hopping over the barrier of the double well potentials [23] with a rate

$$\tau_{\mathrm{th}}^{-1} = \nu_0 \exp\left(-\frac{V}{k_{\mathrm{B}}T}\right) \tag{7}$$

that has to be added to the one-phonon rate (3) to obtain the total relaxation rate. The prefactor ν_0 denotes an attempt frequency which is proportional to the ground state energy E_0. The exact relationship between ν_0 and E_0 will depend on the exact shape of the potential barriers and on the entropy associated with the double well defect states [11].

The distribution of barrier heights cannot extend to arbitrarily large values of V. Tielbürger *et al.* [11] proposed a gaussian distribution function of width V_0 for the barrier heights

$$P(\Delta,V) = \frac{\overline{P}}{E_0} \exp\left(-\frac{V^2}{2V_0^2}\right) \tag{8}$$

which in combination with the additionally assumed correlation $\lambda \propto V$ replaces Eq. 1. For low temperatures ($T < 1$ K) when only systems with small barriers (or tunneling parameters) are relevant the use of Eq. 8 leads to the same predictions (Eqs. 4–6) as the simple distribution function Eq. 1. At temperatures $T > E_0/2k_{\mathrm{B}}$ thermal relaxation sets in and causes an increase of the internal friction with increasing temperature and a decrease of the sound velocity which are both linear in temperature. When the temperature is raised systems with higher barriers contribute to the internal friction. However, since the width V_0 of the barrier distribution (Eq. 8) is finite the internal friction does not further increase but passes a maximum and decreases afterwards with increasing temperature. The position and the height of the maximum of Q^{-1} are approximately proportional to V_0. These predictions are in fair although not perfect quantitative agreement with experimental observations made on several glasses [5, 11, 22] and will be compared to our new experimental results further below.

A distribution function $P(V)$ different from Eq. 8 is used in the so-called "soft potential model" [24] which is based on a more general approach for the potentials of the defect systems and also includes contributions from quasiharmonic low-energy excitations. The soft potential model gives qualitatively similar but quantitatively slightly different predictions for the thermally activated regime. As the differences concerning the internal friction are only rather subtle, we don't want to discuss the soft potential model here in more detail.

At very low temperatures, usually well below 100 mK, interaction effects between neighboring tunneling states may become important as several experiments indicate [12, 13, 14, 15, 16, 17, 18]. Various theories have been suggested to take into account the mutual interaction of tunneling states in glasses [25, 26]. Burin and

Kagan [27, 28] proposed the occurrence of pair excitations due to strain-mediated interaction between tunneling states. The interaction between pairs with similar tunnel splitting leads to an additional relaxation contribution with a rate [28]

$$\tau_{\mathrm{p}}^{-1} \simeq \frac{10\, k_{\mathrm{B}}\, C^3}{\hbar} \left(\frac{\Delta_0}{E}\right)^2 T \; . \tag{9}$$

Below the temperature $T^{\star} \simeq (10\, k_{\mathrm{B}}\, C^3/\hbar A)^{1/2}$ [28] this rate will become larger than the one-phonon rate Eq. 3, and a cross-over from the one-phonon dominated relaxation to a linear temperature dependence of the relaxation rate — and hence of the internal friction — is expected. However, inserting typical values of the parameters C and A leads to a transition temperature $T^{\star} < 1$ mK which is clearly smaller than the temperatures achieved in our experiments (and also much smaller than the overestimate $T^{\star} \sim 10 - 100$ mK given in [28]).

A somewhat different approach to explain deviations from the tunneling model at very low temperatures is based on the idea that interaction between tunneling states may lead to an incoherence of the tunneling motion at very low temperatures [16]. As a result, the resonant contribution to $\delta v/v$ is reduced but additional relaxation effects occur and modify the temperature dependence of $\delta v/v$ and Q^{-1}. However, a complete theory for incoherent tunneling in glasses at very low temperatures has not been worked out yet, and therefore reliable quantitative predictions cannot be made at present.

3 Experimental

The double paddle oscillators used for the experiments were laser cut from a 0.4 mm thick plate of vitreous silica [29]. The geometry, very similar to that of silicon oscillators successfully applied by the Pohl group [7], is shown in the upper left corner of Fig. 1. Oscillators of two different sizes were used, with lateral dimensions of $28 \times 20\ \mathrm{mm}^2$ and $16.8 \times 12\ \mathrm{mm}^2$, respectively. These mechanical resonators can be operated in different torsional (T) and bending (B) modes, i.e., at different frequencies and elastic polarizations. The eigenmode spectrum of the small oscillator was shifted by about a factor of three to higher frequencies compared to the large oscillator. The latter was operated at 0.33 (T), 1.26 (B), 2.52 (B), and 5.03 kHz (T) while the small oscillator was investigated at 0.63 (B), 1.03 (T), 3.88 (B), and 14.0 kHz (T).

A major advantage of the double paddle is that only very small strain amplitudes occur at the clamping position. Finite-Element calculations show that for all modes investigated the strain amplitudes at the clamp are reduced by more than one order of magnitude compared to the maximum strain amplitudes occurring in the sample. This is in marked contrast to vibrating reed or vibrating wire experiments where the maximum strain occurs right at the clamp. Measurements in our group with almost identically shaped silicon oscillators have revealed quality factors Q larger than 10^6 for at least five different modes. Hence we may expect a similarly small background loss for the glass paddles.

Excitation and detection of the oscillator motion was done capacitively [5]. All measurements were performed at excitation levels small enough to avoid nonlinear behavior as was proven by the symmetric Lorentzian line shape of resonance curves. The maximum strain amplitudes in the samples were $e \sim 1 \times 10^{-7}$ or smaller; at significantly higher strain levels a strongly nonlinear behavior was observed, with resonance curves shifting and bending towards lower frequencies as was observed before in vibrating reed experiments on glasses [5, 6]. Strain amplitudes on the order of 10^{-7} appear to be a sensible crossover value between nonlinear and linear behavior at temperatures around 10 mK: For a deformation potential $\gamma \sim 1$ eV the modulation of the asymmetry energy $\delta\Delta = 2\gamma e$ is approximately 2 mK, i.e. still smaller than thermal energy $k_{\mathrm{B}}T$. Hence the thermal occupation and the dynamics of the tunneling states are not drastically changed by the strain fields [30].

The large sample was covered with a 1.4 μm and the small paddle with a 1.1 μm thick silver film. A large film thickness was required to ensure thermalization even at lowest temperatures. Some measurements were carried out with glass paddles covered with only 30 nm gold or 130 nm silver. In these cases the lowest achievable sample temperatures were ~ 60 and ~ 20 mK, respectively. Above these temperatures, no difference of the absolute value of Q^{-1} was observed compared to the paddles covered with the thick silver films, i.e., the thick films appeared to have no significant influence on overall damping of the oscillator. This is also supported by the following argument: Even if one assumes (as an upper limit) that the metal film has a similar internal friction as the glass its influence on the total damping of the oscillator will be not more than 1% as its thickness is smaller by more than two orders of magnitude than the thickness of the glass plate.

4 Results

4.1 Internal friction

Figure 1 shows the temperature dependence of the internal friction at four frequencies over a temperature range of almost four decades. The overall behavior is the same as observed before [4, 5] and described in Sec. 2: The internal friction strongly increases at very low temperatures, exhibits a plateau around 1 K and increases again above ~ 5 K before a maximum at ~ 30 K occurs. In the following paragraphs we want to discuss some features of the internal friction in greater detail as they provide interesting information on the dynamics of tunneling states and on the distribution functions of the double well parameters.

We start with the discussion of the plateau region which is shown in Fig. 2 on an expanded ordinate. The 2.52 kHz data from Fig. 1 have been omitted in this graph for clarity; they lie just in between the 1.26 and the 5.03 kHz results. Instead the 14 kHz measurements is shown where data were taken only up to 1 K. One can clearly see that the height of the plateau (slightly) depends on frequency, in contrast to the prediction of Eq. 5. Apparently, the distribution functions Eq. 1 and Eq. 8 are not strictly valid — even for small values of λ or V the distribution of tunneling parameters or barrier heights must decrease with increasing λ or V. However, we have made no effort yet to try to deduce a

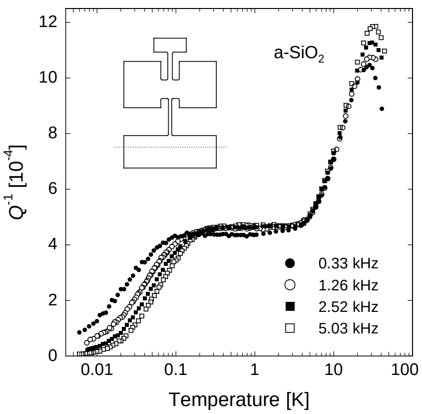

Figure 1 Temperature dependence of the internal friction of vitreous silica at four frequencies. The geometry of the oscillator is depicted in the upper left corner, with the thin dotted line marking the clamping position.

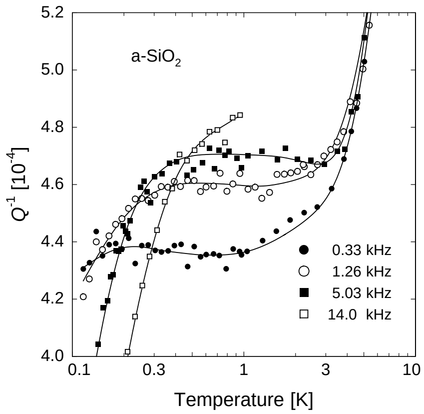

Figure 2 Temperature dependence of the internal friction of vitreous silica in the plateau region on an expanded scale. The lines are only guides to the eye.

more appropriate distribution function from our data by numerical calculations. The results are in excellent agreement with measurements of the internal friction of vitreous silica using a quartz composite oscillator at frequencies 66, 90, and 160 kHz where values of the internal friction slightly higher than 5×10^{-4} were observed in the plateau region [31, 32]. Other measurements [4, 6] indicate an increase of internal friction with increasing frequency as well but the absolute values of Q^{-1} determined in these experiments are about $10 - 30\%$ smaller than in Fig. 2. Another observation worth noting is that the plateau value does not depend on the elastic polarization: Torsional and bending modes show no systematic difference in internal friction, i.e., for vitreous silica $\gamma_\mathrm{t}/v_\mathrm{t} = \gamma_\mathrm{l}/v_\mathrm{l}$. For most other glasses this relation is not valid.

Above helium temperatures the internal friction strongly increases and approaches a maximum around 30 K. The region around the maximum is shown in detail in Fig. 3. Both height and position of the peak shift with increasing frequency to larger values as shown in Fig. 4. This behavior is in good agreement with the extended tunneling model described in Sec. 2.2 where thermally activated processes are supposed to dominate the relaxation dynamics of the double well defect states at temperatures above 10 K. The solid lines in Fig. 4 represent the results of numerical calculations using the parameters $C = 2.8 \times 10^{-4}$, $E_0 = 14.3$ K, $V_0 = 610$ K, and $\nu_0 = 4.8 \times 10^{12}$ s, E_0 and V_0 being slightly larger than the values estimated by Tielbuerger *et al.* [11]. As predicted by the model the peak height and the peak position are proportional to each other [11].

The most remarkable observations of our experiments were made at very low temperatures. Figure 5 shows the internal friction below 1 K at five frequencies on a double-logarithmic scale. Well below the plateau region, in the temperature range $6 - 30$ mK, the internal friction of all modes varies in good approximation as $Q^{-1} \propto T^\alpha$, with the exponent α increasing monotonically with increasing fre-

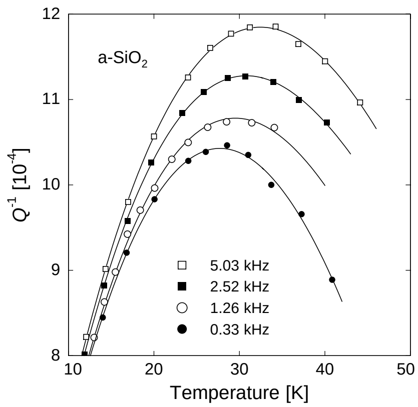

Figure 3 Internal friction of vitreous silica in the maximum region. The lines are only guides to the eye.

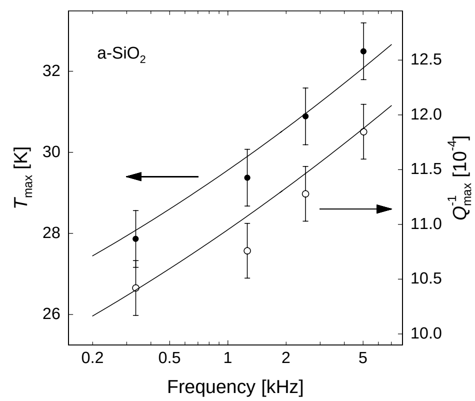

Figure 4 Position (full circles, left scale) and height (open circles, right scale) of the maximum of internal friction as a function of frequency. The solid lines are numerical calculations according to the extended tunneling model.

quency from values smaller than unity below 1 kHz to values larger than 2 at 14 kHz [9]. This is in clear disagreement with numerical calculations according to the tunneling model which are shown in Fig. 5 as solid lines for frequencies 0.33 and 14.0 kHz, respectively. Two parameters enter these calculations, namely $C = 2.8 \times 10^{-4}$ and the prefactor of the one-phonon rate $A = 8 \times 10^{7}\ \mathrm{K}^{-3}\mathrm{s}^{-1}/k_{\mathrm{B}}^{3}$.

An interesting result is obtained in Fig. 6 by plotting the experimental data of Fig. 5 divided by the tunneling model prediction. The same parameters C and A were used for all frequencies. Quite remarkably, all curves show below $\sim$ 30 mK even quantitatively a very similar strong increase towards low temperatures, i.e., the ratio $Q^{-1}/Q^{-1}_{\mathrm{STM}}$ appears not to depend systematically on frequency [33]. It seems very unlikely that the results of Figs. 5 and 6 can be explained solely by a sophisticated modification of the distribution functions (1) or (8). The data rather suggest that the relaxation dynamics of the tunneling states changes substantially below 30 mK. This temperature is of the same order of magnitude as the elastic interaction energy $U \simeq \gamma^2/\varrho v^2 r^3$ between neighboring tunneling states at an average distance $r \sim 10$ nm. Therefore it seems plausible that the mutual interaction plays a distinct role in our experiments.

We have tried to describe our data using the rate Eq. 9 for the relaxation of interacting resonant pairs of tunneling states [28]. However, as discussed in Sec. 2.2 this rate is negligibly small in the temperature range of our measurements. It is nevertheless worth noting that an additional relaxation contribution of the form $\tau^{-1} \propto (\varDelta_0/E)^2 T$ does in principle describe the data fairly well — but only if the prefactor of the rate (9) is arbitrarily increased by $3-4$ orders of magnitude. At present, there is no theoretical justification for such an enormous enhancement of the relaxation rate nor appears there to be any other theory that would quantitatively account for the observed excess internal friction.

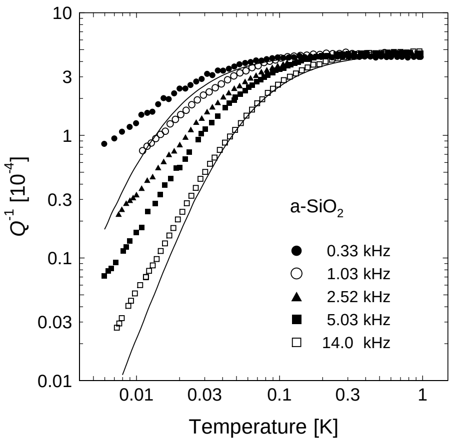

Figure 5 Internal friction of vitreous silica below 1 K at five frequencies. The solid lines are numerical calculations according to the tunneling model for frequencies 0.33 and 14.0 kHz, respectively.

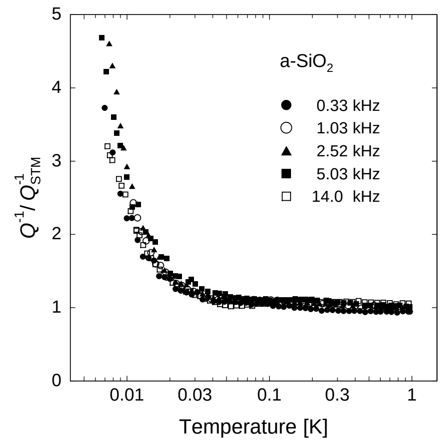

Figure 6 Experimental data of Fig. 5 divided by numerical fits according to the standard tunneling model. The parameters used for the numerical calculations are discussed in the text.

4.2 Sound velocity

Figure 7 shows the temperature dependence of the sound velocity $\delta v/v$ at four frequencies below 1 K. For all modes the sound velocity increases at low temperatures, passes a maximum and decreases at higher temperatures where one-phonon relaxation contributes significantly. As in previous experiments on glasses [5, 22, 31] the increase of the sound velocity below the maximum has a similar slope as the decrease above the maximum, in contrast to the expectation of the tunneling model where a slope ratio of 2 : (−1) is predicted (see Eqs. 4 and 6). A new observation

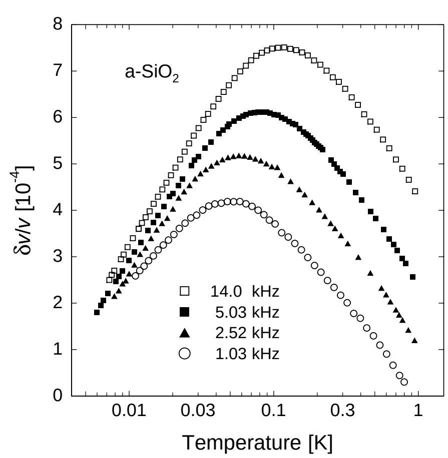

Figure 7 Temperature dependence of the sound velocity of vitreous silica at four frequencies.

is that the low-temperature slope of the sound velocity appears to slightly vary with experimental frequency. The low-temperature part of $\delta v/v$ is usually almost entirely determined by the resonant interaction between phonons and tunneling states; hence the observed frequency dependence either indicates a frequency dependence of the resonant interaction, or, more plausible, the presence of an additional relaxation contribution. Additional measurements over a wider range of frequencies and temperatures are clearly desirable to establish this observation.

The deviation of the ratio of slopes of $\delta v/v$ below and above the maximum from the prediction of the tunneling model has been tentatively explained by the incoherence of the tunneling motion [16] which may be evoked at sufficiently low temperatures by interaction effects between tunneling states. Here we would like to discuss another possible scenario that may be closely, though less directly, related to mutual interaction of tunneling systems as well.

Recently, it has been suggested [18, 26, 34] that the interaction between tunneling states might lead to a low-energy cutoff $\Delta_{0,\mathrm{min}}$ in the distribution of tunnel splittings on the order of several mK. The cutoff was introduced to explain the "saturation" of the dielectric constant at very low temperatures. In fact, such a cutoff would influence the temperature dependence of the dielectric constant and of the sound velocity (apart from the sign both quantities should have the same qualitative temperature dependence, see, e.g. [5]) even at temperatures well above $\Delta_{0,\mathrm{min}}$ and would modify the ratio of slopes rather closely towards the experimentally observed value of $1:-1$. This can be seen in the lower part of Fig. 8 where the dotted line represents a numerical calculation with $\Delta_{0,\mathrm{min}} = 6$ mK that may be compared to a fit according to the standard tunneling model (solid line). It becomes clear from both panels of Fig. 8, however, that the assumption of a minimum tunnel splitting of several Millikelvin, if naively applied, leads to obvious incon-

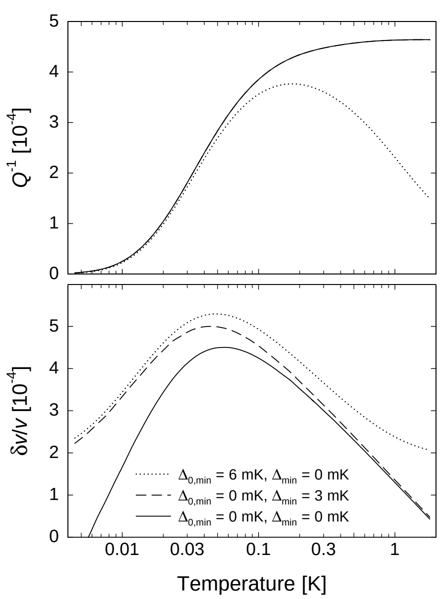

Figure 8 Comparison of different models to describe the temperature dependence of the sound velocity and internal friction. Solid line: Standard tunneling model, no cut-off in Δ and Δ_0. Dotted line: No cut-off in Δ, cut-off 6 mK in Δ_0. Dashed line: Cut-off 3 mK in Δ, no cut-off in Δ_0. In the upper panel the dashed line falls on top of the solid line, i.e., the cut-off in Δ has almost no influence on the internal friction. Parameters common to all curves: $C = 2.8 \times 10^{-4}$, $A = 8 \times 10^7\ \mathrm{K}^{-3}\mathrm{s}^{-1}$, $f = 1$ kHz.

sistencies with the experimental results of Figs. 1, 2, and 7: At temperatures of several 100 mK the relaxation contribution to $\delta v/v$ and Q^{-1} would be significantly suppressed due to the reduced number of tunneling systems with large tunneling parameters λ or barrier heights V. The mere existence of the plateau and of the damping peak (Fig. 3) clearly show that there must exist double well defect states with fairly large values of λ and V, i.e. values of Δ_0 much smaller than 1 mK. The inconsistency can be avoided by postulating a temperature dependent effective density of states or distribution function $P(\Delta,\Delta_0,T)$. One might argue that at very low temperatures the dynamics of thermal tunneling states ($\Delta_0 \sim k_{\mathrm{B}}T$) change drastically as coupled excitations emerge when the elastic or dielectric interaction energy U between tunneling states exceeds $k_{\mathrm{B}}T$. In contrast, at high temperatures ($k_{\mathrm{B}}T \gg U$) the tunneling states can move entirely independently so that no renormalization of the distribution function $P(\Delta,\Delta_0)$ is required.

A simpler way towards a satisfactory description of the experimental data might be a modification of the distribution function of asymmetry energies Δ. Included in Fig. 8 as dashed lines are numerical calculations where a minimum asymmetry energy $\Delta_{\mathrm{min}} = 3$ mK was assumed while $\Delta_{0,\mathrm{min}}$ was set to a value smaller than 1 μK, denoted in Fig. 8 for simplicity as "0 mK". The temperature dependence of the sound velocity below the maximum is very similar for both cutoff models. The advantage of the cutoff in Δ is that both the sound velocity and the internal friction at higher temperatures can be described much better than by a cutoff in Δ_0. The prediction for Q^{-1} is not even noticeably affected by introducing a Δ_{min} of several Millikelvin; the dashed line lies on top of the solid line which represents the standard tunneling model. However, even though the numerical calculations using a low-energy cutoff in Δ seem to be quite promising, in particular with respect to the sound velocity data, at present there appears to be no convincing theoretical argument why in glasses a strong suppression of tunneling systems with small asymmetry energies should indeed occur.

5 Concluding remarks

We have presented results of new measurements of the elastic properties of vitreous silica over a wide range of temperatures and frequencies. The study was motivated both by progress made in experimental technique and by increasing evidence — derived from a large variety of experiments — of the relevance of interaction effects of tunneling states in glasses at very low temperatures. The data at temperatures above 100 mK overall confirm previous findings but the ability to perform elastic measurements at different frequencies with the same sample has offered new possibilities to study subtle effects like a slight frequency dependence of the internal friction even in the damping "plateau" or details of the maximum region of Q^{-1} around 30 K. At very low temperatures, both sound velocity and internal friction showed significant and systematic deviations from the tunneling model. Several ideas have been discussed to provide a first approach to an understanding of the experimental data. Although a truly satisfactory explanation of the low temperature results is still lacking most of these suggestions have in common that the mutual interaction appears to play a decisive role for the dynamics and/or the density of states of tunneling states. The interaction of tunneling states in glasses remains an exciting subject for further experimental and theoretical studies.

Acknowledgement

We are grateful to S. Hohmann for support with the experiments. We would also like to thank R.O. Pohl, A. Burin, and I. Polyshchuk for many valuable discussions, and S. Wächter from IFW Jena, Germany, for the excellent quality of the laser cuts. This work was supported in part by the Deutsche Forschungsgemeinschaft (Grant No. Hu359/11).

Bibliography

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[1] W. A. Phillips (ed.) *Amorphous Solids — Low Temperature Properties* (Springer, Berlin, 1981).

[2] P. Esquinazi (ed.) *Tunneling Systems in Amorphous and Crystalline Solids* (Springer, Berlin, 1998).

[3] W.A. Phillips, J. Low Temp. Phys. **7**, 351 (1972); P.W. Anderson, B.I. Halperin, C.M. Varma, Phil. Mag. **25**, 1 (1972).

[4] A.K. Raychaudhuri, S. Hunklinger, Z. Phys. B **57**, 113 (1984).

[5] J. Classen, C. Enss, C. Bechinger, G. Weiss, S. Hunklinger, Ann. Phys. (Leipzig) **3**, 315 (1994).

[6] P. Esquinazi, R. König, F. Pobell, Z. Phys. B **87**, 305 (1992).

[7] B.E. White, Jr., R.O. Pohl, Phys. Rev. Lett. **75**, 4437 (1995).

[8] T. Burkert, J. Classen, C. Enss, S. Hunklinger, Contribution to the XXII. Int. Conf. on Low Temp. Phys. (LT22, August 99, Helsinki), to appear in Physica B.

[9] J. Classen, T. Burkert, C. Enss, S. Hunklinger, Phys. Rev. Lett. **84**, 2176 (2000).

[10] J. Jäckle, L. Piché, W. Arnold, S. Hunklinger, J. Non-Cryst. Solids **20**, 365 (1976).

[11] D. Tielbürger, R. Merz, R. Ehrenfels, S. Hunklinger, Phys. Rev. B **45**, 2750 (1992).

[12] W. Arnold, S. Hunklinger, Solid State Commun. **17**, 833 (1975).

[13] J.E. Graebner, B. Golding, Phys. Rev. B **19**, 964 (1979).

[14] C. Enss, S. Ludwig, R. Weis, S. Hunklinger, Czech. J. Phys. **46**, 2247 (1996).

[15] S. Rogge, D. Natelson, D.D. Osheroff, Phys. Rev. Lett. **76**, 3136 (1996), D. Natelson, D. Rosenberg, D.D. Osheroff, ibid. **80**, 4689 (1998).

[16] C. Enss, S. Hunklinger, Phys. Rev. Lett. **79**, 2831 (1997).

[17] P. Strehlow, C. Enss, S. Hunklinger, Phys. Rev. Lett. **80**, 5361 (1998).

[18] P. Strehlow *et al.*, Phys. Rev. Lett. **84**, 1938 (2000).

[19] J. Jäckle, Z. Phys. B **257**, 212 (1972).

[20] P. Doussineau, C. Frénois, R.G. Leisure, A. Levelut, J.-Y. Prieur, J. Phys. (Paris) **41**, 1193 (1980).

[21] P. Neu, A. Würger, Z. Phys. B **95**, 385 (1994).

[22] S. Rau, C. Enss, S. Hunklinger, P. Neu, A. Würger, Phys. Rev. B **52**, 7179 (1995).

[23] P. Neu, A. Heuer, J. Chem. Phys. **106**, 1749 (1997).

[24] V.G. Karpov, M.I. Klinger, F.N. Ignatev, Zh. Eksp. Teor. Fiz. **84**, 760 (1983) [Sov. Phys. JETP **57**, 439 (1983)]; Y.M. Galperin, V.G. Karpov, V.I. Kozub, Adv. Phys. **38**, 669 (1989); U. Buchenau, Yu.M. Galperin, V.L. Gurevich, D.A. Parshin, M.A. Ramos, H.R. Schober, Phys. Rev. B **46**, 2798 (1992); D.A. Parshin, Phys. Rev. B **49**, 9400 (1994).

[25] A.J. Leggett, S. Chakravarty, A.T. Dorsey, M.P.A. Fisher, A. Garg, W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987); C.C. Yu, A.J. Leggett, Comments in Cond. Mat. Phys. **14**, 231 (1988); K. Kassner, R.J. Silbey, J. Phys. Cond. Matter **1**, 4599 (1989).

[26] S. Kettemann, P. Fulde, P. Strehlow, Phys. Rev. Lett. **83**, 4325 (1999).

[27] A.L. Burin, Yu. Kagan, Physica B **194-196**, 393 (1994).

[28] A.L. Burin, J. Low Temp. Phys. **100**, 309 (1995).

[29] Suprasil 300, manufactured by Heraeus Hanau, Germany. Except for Cl impurities it is a chemically extremely pure glass, similar to Suprasil W used in previous experiments [5].

[30] J. Stockburger, M. Grifoni, M. Sasseti, U. Weiss, Z. Phys. B **94**, 447 (1994).

[31] J.E. Van Cleve, Ph.D. thesis, Cornell University, 1991 (unpublished).

[32] E. Thompson, G. Lawes, J.M. Parpia, R.O. Pohl, Contribution to the XXII. Int. Conf. on Low Temp. Phys. (LT22, August 99, Helsinki), to appear in Physica B; E. Thompson, G. Lawes, J.M. Parpia, R.O. Pohl, to appear in Phys. Rev. Lett.

[33] In Ref. [9] a similar graph was shown. Erroneously, in three of the curves (1.03, 2.52, and 5.03 kHz) a value of $A = 1 \times 10^8$ $\mathrm{K^{-3}s^{-1}}$ rather than $A = 8 \times 10^7$ $\mathrm{K^{-3}s^{-1}}$ was used. However, the qualitative picture is not significantly changed by this error.

[34] S. Rogge, D. Natelson, B. Tigner, D.D. Osheroff, Phys. Rev. B **55**, 11256 (1997).